Electron Ballistics

Uniform Electric Field : Zero Initial Velocity

Two large plane parallel plates *A* and *B* situated in vacuum at a distance of *d* meters from each other and having a potential difference of *V* volt between them. Obviously, there will be a uniform electric field of strength *E=V/d* volt/meter between the two plates. An electron placed at plate *A* will be attracted towards the positively-charged plate *B*. If free to move, the electron will be accelerated towards plate *B* along.Y-axis as shown.

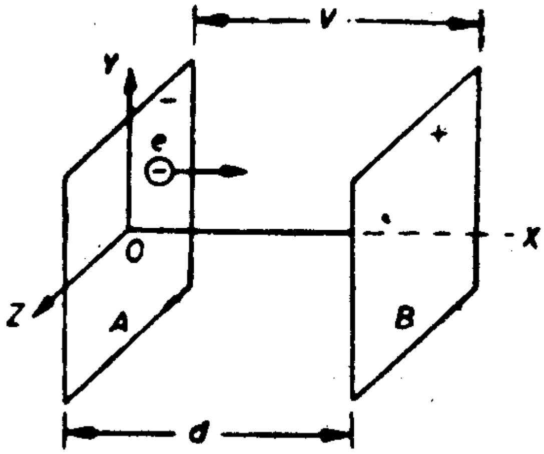


Fig.2.1. Two parallel plates situated in vacuum having a potential difference between them.

Since the negatively-charged electron is situated in an electric field, it is acted upon by a force given by

$$F = eE$$ Also

$$F = ma$$

where *a* is the acceleration along .Y-axis, *m* the mass of the electron in kg and *e* its charge in coulomb.

$$ma = eE$$

$$a = \frac{eE}{m} = \frac{e}{m} \cdot \frac{V}{d}$$

(i) Velocity of the electron at any time *t* is given by

$$v = at = \frac{eE}{m} t = \frac{e}{m} \cdot \frac{V}{d} t$$

(ii) Distance traveled by the electron during that time is

$$x = \frac{1}{2} at^2 = \frac{eE}{2m} t^2 = \frac{e}{2m} \cdot \frac{V}{d} t^2$$

$$t^2 = \frac{2xmd}{eV}$$

$$v = at = \sqrt{\frac{2eVx}{md}}$$

Now, *elm* = l.6× 10^{-19}/9.1 ×10^{-31} = l.76× 10^{-11} C/kg

(iii) The impact velocity v_i of the electron as it strikes the plate *B* can be found by putting *x* =*d* in Equation above

$$v_i = at = \sqrt{\frac{2eVd}{md}} = 5.93 \times 10^5 \sqrt{V}$$

(iv) The time of transit from plate *A* to plate *B* may be found by dividing the distance with the average speed.

$$t = \frac{d}{\frac{0 + v_i}{2}} = \frac{2d}{v_i}$$

Example

Two parallel plates are situated 4 cm apart in vacuum and have a potential difference of 200 V between them. Calculate(i) force on an electron situated between the plates (ii) velocity of arrival at the positive plate (in) transit time (iv) velocity when the electron is half-way between the plates.

Solution

(i)

$$F = eE = \frac{eV}{d} = \frac{1.6\times10^{-19}\times200}{0.04} = 8\times10^{-16}\,N$$

(ii)

$$v_i = 5.93\times10^{5}\sqrt{V} = 5.93\times10^{5}\sqrt{200} = 8.4\times10^{4}\,m/s$$

(iii)

$$t = \frac{d}{\frac{0+v_i}{2}} = \frac{2d}{v_i} = \frac{2\times0.04}{8.4\times10^{4}} = 9.5\times10^{-9}\,s$$

(iv)

$$v = \sqrt{\frac{2eVx}{md}} = 5.93\times10^{5}\sqrt{\frac{Vx}{d}}$$

$$x = \frac{d}{2} = 0.02m$$

$$v = 5.93\times10^{5}\sqrt{\frac{200\times0.02}{0.04}} = 5.93\times10^{6}\,m/s$$

Uniform Electric Field : Initial Velocity in the Direction of Field

In this case, the acceleration of the electron is the same and is, as before, given by

$$a = \frac{eE}{m} = \frac{e}{m} . \frac{V}{d} = 1.76 \times 10^{11} \frac{V}{d}$$

The other quantities are as follows :

