3-Phase Circuits

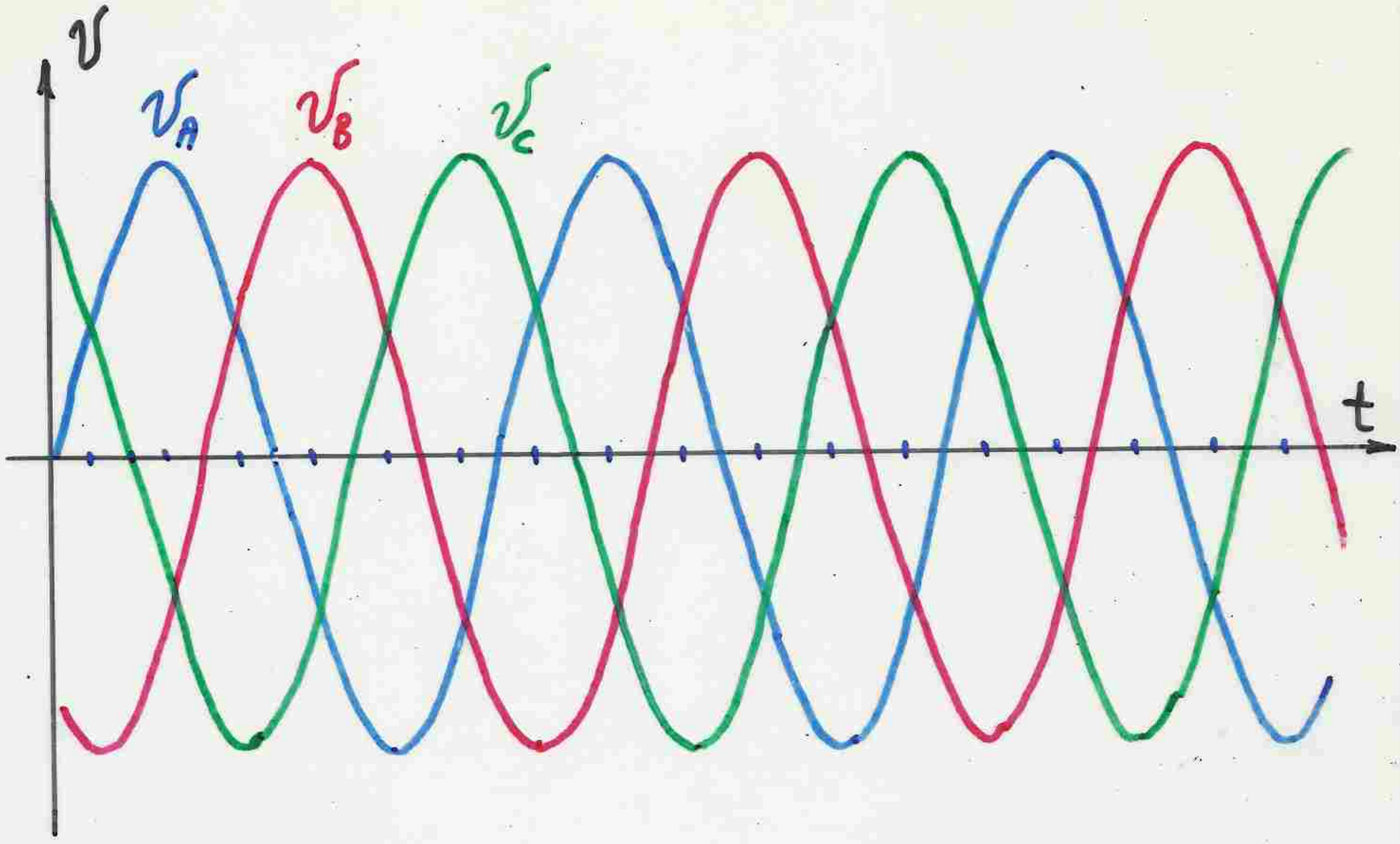
= THREE PHASE CIRCUITS.

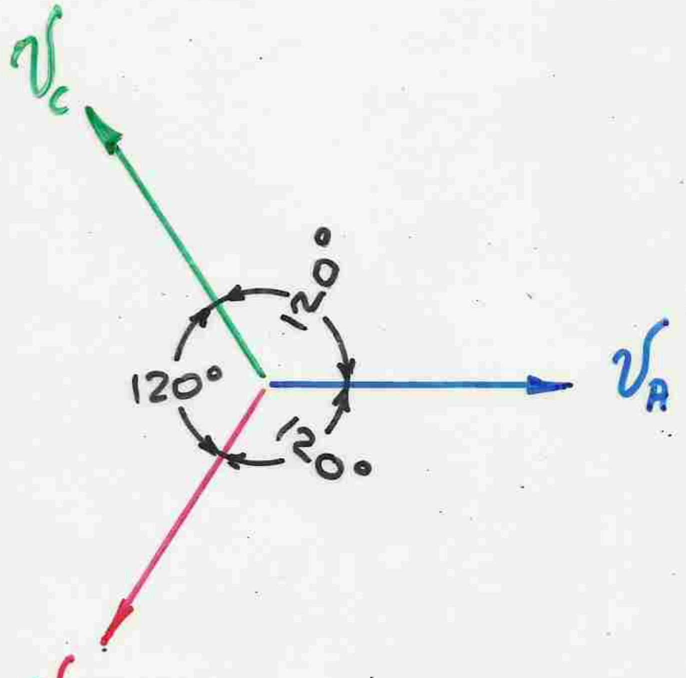
Generators with two or more windings in which the induced e.m.f are of the same amplitude and frequency but shifted in phase are called (Poly phase Generators).
A Circuit Containing ploy phase generators and Loads is called a poly phase Circuit.

* The most Common in practice for the transmission and distribution of electrical power is the three-phase system for economical, technical and simplicity reasons.

In a three-phase generator a set of balanced voltages that have identical amplitudes and frequency but are shifted in respect with each other by 120° are produced.

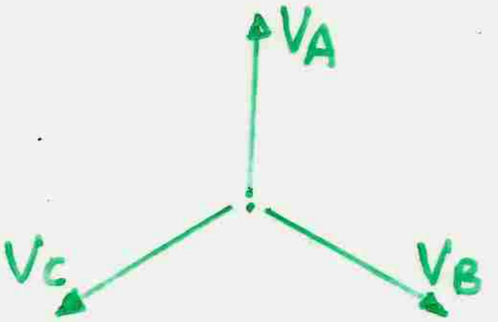
In discussing 3-phase Circuits, it is a stand practice to refer to the three phase as:
(A, B, and C) or (R, S, and T) or (U, V, and W)





Three-phase voltage system

The order in which the voltages of a 3-ph-ase generator go through the same value is called <u>phase_ Sequence</u> .



ABC System .

(Positive Sequence)

If $V_A = 50 \angle 90^\circ$ volt .

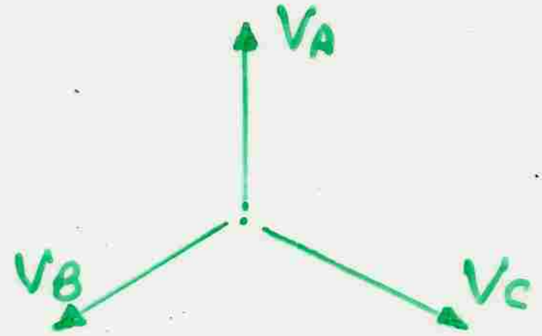
$\therefore V_B = 50 \angle -30^\circ$ " .

& $V_C = 50 \angle -150^\circ$ " .

If $V_A = 100 \angle 0^\circ$ volt .

$V_B = 100 \angle -120$ " .

$V_C = 100 \angle +120^\circ$ " .



CBA System .

(Negative Sequence) .

If $V_A = 50 \angle 90^\circ$ volt .

$\therefore V_B = 50 \angle -150^\circ$ " .

& $V_C = 50 \angle -30^\circ$ " .

If $V_A = 100 \angle 0^\circ$ volt .

$V_B = 100 \angle 120^\circ$ " .

$V_C = 100 \angle -120^\circ$ " .

* If the phase sequence is known and one voltage in the set, we can find the entire set.

An important characteristic of a set of balanced three-phase voltages is that at any instant of time the sum of the three voltages adds to zero.

$$\boxed{V_A + V_B + V_C = 0}$$

* Fig. 5 shows a system in which the power is carried from the generator windings to the load along six wires without interlinking the phases.

* Fig. 6 shows a change made for the sake of economy. Instead of having a return wire from each load to each winding, a single wire is used for the return current of all three. This wire is called the (Neutral Wire).

If the three loads are equal ($Z_A = Z_B = Z_C$), the three currents I_A, I_B, and I_C will be balanced and the sum of these currents is zero.

$$\boxed{I_N = I_A + I_B + I_C = 0}$$

when $Z_A = Z_B = Z_C$.

