Integrating both sides, we get

$$\int dv = 8.8\times10^{13}\int \sin(6.28\times10^{6}t)dt$$

$$v = -1.4\times10^{7}\cos(6.28\times10^{6}t) + K_1$$

where K_1 is a constant of integration whose value can be found from known initial conditions; t=0, v=1.5×10^6 m/s

Substituting these values, we get $1.5\times10^{6}=-1.4\times10^{7}+K_1$

$K_1=1.55\times10^{7}$m/s

Putting this value of K_1, we get

$$v = 1.55\times10^{7} - 1.4\times10^{7}\cos(6.28\times10^{6}t)$$

$$v = \frac{dx}{dt} = 1.55\times10^{7} - 1.4\times10^{7}\cos(6.28\times10^{6}t)$$

Integrating both sides, we get

$$\int dx = \int\left(1.55\times10^{7} - 1.4\times10^{7}\cos(6.28\times10^{6}t)\right)dt$$

$$x = 1.55\times10^{7}t - 2.229\sin(6.28\times10^{6}t) + K_2$$

When $t=0, x=0$

$K_2=0$

$$x = 1.55\times10^{7}t - 2.229\sin(6.28\times10^{6}t)$$

Uniform Electric Field : Initial Velocity Perpendicular to the Field

Let an electron having an initial velocity of u along *X-axis* enter at point *0* the space between two plane parallel plates *A* and *B* where an electric field *E* exists along the Y-axis as shown. While moving between the two plates, the electron experiences a vertical acceleration along Y-axis but none along *X-axis*. It is worth emphasizing that since there is no force along *X-axis*, the electron velocity *remains constant along this direction.*

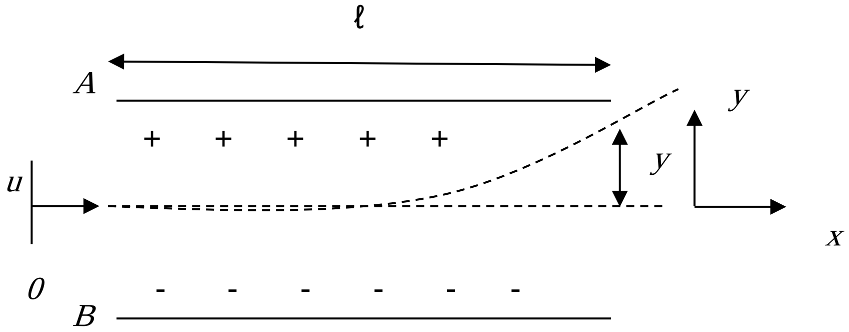


Fig. 2.2 Initial velocity perpendicular to the field
The axial distance travelled by the electron is:

$$x = ut \qquad \text{(i)}$$

There is no initial electron velocity along y-axis but as the electron moves between the plates, its velocity along Y-axis keeps on increasing.

$$a_y = \frac{eE}{m} = \frac{e}{m} \cdot \frac{V}{d}$$

The velocity and displacement along y-axis after time t are given by

$$v = a_y t$$

$$y = \frac{1}{2} a_y t^2 \quad \text{………….(ii)}$$

Substituting value of t from Eq. (i) in Eq. (ii), we get

$$y = \frac{1}{2} a_y \left(\frac{x}{u} \right)^2 = \left(\frac{1}{2} \cdot \frac{a_y}{u^2} \right) x^2$$

It shows that the electron moves along a parabolic path in the region between the two plates.

If an electron enter at angle θ as shown. A few characteristics of this motion are worth noting

(i) the velocity along x-axis remains constant and equal to the initial axial velocity because there is no force and hence no acceleration along x-axis.