(i) the velocity at any time is given by

$$v = u + at = u + \frac{eE}{m} t = u + \frac{e}{m} . \frac{V}{d} t$$

Where u is the initial velocity

(ii) the distance travelled by the electron in time t is

$$x = ut + \frac{1}{2} at^2 = ut + \frac{eE}{2m} t^2 = ut + \frac{e}{2m} . \frac{V}{d} t^2$$

(iii) the electron velocity at any distance *x* from the negative plate can be found by

$$v = \sqrt{u^2 + \frac{2eVx}{md}}$$

(iv) the impact velocity or velocity of arrival at the positive plate B can be found by putting x = d in Eq. above. The result so obtained is

$$v_i = \sqrt{u^2 + \frac{2eV}{m}}$$

(v) The time of transit from plate *A* to plate *B* may be found by dividing the distance with the average speed.

$$t = \frac{d}{\frac{u+v_i}{2}} = \frac{2d}{u+v_i}$$

Example

In a parallel-plate diode, the anode is at 250 V with respect to the cathode which is 4 mm away from it. An electron is emitted from cathode with an initial velocity towards cathode of 2×10^{6} m/s. Calculate ;

(i) the arrival velocity of the electron at anode

(ii) time of transit

(iii) the velocity and distance travelled by the electron after 0.5 $\times 10^{-9}$ second.

Solution

(i)

$$v_i = \sqrt{u^2 + \frac{2eV}{m}} = 9.59\times10^{4} m/s$$

(ii)

$$t = \frac{2d}{u+v_i} = 6.91\times10^{-10} s$$

(iv)

$$v = u + \frac{e}{m}.\frac{V}{d}t = 2\times10^{6} + \frac{1.76\times10^{11}\times250\times0.5\times10^{-9}}{4\times10^{-3}} = 7.5\times10^{6} m/s$$

$$x = ut + \frac{e}{2m}.\frac{V}{d}t^2 = 2.375 mm$$

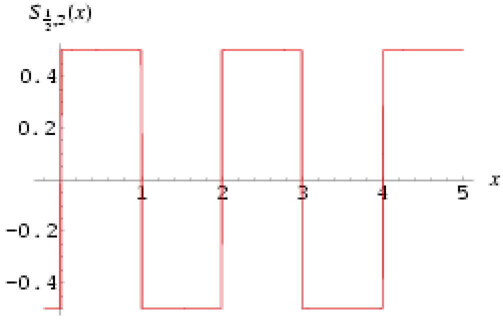
Example

Two parallel plates are spaced 4 mm in vacuum. An alternating square wave of voltage having amplitude of 0.5 V and frequency 100 MHz is applied across the two plates. An electron is at rest at the negative plate at the instant when the voltage is zero and is increasing in the positive direction. Calculate the velocity and position of the electron at the end of (a) positive half-cycle (b) negative half-cycle (c) when amplitude is 10V.

Solution

(a)

$$T = \frac{1}{f} = \frac{1}{100*10^6} = 10^{-8}s$$



Half time-period=$T/2$=0.5×10^{-8}s

with zero initial velocity of the electron,

$$x = \frac{e}{2m}.\frac{V}{d}t^2 = \frac{1.76\times10^{11}}{2}\frac{0.5}{4\times10^{-3}}(0.5\times10^{-8})^2 = 0.275mm$$

(b) During the negative half-cycle, the electron will experience an equal

$$v = \frac{e}{m}.\frac{V}{d}t = 1.76\times10^{11}\frac{0.5}{4\times10^{-3}}0.5\times10^{-8} = 1.1\times10^5 m/s$$

retardation in the opposite direction and will come back to the negative plate with zero velocity.

$$x = \frac{e}{2m}.\frac{V}{d}t^2 = \frac{1.76\times10^{11}}{2}\frac{10}{4\times10^{-3}}(0.5\times10^{-8})^2 = 5.5mm$$

(c) When Voltage Amplitude is 10V

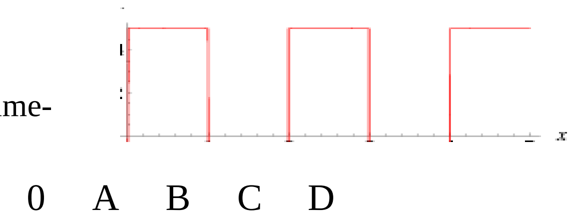
This distance is more than the plate separation. Hence, the electron will be absorbed by the positive plate and will be lost.