But when $Z_A \neq Z_B \neq Z_C \longrightarrow I_N \neq 0$

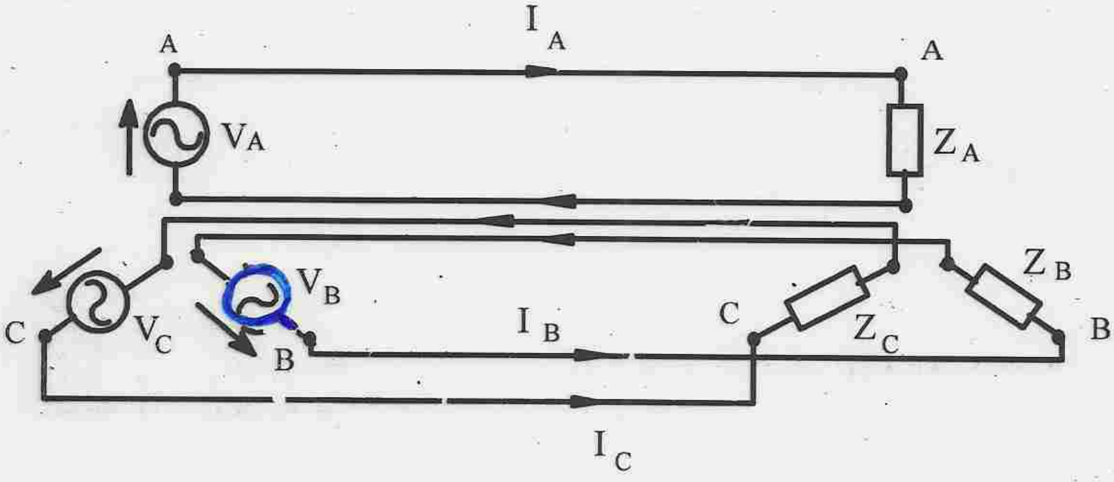


Fig. 5 Non-interlinked 3-phace SIX wire system

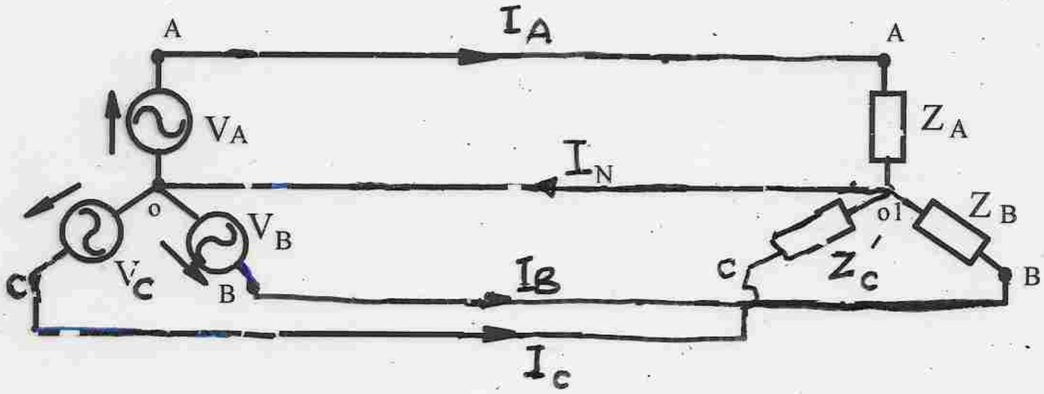


Fig. 6 Three phase FOUR wire star-star system

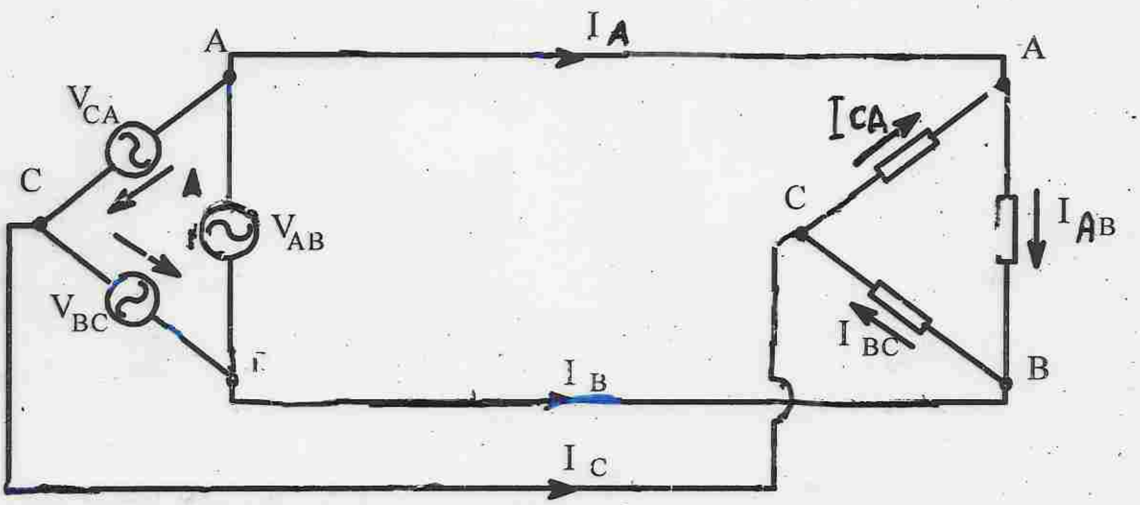


Fig. 7 Three phase THPEE wire delta-delta system

Fig.7 shows another method used for connecting the generator and the load. This system is called 3-phase 3-wire Δ-Δ system.

A more general method of 3-phase connection which is used for power distribution is shown in Fig. 8.

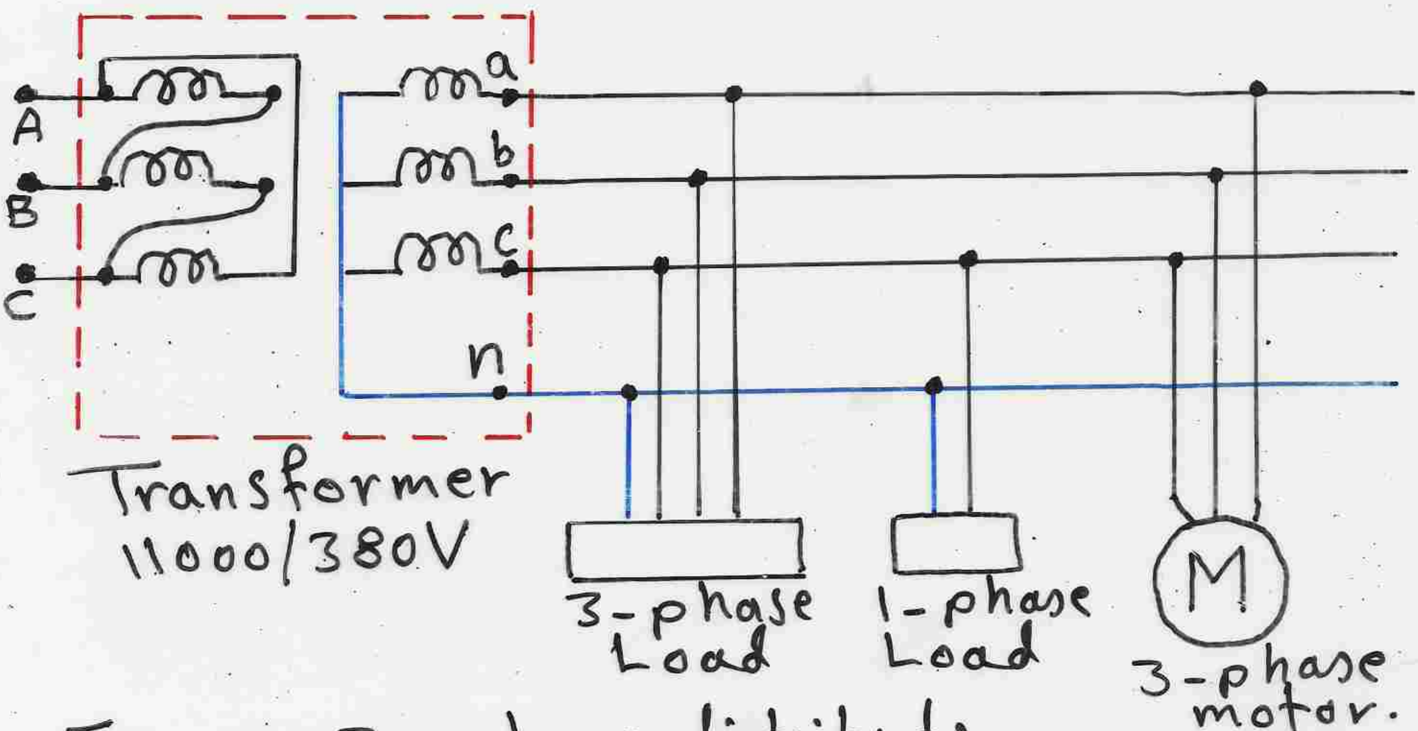


Fig. 8. 3-phase distribution method.

** The voltage across any generator winding or any load impedance is called the phase voltage of the generator or the load (V_{ph}).