(ii) the time taken to travel the parabolic path is equal to that taken to travel the axial distance *AB.*

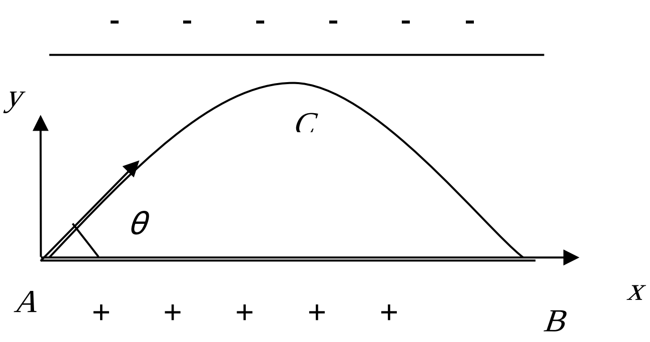
(iii) time taken by the electron to rise from *A to C* is equal to that taken by it to fall from C to *B* and each is equal to half the time taken to travel the axial distance *A B.*

(iv) velocity of impact or arrival at point *B is* exactly the same as the initial velocity at point *A.*

(v) the net vertical distance traveled by the electron is zero because the distance traveled upwards is exactly equal and opposite to that traveled downwards. Hence, the two cancel out.

Example

A 500-V electron enters at an angle of 60° to the electric field existing between two plates separated in vacuum by a distance of 3 cm and having a fixed potential difference of V volt between them. The electron reaches point B where AB =10 cm. Calculate (a) the time taken by the electron to go from A to B (b) the value of V if the electron is **to** reach point B (c) highest point of ascent of the electron.



Solution

By a 500-V electron is meant an electron which has been accelerated through 500 V so that its

kinetic energy is

$$E_k = \frac{mu^2}{2} = eV$$

$$u = \sqrt{\frac{2eV}{m}} = 5.93*10^5\sqrt{V} = 5.93*10^5\sqrt{500} = 1.326*10^7\, m/s$$

Initial velocity along x-axis $u_x = u \cos 60° = 6.6308 \times 10^6$ m/s

Initial velocity along y-axis $u_y = u \sin 60° = 1.148 \times 10^7$ m/s

(a) As u_x remains constant, the time taken by the electron to travel the distance *AB* is $t = 0.1/ u_x = 0.1/6.6308 \times 10^6 = 1.5081 \times 10^{-8}$ s

(b) The vertical distance traveled by the electron can be found by using the well-known relation

$$S = u_y t + \frac{1}{2} a_y t^2$$

Since the vertical distance traveled by the electron upwards is equal and opposite to that traveled by it downwards, the two cancel out. Hence, putting $S = 0$ in the above equation, we get

$$a_y = \frac{-2u_y}{t} = -1.5231 \times 10^{15}$$

The distances and velocity etc. directed upwards are taken as negative whereas those directed downwards are taken as positive. The negative sign merely indicates that u_y and a_y are oppositely-directed.

$$a_y = \frac{eE}{m} = \frac{e}{m} \cdot \frac{V}{d}$$

$$V = \frac{a_y md}{e} = \frac{1.5231 \times 10^{15} \times 9.1 \times 10^{-31} \times 0.03}{1.602 \times 10^{-19}} = 259.8\text{V}$$

(c) For finding the point of highest ascent *i.e.* point **C,** the following well-known relation may be used

$$v^2 - u^2 = 2aS$$

Now, remembering that at point C, $V_y = 0$,

$$0 - u_y{}^2 = 2a_y S$$

$$S = \frac{-u_y{}^2}{2a_y}$$

$S = -0.0433\text{m} = -4.33\text{cm}$

The negative sign again indicates that the distance is travelled upwards

$y_{max} = 4.33\text{cm}$

PROBLEMS

Q1: An electron starts from rest at the negative plate separated by 2 cm and having a potential difference of 1500 volts. How long does it take to reach a speed of 10^7 m/s and what position does it reach at this speed? Find the kinetic energy of the electron when it hits the anode?

(Ans;0.38cm, 1500eV)

Q2: Electrons accelerated from rest through 400 V are introduced at A into a uniform electric field E of intensity 150 V/ cm. If the electrons emerge at B 5×10^{-9}s later, determine distance AB.

(Ans;4.98 cm)

Q3: Two parallel plates A and B are spaced 1 cm apart. A stream of electrons is projected at an accelerating voltage of 2 kV into the space between the plates through a hole in plate A at an angle of 30° to it. Find the value and polarity of the potential difference which is required between A and B in order that the electron just touches plate B.

(Ans; 500 V)