Example

A pulsating square voltage of amplitude 2 V and frequency 50 MHz is impressed across two plane parallel plates kept 2.5 cm apart in vacuum. An electron starts with zero initial velocity from the surface of the negative plate at the beginning of a cycle. Determine its position at the end of (a) first complete cycle and (b) the second complete cycle.

<u>Solution</u>

(a) $\frac{T}{2} = \frac{1}{2f} = \frac{1}{2\times 50\times 10^{6}} = 10^{-8}\, s$ (Half time-period)



Velocity and displacement at the end of 10^{-8}s at A are

$$v = \frac{e}{m}.\frac{V}{d}t = 1.76\times 10^{11}\frac{2}{0.025}10^{-8} = 1.4\times 10^{5}\, m/s$$

$$x = \frac{e}{2m}.\frac{V}{d}t^{2} = \frac{1.76\times 10^{11}}{2}\frac{2}{0.025}(10^{-8})^{2} = 0.7mm$$

During the next half-cycle *i.e.* between points *A* and *B*, there is no accelerating voltage, the electron travels with a constant speed of 1.4×10^{5} m/s for a period of 10^{-8} s.

Distance traveled is $=1.4\times10^{5}\times10^{-8}$ =1.4 mm.

Total distance traveled at the end of first complete cycle *i.e.* up to point *B* = 1.4+0.7**=2.**1 mm

(b) During first half-cycle of the second cycle, the accelerating voltage will be again available. The electron velocity will increase from its value of *1.4×10^{5}* (at point B) to

$$v = u + at = 1.4\times10^{5} + 1.76\times10^{11}\frac{2}{0.025}10^{-8} = 2.8\times10^{5}ms^{-1}$$

Distance traveled from point *B* to C

$$x = ut + \frac{1}{2}at^{2} = 2.1\times10^{-3}m$$

During the next half-cycle, the velocity remains constant at 2.8×10^5 m/s because there is no accelerating voltage. Distance traveled from point C to *D* = $2.8\times10^5 \times10^{-8}$= 2.8 mm.

Total distance traveled at the end of second cycle is =2.1+2.1+2.8= 7mm.

Example

A sinusoidal voltage wave having an amplitude of 10 V and frequency 1MHz is applied across two plane parallel plates kept 2 cm apart in vacuum. An electron is released with an initial velocity of 1.5×10^6 m/second along the lines of force at an instant when the applied voltage is zero. Find an expression for the speed and distance of the electron at any subsequent time t.

Solution

Electric intensity,

$$E = \frac{V}{d} = \frac{10\sin(2\pi\times10^{6}t)}{0.02} = 500\sin(6.28\times10^{6}t)$$

Acceleration,

$$a = \frac{eE}{m} = 1.76\times10^{11}E = 1.76\times10^{11} * 500\sin(6.28\times10^{6}t)$$

$$= 8.8\times10^{13}\sin(6.28\times10^{6}t)$$

$$a = \frac{dv}{dt} \qquad \therefore \frac{dv}{dt} = 8.8\times10^{13}\sin(6.28\times10^{6}t)$$

Since acceleration

Integrating both sides, we get

$$\int dv = 8.8\times10^{13}\int \sin(6.28\times10^{6}t)dt$$

$$v = -1.4\times10^{7}\cos(6.28\times10^{6}t) + K_1$$

where K_1 is a constant of integration whose value can be found from known initial conditions; t=0, v=1.5×10^6 m/s

Substituting these values, we get $1.5\times10^{6}=-1.4\times10^{7}+K_1$

$K_1=1.55\times10^{7}$m/s

Putting this value of K_1, we get

$$v = 1.55\times10^{7} - 1.4\times10^{7}\cos(6.28\times10^{6}t)$$

$$v = \frac{dx}{dt} = 1.55\times10^{7} - 1.4\times10^{7}\cos(6.28\times10^{6}t)$$

Integrating both sides, we get

$$\int dx = \int\left(1.55\times10^{7} - 1.4\times10^{7}\cos(6.28\times10^{6}t)\right)dt$$

$$x = 1.55\times10^{7}t - 2.229\sin(6.28\times10^{6}t) + K_2$$

When $t=0$, $x=0$

$K_2=0$

$$x = 1.55\times10^{7}t - 2.229\sin(6.28\times10^{6}t)$$

Uniform Electric Field : Initial Velocity Perpendicular to the Field

Let an electron having an initial velocity of u along *X-axis* enter at point *0* the space between two plane parallel plates *A* and *B* where an electric field *E* exists along the Y-axis as shown. While moving between the two plates, the electron experiences a vertical acceleration along Y-axis but none