** The Voltage between any two of the wires (ABC) is Called the (Line Voltage V_ℓ).

** The Current passing through any one of the three (ABC) wires is Calle the (Line Current I_ℓ).

** The Current passing through any generator winding (or Load impedance) is Called the (phase current I_{ph}).

** The point Connecting all the ends of generator windings (XYZ) is called the neutral point of the generator.

** The point Connecting all the ends of Load impedances is Called the neutral point of the Load.

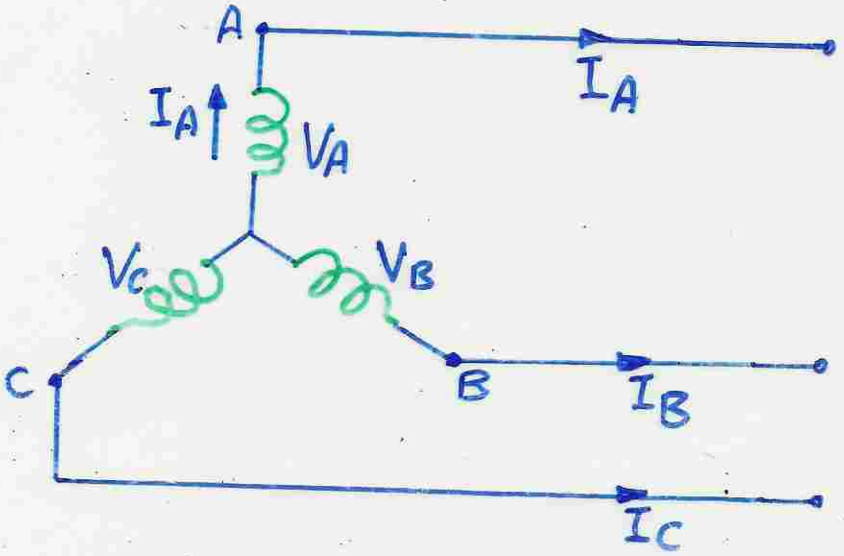
** If the phase sequence is not mentioned then apositive ABC system is assumed.

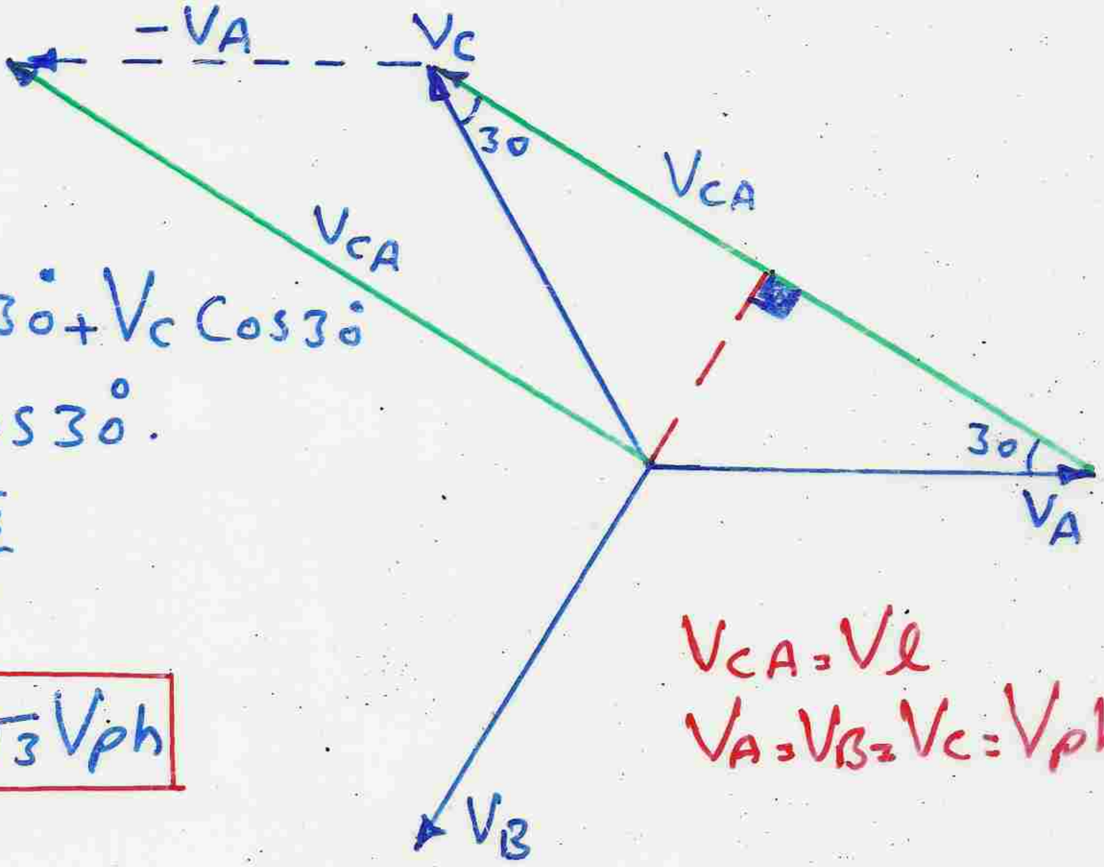
** Unless Otherwise specifically stated, a Voltage given for a 3-phase system Shall be assumed to be the Line Voltage (V_L).

Relation Ship between Line and Phase Quantities

① In Y Connected generator and in balanced Y Connected Load;-

$$\boxed{I_l = I_{ph}} \qquad \boxed{V_l = \sqrt{3}\, V_{ph}.}$$





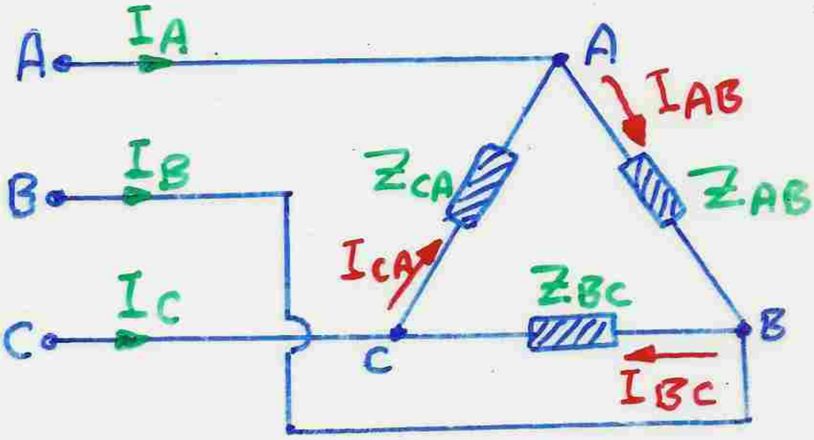
$$V_{CA} = V_A \cos 30^\circ + V_C \cos 30^\circ$$
$$= 2 V_A \cos 30^\circ.$$
$$= 2 V_A \frac{\sqrt{3}}{2}$$

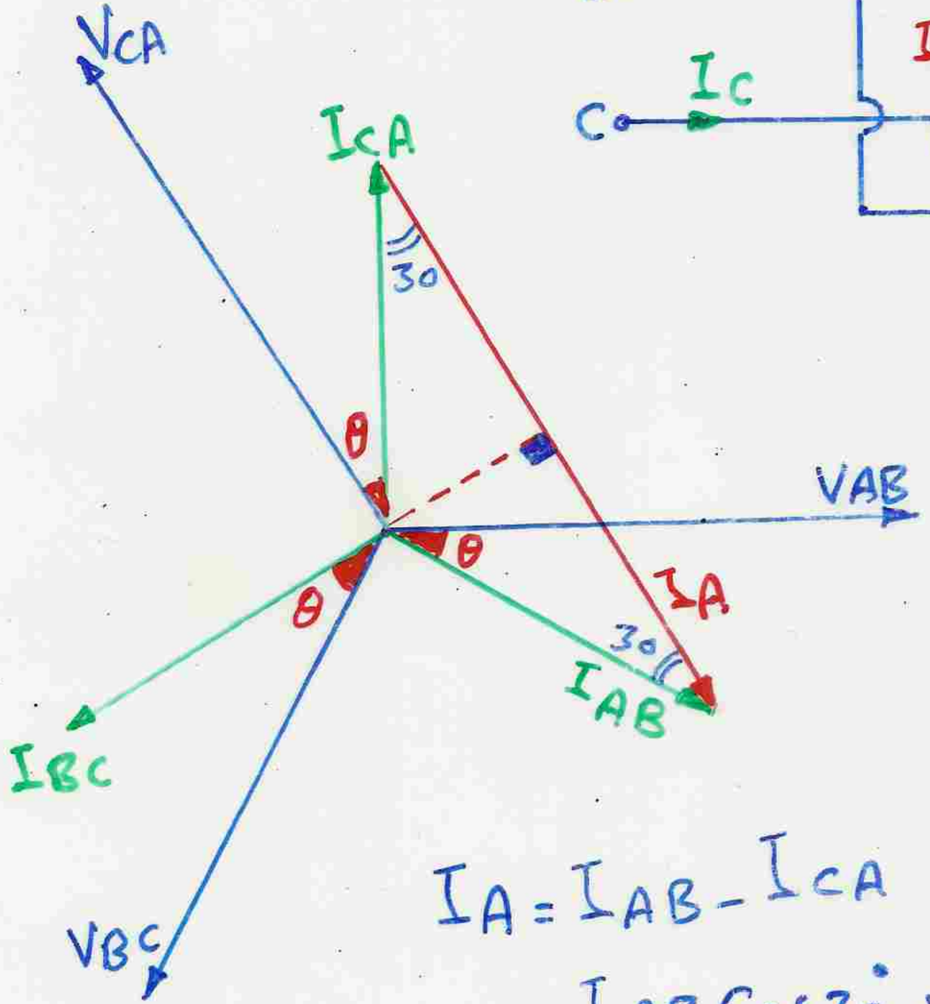
$$\therefore \boxed{V_l = \sqrt{3}\, V_{ph}}$$

