Antenna Equivalent Areas

<u>Antenna Effective Aperture</u> (Area)

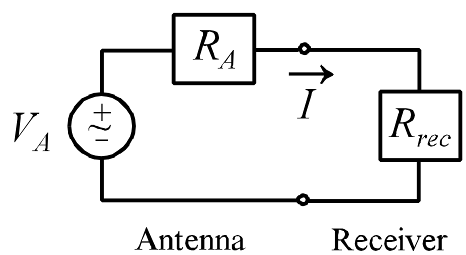
Given a receiving antenna oriented for maximum response, polarization matched to the incident wave, and impedance matched to its load, the resulting power delivered to the receiver (P_{rec}) may be defined in terms of the *antenna effective aperture* (A_e) as

$$P_{rec} = SA_e \qquad \text{(W)}$$

where S is the power density of the incident wave (magnitude of the Poynting vector) defined by

$$S = \frac{1}{2} | \boldsymbol{E_i} \times \boldsymbol{H_i}^* | \qquad \text{(W/m}^2\text{)}$$

According to the equivalent circuit under matched conditions,



$$R_A = R_r + R_L = R_{rec}$$

$$P_{rec} = \frac{1}{2} V_{rec} I^* = \frac{1}{2} \frac{V_A}{2} \left(\frac{V_A}{2R_A} \right)^* = \frac{|V_A|^2}{8R_A}$$

We may solve for the antenna effective aperture which gives

$$A_e = \frac{P_{rec}}{S} = \frac{|V_A|^2}{8SR_A} = \frac{|V_A|^2}{8S(R_r + R_L)}$$

Antenna Scattering Area

The total power scattered by the receiving antenna is defined as the product of the incident power density and the *antenna scattering area* (A_s).

$$P_s = SA_s$$

From the equivalent circuit, the total scattered power is

$$P_s = \frac{|V_A|^2 R_r}{8(R_r + R_L)^2}$$

which gives

$$A_s = \frac{P_s}{S} = \frac{|V_A|^2 R_r}{8S(R_r + R_L)^2}$$

Antenna Loss Area

The total power dissipated as heat by the receiving antenna is defined as the product of the incident power density and the *antenna loss area* (A_L).

$$P_L = SA_L$$

From the equivalent circuit, the total dissipated power is

$$P_L = \frac{|V_A|^2 R_L}{8(R_r + R_L)^2}$$

which gives

$$A_L = \frac{P_L}{S} = \frac{|V_A|^2 R_L}{8S(R_r + R_L)^2}$$

Antenna Capture Area

The total power captured by the receiving antenna (power delivered to the load + power scattered by the antenna + power dissipated in the form of heat) is defined as the product of the incident power density and the *antenna capture area* (A_c).

$$P_c = SA_c$$

The total power captured by the antenna is

$$P_c = P_{rec} + P_s + P_L = \frac{|V_A|^2}{4(R_r + R_L)}$$

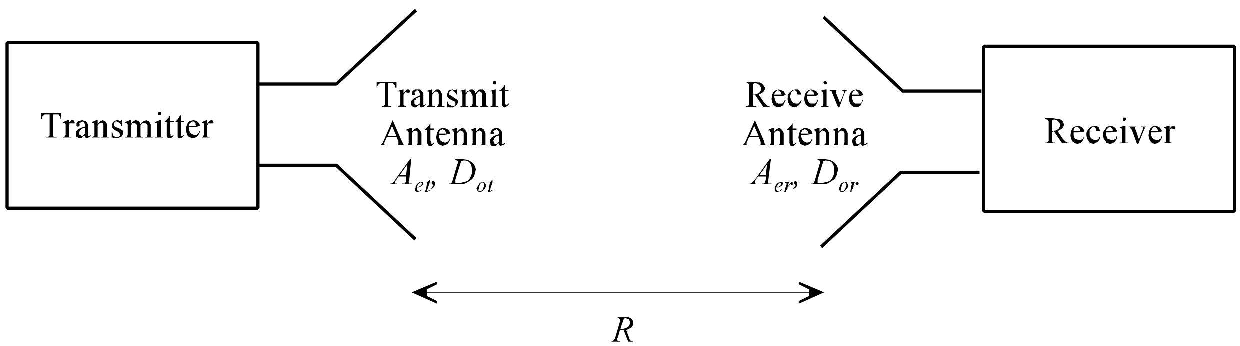
which gives

$$A_c = \frac{P_c}{S} = \frac{|V_A|^2}{4S(R_r + R_L)}$$

Note that $A_c = A_e + A_s + A_L$.

Maximum Directivity and Effective Aperture

Assume the transmitting and receiving antennas are lossless and oriented for maximum response.



A_{et}, D_{ot} - transmit antenna effective aperture and maximum directivity
A_{er}, D_{or} - receive antenna effective aperture and maximum directivity

If we assume that the total power transmitted by the transmit antenna is P_t, the power density at the receive antenna (W_r) is

$$W_r = \frac{P_t}{4\pi R^2} D_{ot}$$

The total power received by the receive antenna (P_r) is

$$P_r = W_r A_{er} = \frac{P_t D_{ot} A_{er}}{4\pi R^2}$$

which gives

$$D_{ot} A_{er} = \frac{P_r}{P_t} 4\pi R^2$$

If we interchange the transmit and receive antennas, the previous equation still holds true by interchanging the respective transmit and receive quantities (assuming a linear, isotropic medium), which gives

$$D_{or} A_{et} = \frac{P_r}{P_t} 4\pi R^2$$

These two equations yield

$$D_{ot} A_{er} = D_{or} A_{et}$$

or

$$\frac{D_{ot}}{A_{et}} = \frac{D_{or}}{A_{er}}$$

If the transmit antenna is an isotropic radiator, we will later show that

$$D_{ot} = 1 \qquad\qquad A_{et} = \frac{\lambda^2}{4\pi}$$

which gives

$$\frac{D_{or}}{A_{er}} = \frac{4\pi}{\lambda^2} \qquad\qquad (\textbf{for any antenna})$$

Therefore, the equivalent aperture of a lossless antenna may be defined in terms of the maximum directivity as

$$A_e = \left(\frac{\lambda^2}{4\pi} \right) D_o$$

The overall antenna efficiency (e_o) may be included to account for the ohmic losses and mismatch losses in an antenna with losses.

$$A_e = e_o \left(\frac{\lambda^2}{4\pi} \right) D_o = e_{cd}(1 - |\Gamma|^2) \left(\frac{\lambda^2}{4\pi} \right) D_o$$

The effect of polarization loss can also be included to yield

$$A_e = e_{cd}(1 - |\Gamma|^2) \left(\frac{\lambda^2}{4\pi} \right) D_o |\boldsymbol{a}_t \cdot \boldsymbol{a}_r|^2$$