$V_{CA} = V_l$
$V_A = V_B = V_C = V_{ph}$

② In Δ Connected generator and in balanced Δ Connected Load:-

$$\boxed{V_l = V_{ph}} \qquad \boxed{I_l = \sqrt{3}\, I_{ph}}$$





$$I_A = I_{AB} - I_{CA}$$

$$= I_{AB}\cos 30^\circ + I_{CA}\cos 30^\circ.$$

$$= 2\, I_{AB}\cos 30^\circ.$$

$$= 2\, I_{AB}\,\frac{\sqrt{3}}{2}.$$

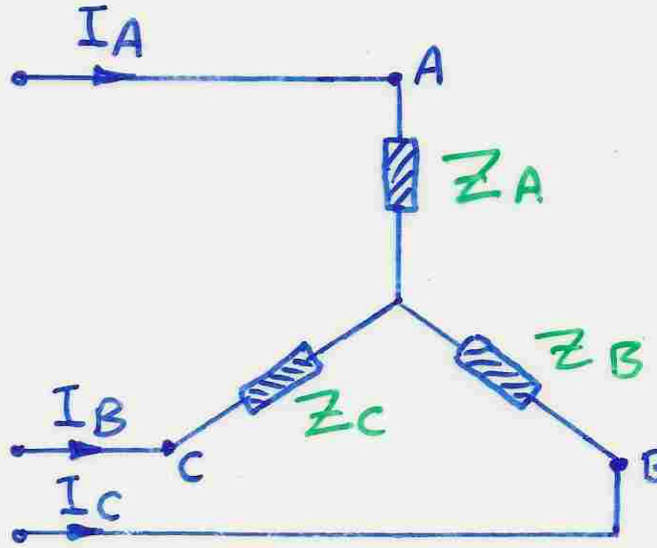
$I_{AB} = I_{ph}$

$I_A = I_l.$

$$\therefore \quad \boxed{I_l = \sqrt{3}\, I_{ph}}$$

Power in 3-phase Circuits

① In Y-Connected Load :-



* When $Z_A \neq Z_B \neq Z_C$.

$\therefore P = V_A I_A \cos\theta_A + V_B I_B \cos\theta_B + V_C I_C \cos\theta_C$.

* When $Z_A = Z_B = Z_C$.

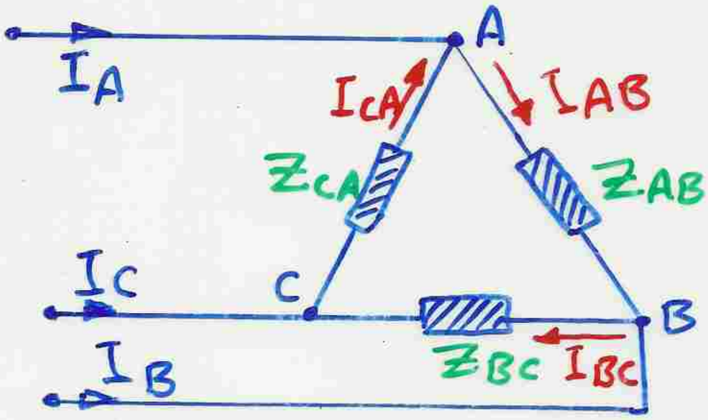
$\therefore P = 3 V_A I_A \cos\theta_A$.

$= \sqrt{3} \cdot \sqrt{3} \cdot \frac{V_l}{\sqrt{3}} \cdot I_l \cos\theta$.

$\therefore \boxed{P = \sqrt{3} V_l \cdot I_l \cos\theta.}$

& $\boxed{Q = \sqrt{3} V_l \cdot I_l \cdot \sin\theta.}$

② In Δ - Connected Load :-



* When $Z_{AB} \neq Z_{BC} \neq Z_{CA}$.

$$P = V_{AB} I_{AB} \cos\theta_{AB} + V_{BC} I_{BC} \cos\theta_{BC} + V_{CA} I_{CA} \cos\theta_{CA}$$

* When $Z_{AB} = Z_{BC} = Z_{CA}$.

$$P = 3 V_{AB} . I_{AB} \cos\theta_{AB} .$$

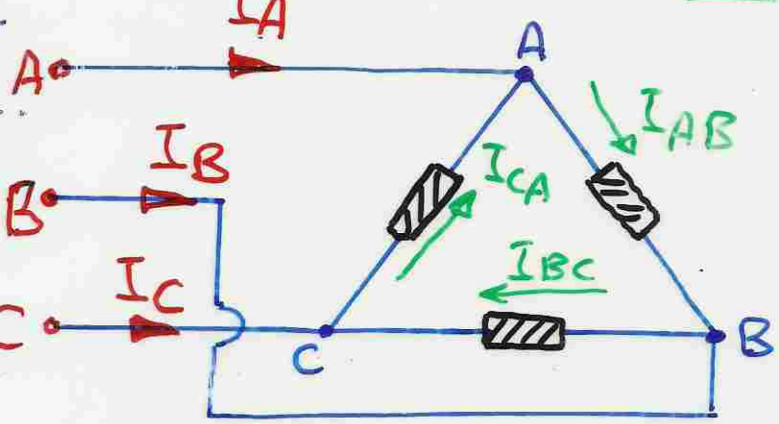
$$= \sqrt{3} . \sqrt{3} . V_l . \frac{I_l}{\sqrt{3}} \cos\theta .$$

$$\therefore \boxed{P = \sqrt{3} V_l . I_l . \cos\theta .}$$

$$\boxed{Q = \sqrt{3} . V_l . I_l . \sin\theta .}$$

Ex: For the Circuit Shown, V=200 Volt. Find the Line currents

$Z_{AB}=Z_{BC}=Z_{CA}=10\angle 75\ \Omega$.



** let $V_{AB}=200\angle 0$ V.

$\therefore V_{BC}=200\angle -120$ V.

$V_{CA}=200\angle +120$ V.

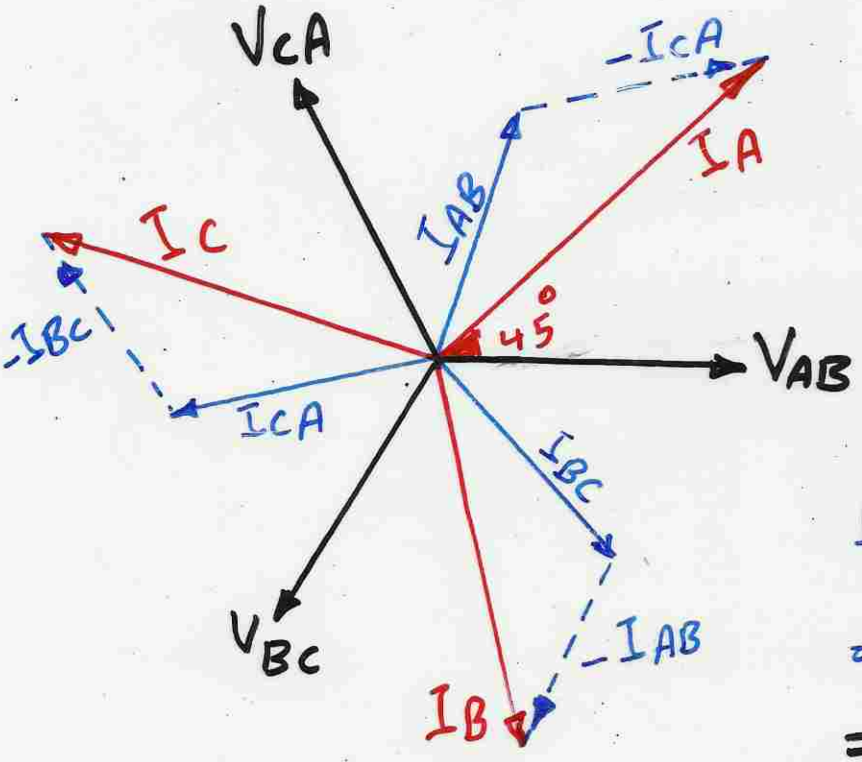
$\therefore I_{AB}=\dfrac{V_{AB}}{Z_{AB}}=\dfrac{200\angle 0}{10\angle -75}=20\angle 75$ Amp.

and $I_{BC}=20\angle -45$ A , $I_{CA}=20\angle 195$ Amp.

$\therefore I_A=I_{AB}-I_{CA}=20\angle 75-20\angle 195=34.6\angle 45$ Amp.

$I_B=I_{BC}-I_{AB}=34.6\angle -75$ Amp.

$I_C=I_{CA}-I_{BC}=34.6\angle 165$ Amp.

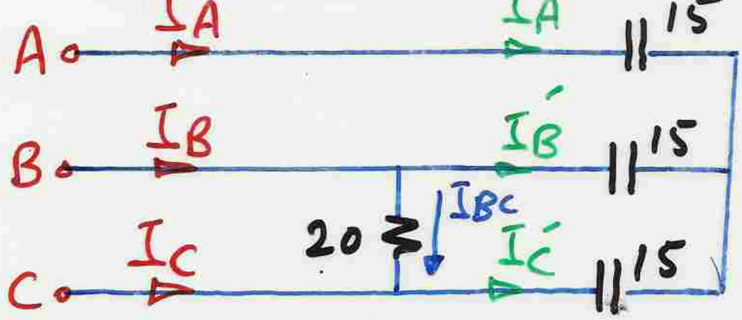


$P=\sqrt{3}\,V_l I_l \cos\theta$

$=\sqrt{3}\times 200\times 34.6\cos 75$

$=3102$ Watts.

Ex For the Circuit
$V = 260\,V$
Find line currents.



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$$V_{ph} = \frac{260}{\sqrt{3}} = 150\,V.$$

$\therefore V_A = 150\angle 0\,V \quad , V_B = 150\angle -120\,V \quad , V_C = 150\angle +120\,V.$

and
$$I_A' = \frac{150\angle 0}{15\angle -90} = 10\angle 90\,A$$

$$I_B' = \frac{150\angle -120}{15\angle -90} = 10\angle -30\,A \quad \text{and} \quad I_C' = 10\angle 210\,A$$

But: $\quad I_A = I_A' = 10\angle 90^\circ\,Amp.$

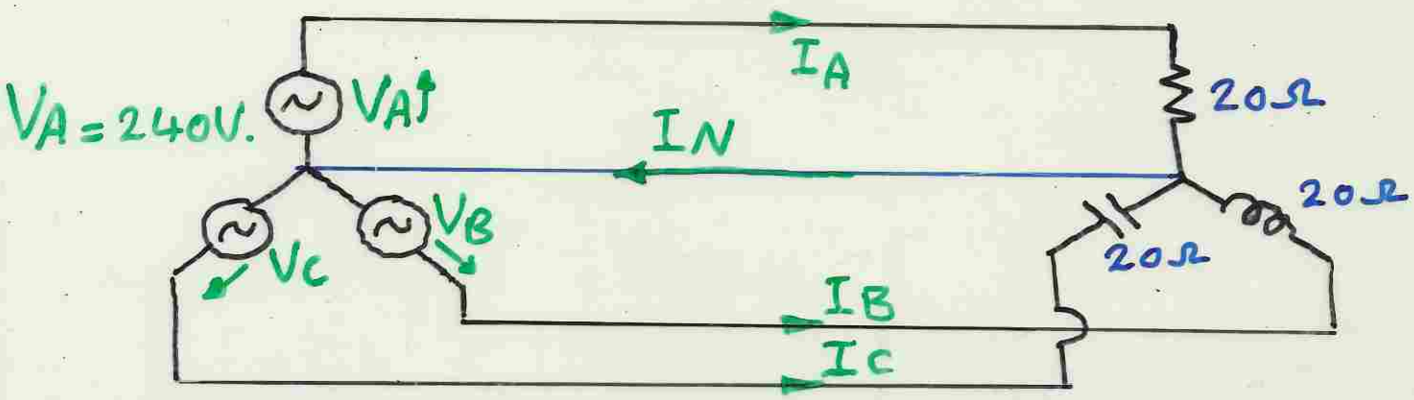
$$I_{BC} = \frac{V_{BC}}{20} = \frac{260\angle -90}{20} = 13\angle -90^\circ\,Amp.$$

$$I_B = I_B' + I_{BC} = 10\angle -30 + 13\angle -90 = 20\angle -64.3\,A.$$

$$I_C = I_C' - I_{BC} = 10\angle 210 + 13\angle -90 = 11.79\angle 137.3\,A$$

Example:

In the 3-phase circuit shown :-
Find the current distribution in the circuit when the phase sequence is ABC.
Draw the phasor diagram of Voltages & Current.



If $V_A = 240\angle 0°$ V. $\therefore V_B = 240\angle -120$, $V_C = 240\angle +120$.

$$I_A = \frac{240\angle 0°}{20} = 12\angle 0 \text{ Amp.}$$

$$I_B = \frac{240\angle -120}{20\angle 90} = 12\angle -210 \text{ Amp.}$$

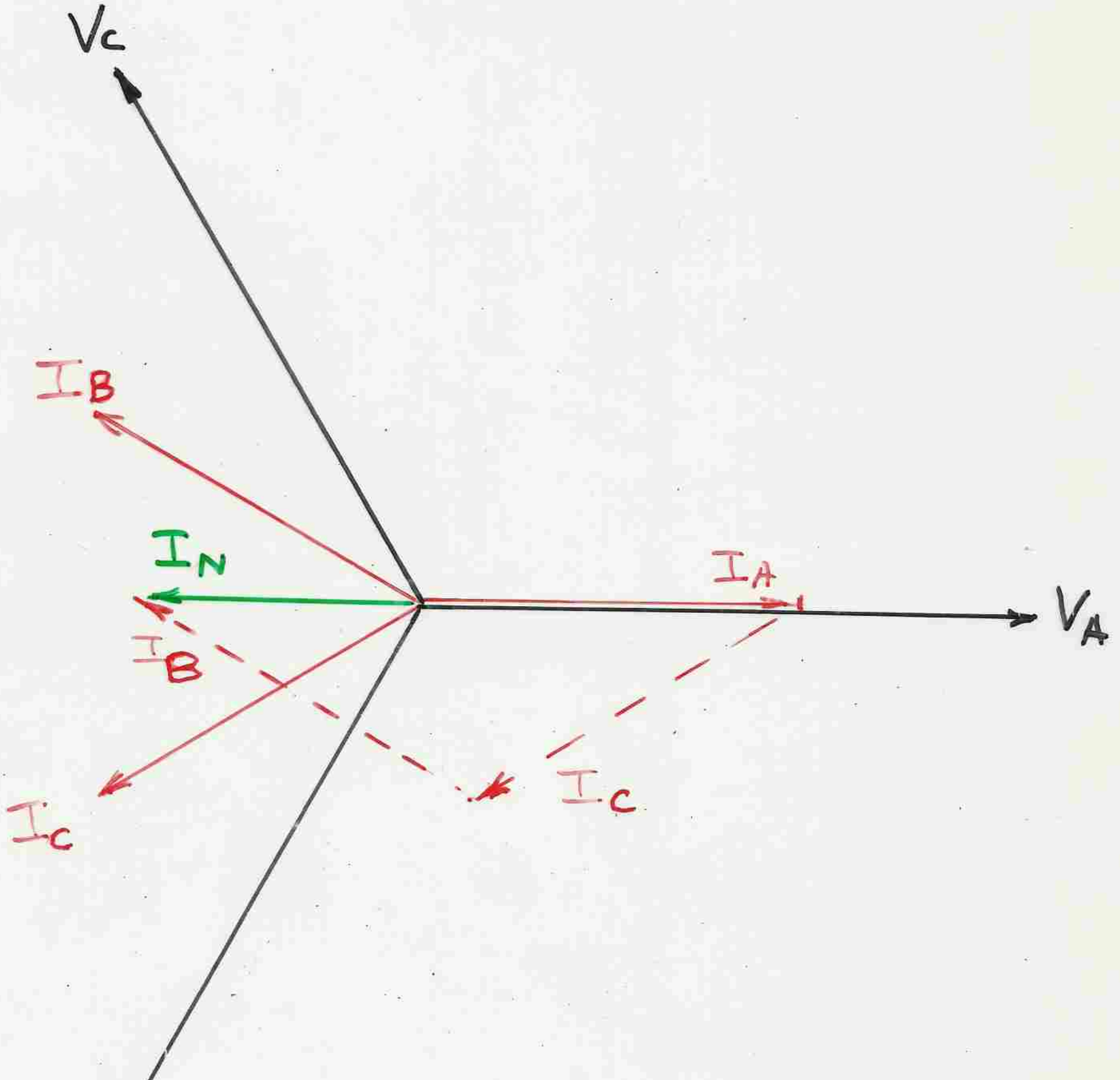
& $$I_C = \frac{240\angle +120}{20\angle -90} = 12\angle 210 \text{ Amp.}$$

$$\therefore I_N = I_A + I_B + I_C$$

$$= 12\angle 0 + 12\angle -210 + 12\angle +210 = 8.78\angle -180$$

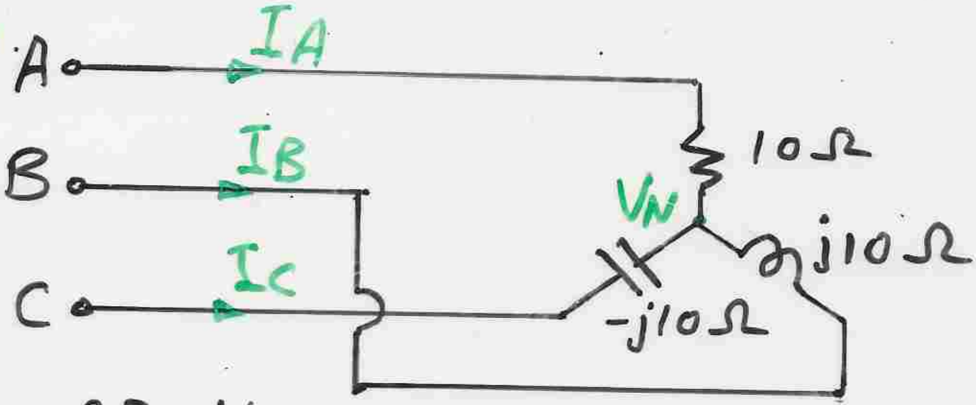
$$= -8.78 \text{ Amp.}$$

Note: Repeat the same question for -ve sequence.


Vc
IB
IN
IA
VA
IB
Ic
Ic
VB

Example:

For the Circuit Shown, find the currents distribution.



Supply Voltage (400V).

Sol:-

$$V_{ph} = \frac{400}{\sqrt{3}} = 230\,V.$$

$$\therefore V_A = 230\angle 0^\circ \ , \ V_B = 230\angle{-120} \ , \ V_C = 230\angle{+120}.$$

$$\left(\frac{1}{10} + \frac{1}{j10} + \frac{1}{-j10}\right)V_N - \frac{V_A}{10} - \frac{V_B}{+j10} - \frac{V_C}{-j10} = 0.$$

$$(0.1 + j0.1 - j0.1)V_N = 0.1V_A - j0.1V_B + j0.1V_C.$$

$$\therefore V_N = \frac{230 \times 0.1 + 230\angle{-120} \times 0.1\angle{-90} + 230\angle{120} \times 0.1\angle{90}}{0.1 + j0.1 - j0.1}.$$

$$= 230 - 200 + j115 - 200 - j115 = \underline{-170}\text{ Volt.}$$

$$\therefore I_A = \frac{V_A - V_N}{10}$$

$$= \frac{230 - (-170)}{10} = \frac{400}{10} = 40\text{ Amp.}$$

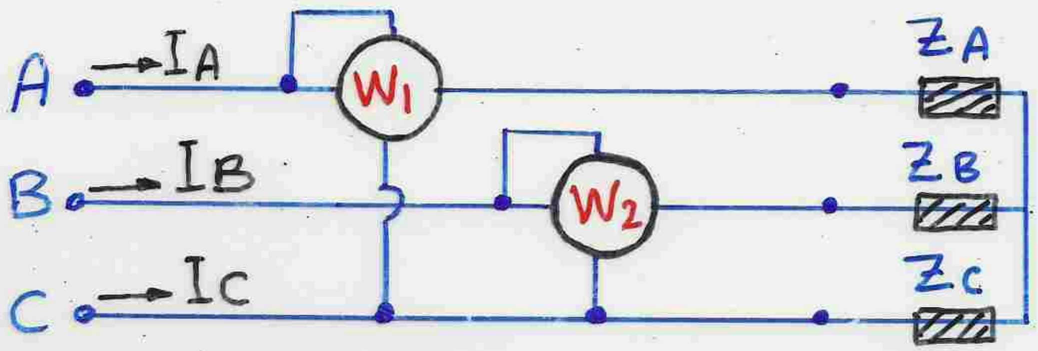
$$I_B = \frac{V_B - V_N}{j10} = \frac{230\angle{-120} - (-170)}{10\angle{90}} = 20.7\angle{-165}\text{ A.}$$

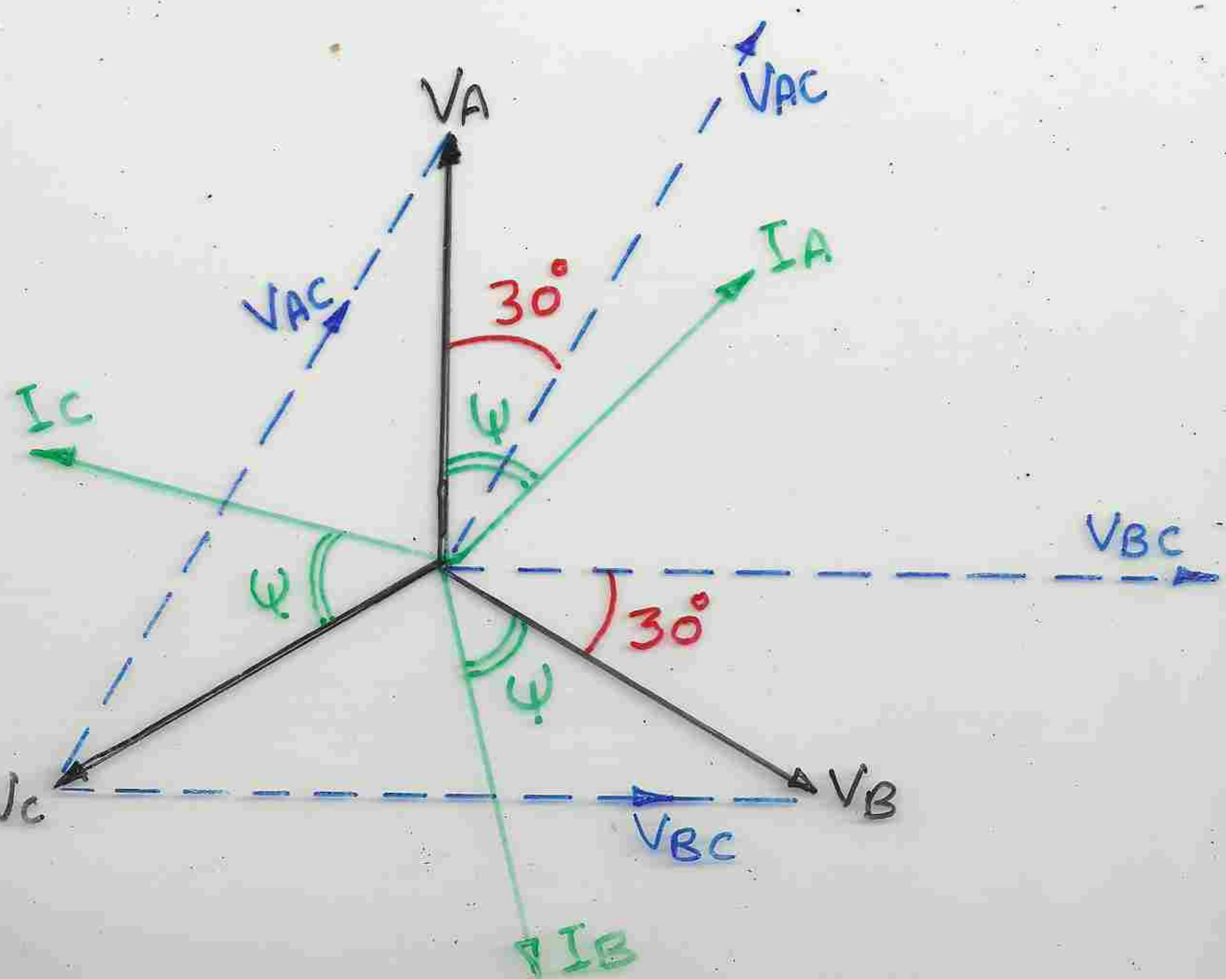
$$I_C = \frac{V_C - V_N}{-j10} = \frac{230\angle{+120} - (-170)}{10\angle{-90}} = 20.7\angle{165}\text{ A.}$$

Note: Repeat the same question by using superposition & Loop current method.

«The Two Wattmeter method»

The active power delivered to a 3-ph - three wire, λ or Δ connected balanced or unbalanced Load can be found using only two wattmeters if the proper connection is used and if the wattmeters readings are properly interpreted.





$$W_1 = V_{AC} . I_A . \cos(\psi - 30°).$$

$$W_2 = V_{BC} . I_B . \cos(\psi + 30°).$$

$$W_1 + W_2 = V_{AC} . I_A . \cos(\psi - 30°) + V_{BC} . I_B . \cos(\psi + 30°).$$

$$= V_l . I_l . \left[\cos(\psi - 30°) + \cos(\psi + 30°)\right].$$

$$= V_l . I_l . \left[\cos\psi . \cos 30° + \sin\psi \sin 30° + \cos\psi \cos 30° - \sin\psi \sin 30°\right].$$

$$\therefore W_1 + W_2 = 2 V_l . I_l . \cos\psi \cos 30°.$$

$$\therefore \boxed{W_1 + W_2 = \sqrt{3}\, V_l . I_l \cos\psi.}$$

which means that the active power delivered to the Load is equal to the Sum of the two wattmeter readings.

* When $\psi = 0$ (resistive load) $W_1 = W_2$.
* When $\psi = 90°$ (pure Inductive or Capacitive load) $W_1 = -W_2$.

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* $$W_1 - W_2 = V_l . I_l \left[\cos 30 \cos\psi + \sin 30 \sin\psi - \cos 30 \cos\psi + \sin 30 \sin\psi\right]$$

$$= 2 V_l I_l \sin 30° \sin\psi.$$

$$\therefore W_1 - W_2 = V_l . I_l \sin\psi.$$

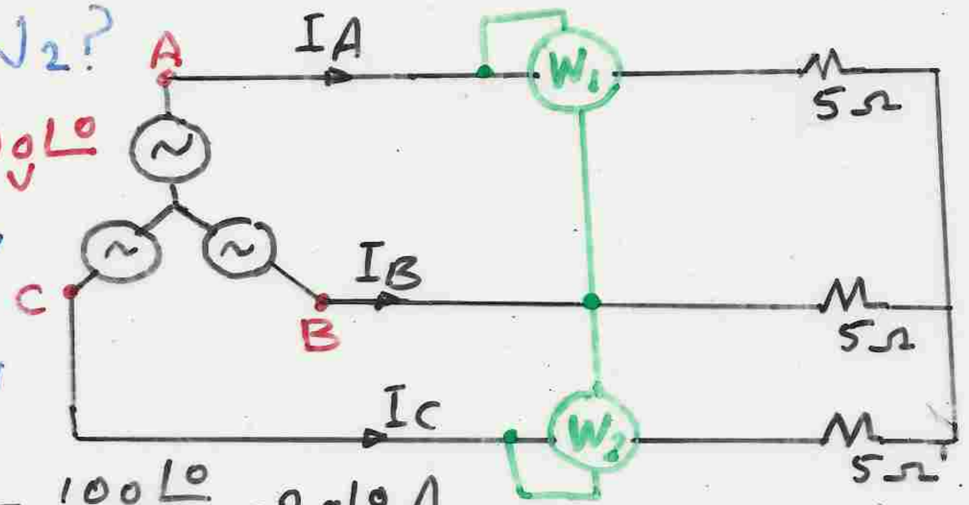
$$\therefore \sqrt{3}\,(W_1 - W_2) = \sqrt{3}\,V_l \cdot I_l \cdot \sin\varphi = Q.$$

$$\boxed{\tan\varphi = \frac{Q}{P} = \sqrt{3}\,\frac{W_1 - W_2}{W_1 + W_2}}$$

Note: This equation used only for Balanced load.

Example

For the Circuit Shown, Find the reading of W_1 & W_2?



$\therefore V_B = 100\angle -120$ V

$V_C = 100\angle +120$ V

$$I_A = \frac{V_A}{5\Omega} = \frac{100\angle 0°}{5} = 20\angle 0° \text{ A.}$$

$\therefore I_B = 20\angle -120°$ A , $I_C = 20\angle +120$ A.

$W_1 = V_{AB} \cdot I_A \cdot \cos\psi$

$= \sqrt{3} \times 100 \times 20 \cos 30 = \underline{3000}$ Watts

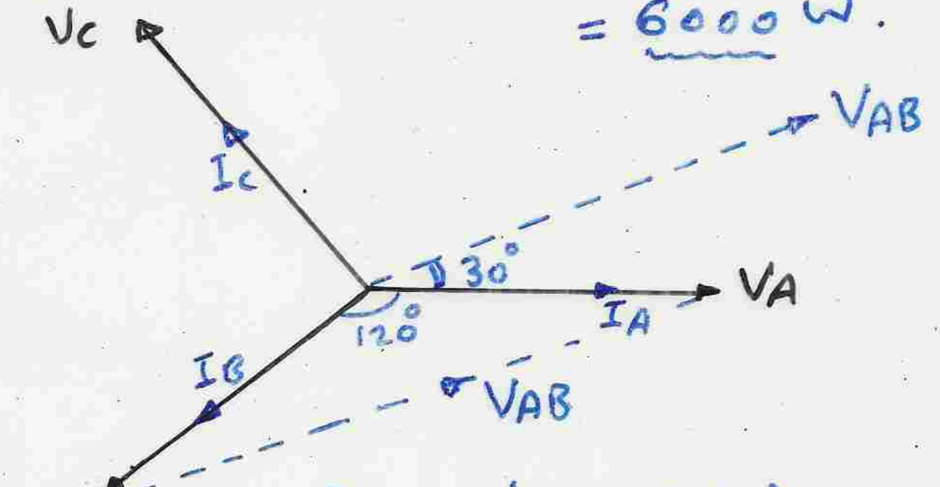
$W_2 = V_{CB} \cdot I_C \cdot \cos\psi$

$= \sqrt{3} \times 100 \times 20 \cos 30 = \underline{3000}$ Watts

$P_T = W_1 + W_2 = 6000$ W.

OR: $P_T = 3 I^2 . R = 3 \times (20)^2 \times 5 = \underline{6000}$ W.

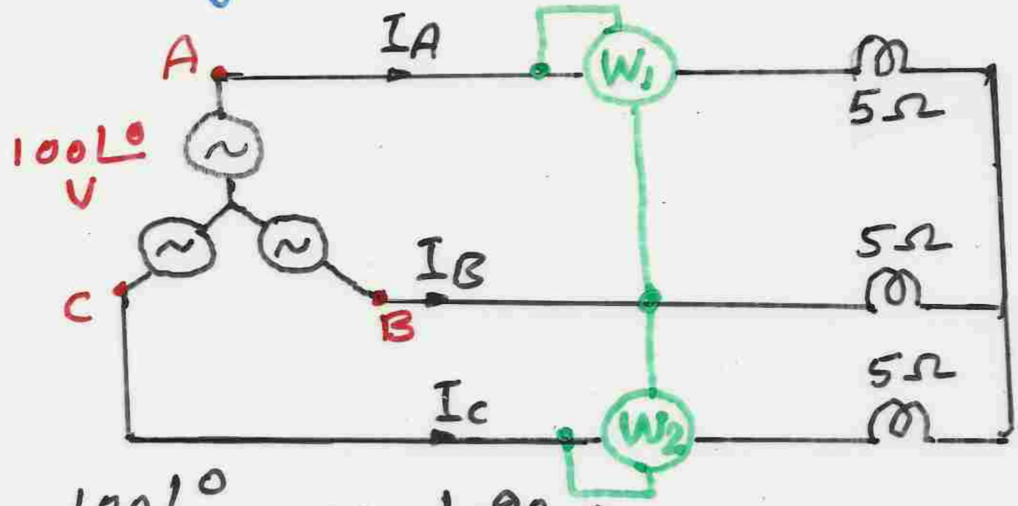
OR: $P_T = \sqrt{3} . V_l . I_l \cos\psi = \sqrt{3} \times \sqrt{3} \times 100 \times 20 \times \cos 0$
$= \underline{6000}$ W.



$Q = \sqrt{3}(W_1 - W_2) = 0$ (Why).

Example

Find the reading of W_1 & W_2, for the Circuit



$$I_A = \frac{V_A}{j5} = \frac{100\angle 0}{5\angle 90} = 20\angle -90 \text{ Amp.}$$

$$I_B = \frac{V_B}{j5} = \frac{100\angle -120}{5\angle 90} = 20\angle -210 \text{ Amp.}$$

$$I_C = \frac{V_C}{j5} = 20\angle 30^\circ \text{ Amp.}$$

$$W_1 = V_{AB} \cdot I_A \cdot \cos\psi$$

$$= \sqrt{3} \times 100 \times 20 \cos 120 = -1732 \text{ W}$$

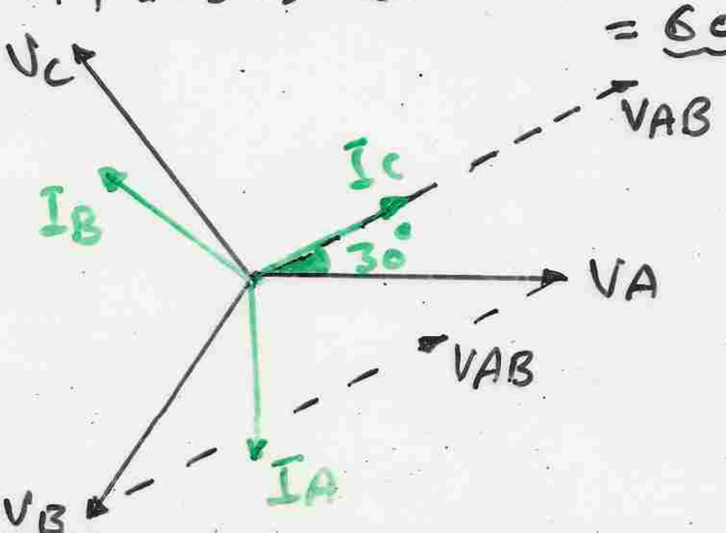
$$W_2 = V_{CB} \cdot I_C \cdot \cos\psi$$

$$= \sqrt{3} \times 100 \times 20 \cos 60 = 1732 \text{ W.}$$

$$P_T = W_1 + W_2 = 0 \quad \text{(why).}$$

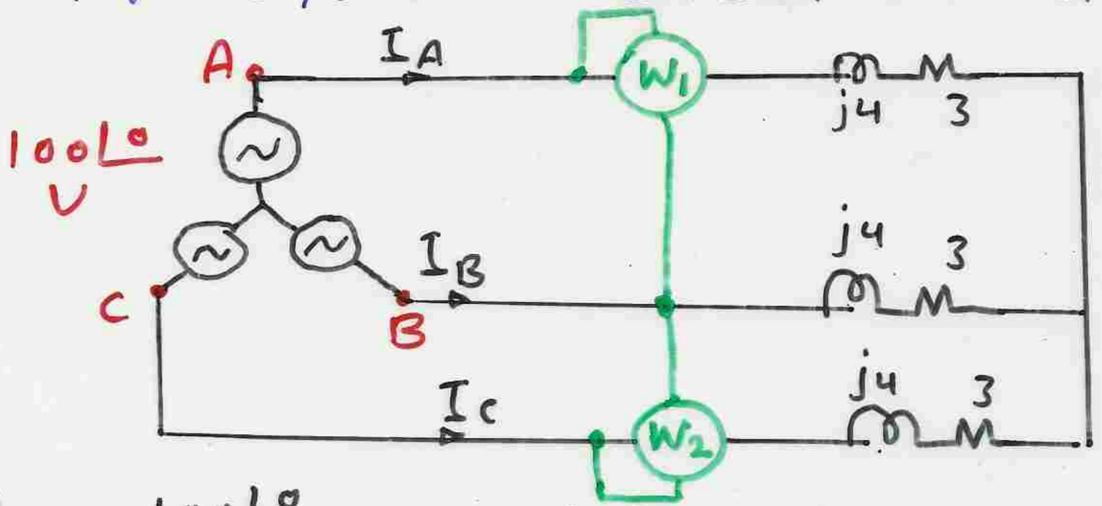
$$Q_T = \sqrt{3}\,(W_1 - W_2) = 6000 \text{ VAR.}$$

OR: $Q_T = \sqrt{3}\, V_l \cdot I_l \sin\psi = \sqrt{3} \cdot \sqrt{3} \cdot 100 \times 20 \times \sin 90$
$= 6000 \text{ VAR.}$



Example

Find W_1 & W_2, for the Circuit shown



$$I_A = \frac{V_A}{3+j4} = \frac{100\angle 0}{5\angle 53.1} = 20\angle{-53.1} \text{ Amp.}$$

$$I_B = \frac{100\angle{-120}}{5\angle 53.1} = 20\angle{-173.1} \text{ Amp.}$$

$$I_C = \frac{100\angle{+120}}{5\angle 53.1} = 20\angle 66.9 \text{ Amp.}$$

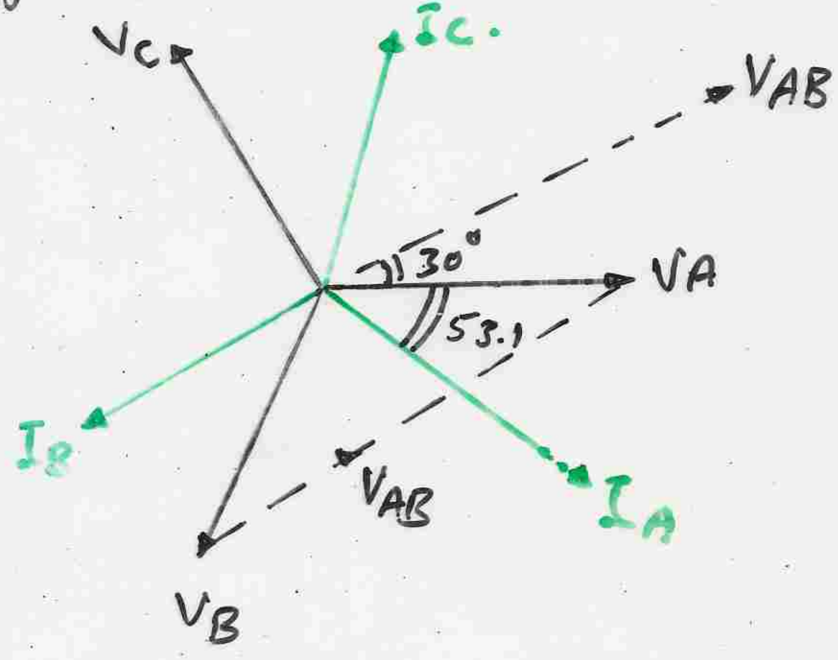
$W_1 = \sqrt{3} \times 100 \times 20 \cos 83.1 = \underline{414.3}$ Watts

$W_2 = \sqrt{3} \times 100 \times 20 \cos 23.1 = \underline{3186.3}$ watts.

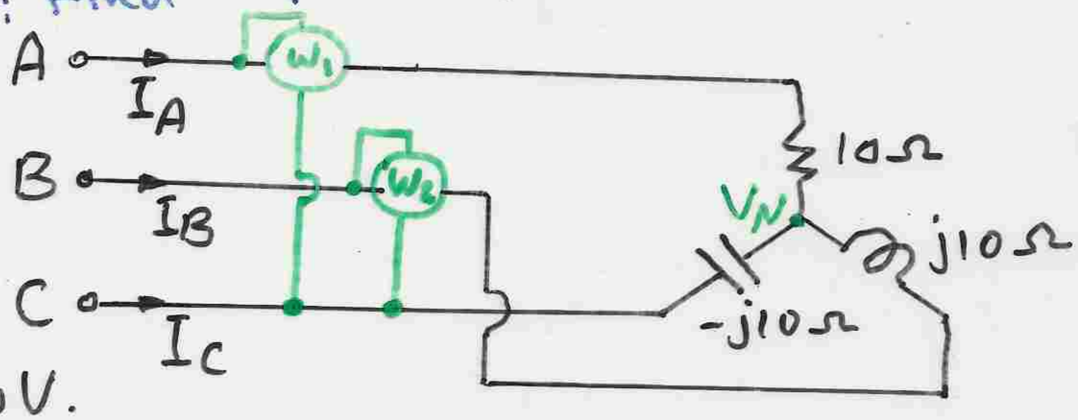
$P_T = W_1 + W_2 = \underline{3600}$ W (check).

$Q_T = \sqrt{3}(W_1 - W_2) = \underline{4800}$ VAR (check).

$S_T = P_T + jQ_T = 6000$ VA $= \sqrt{3} V_l . I_l$. (check)



Example: Find W_1 & W_2, for the circuit shown



$V_l = 400V.$

We have $I_A = 40A$, $I_B = 20.7\angle{-165}$, $I_C = 20.7\angle{165}$ A.

$$W_1 = I_A V_{AC} \cos\psi$$

$$= 40 \times 400 \cos 30 = 13856.4 \, W.$$

$$W_2 = I_B V_{BC} \cos\psi$$

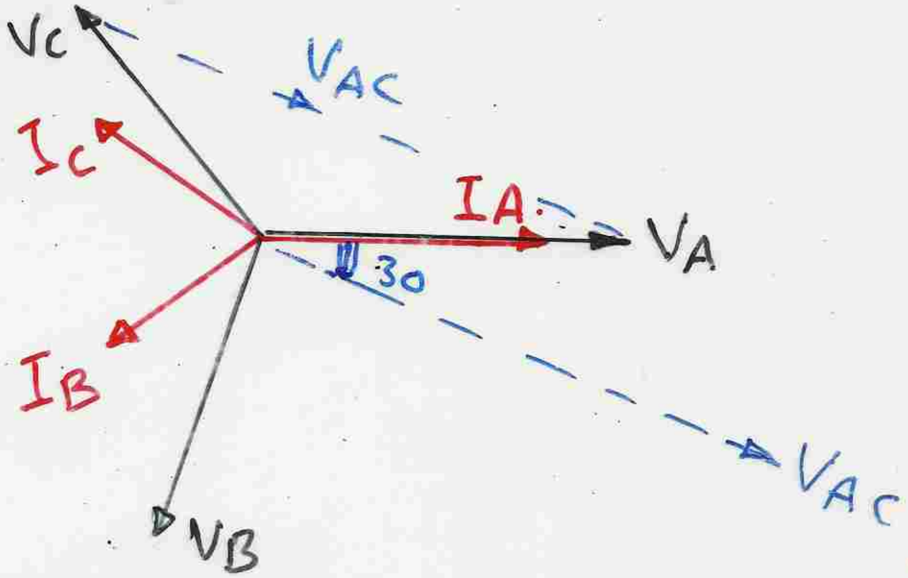
$$= 20.7 \times 400 \cos 75 = 2143 \, W.$$

$$\therefore P_T = W_1 + W_2 = 16000 \, W$$

Check: * $P_T = (I_A)^2 \times 10 = (40)^2 \times 10 = 16000 \, W$

* $P_T = V_A \times I_A \times \cos\psi = 400 \times 40 = 16000 \, W$

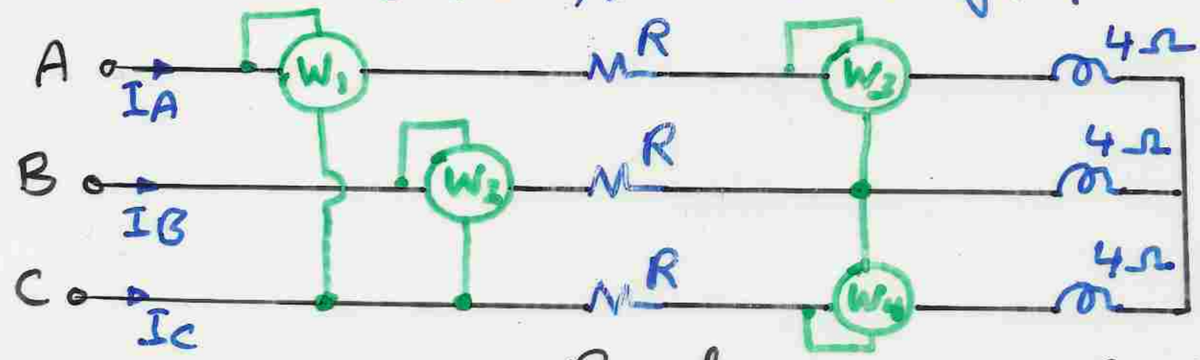
Note: Change the position of W_1 & W_2.
You must find same results.



Example

For the Circuit shown, $W_1 = 12745\,W$, $W_4 = 5542\,W$

Find W_2 & W_3, R, line voltage ?



Since W_3 & W_4 Read the power Consumed by 4Ω. $\therefore W_3 + W_4 = 0$.

$\therefore W_3 = -5542\,W$

Since (W_1 & W_2) & (W_3 & W_4) read the Same reactive power (Q) of the load.

$$\therefore \sqrt{3}(W_1 - W_2) = \sqrt{3}(W_3 - W_4) = Q_T.$$

$\therefore W_2 = 1661\,W.$

* $Q_T = \sqrt{3}(W_1 - W_2) = \sqrt{3}(W_3 - W_4) = 19198$ VAR

$\therefore Q_{ph} = 19198/3 = 6399.35 = I_l^2 . X_l.$

$\therefore I_l = I_A = I_B = I_C = 40$ Amp.

* $P_T = W_1 + W_2 = 14406$ Watts.

$P_{ph} = 14406/3 = 4802 = I_l^2 . R.$

$$\therefore R = \frac{4802}{(40)^2} = 3\,\Omega.$$

* $\tan\varphi = \dfrac{Q_T}{P_T} = \dfrac{X_l}{R} = \dfrac{4}{3}$ $\quad \therefore \varphi = 53.1$

$P_T = \sqrt{3}\, V_l I_l \cos\varphi$ $\quad \therefore V_l = 346.3$ Volts