Conductivity of metals

The most important properties of metals their high thermal and electrical conductivity. In conductor the electrons moving with random velocity depends upon the temperature

$$E_k = \frac{mv^2}{2} = \frac{3kT}{2}$$

$$v_T = \left(\frac{3kT}{m}\right)^{1/2}$$

where k= Boltzman constant=1.38110^{-23}J/K, v_T=Thermal velocity.

When electric field E is applied to the conductor the electron acquire a ***drift velocity*** (**v_D**), the electron experience a force $F=eE$ moving toward positive terminal.

$$v_D = \mu E$$

where μ= Mobility m^2/v.s

Let a block of metals of length (l) and cross section area (A) which contains (n) free electrons. If a voltage (V) is applied a cross it a field ***E=V/l*** cause the electrons to move with drift velocity v_D. Hence the current I is

$$I = nev_D A$$

The current density ***J=I/A, then***

$$J = nev_D = ne\mu E$$

Ohm's law ***V=IR***, where ***R*** is the resistance of block given by

$$R = \rho \frac{l}{A} = \frac{V}{I}$$

where ρ is the resistivity of material in (Ω.m)

$$\frac{V}{I} = \rho \frac{l}{A}$$

$$\frac{1}{\rho} . \frac{V}{l} = \frac{I}{A}$$

$$\frac{1}{\rho} = \sigma$$

σ is conductivity is the (inverse) of electrical resistivity and has the SI units of Siemens per meter ($S \cdot m^{-1}$).

$$J = \sigma E$$

$$ne\mu E = \sigma E$$

$$\sigma = ne\mu$$

If N is the number of atom per m^3

$$N = \frac{N_{Avo} \times Density}{Atomicweight}$$

where N_{Avo} is the Avogadro's number=6.023×10^{26}.

For copper each atom has one valence electron, the number of free electrons n equal to atom number N.

Example

For copper calculate (i) Thermal velocity, (ii) drift velocity, and (iii) current density. If E=100V/m, ρ=1.72×10^{-8} Ω.m, T=25°C, density=8.46×10^{3}kg/m^{3}, and atomic weight=63.5.

Solution

T=25+273=298 K

(i)

$$v_T = \left(\frac{3kT}{m}\right)^{1/2} = \left(\frac{3\times1.38\times10^{-23}\times298}{9.109\times10^{-31}}\right)^{1/2} = 1.16\times10^{5}\,m/s$$

(ii)

$$v_D = \mu E$$

$$\sigma = ne\mu$$

$$\mu = \frac{\sigma}{ne} = \frac{1}{\rho ne}$$

$$n = N = \frac{N_{Avo}\times Density}{Atomicweight} = 8.5\times10^{28}/m^3$$

$$\mu = \frac{1}{\rho ne} = 4.33\times10^{-3}\,m^2/V.s$$

v_D=4.33×10^{-3}×100=0.433 *m/s*

(iii)

$$J = \sigma E = \frac{E}{\rho} = 5.8\times10^{9}\,A/m^2$$

Mean free path (l')

Due to the random motion collisions occur between the electron and the molecule, there is a mean distance an electron can travel before colliding called mean free path (l'). The average time between collisions (τ) known as relaxation time. If the electron drift with speed v_D.

$$l' = v_D \tau$$

$$ma = eE \qquad \because a = \frac{v_D}{\tau}$$

$$m\frac{v_D}{\tau} = eE \qquad \because v_D = \mu E$$

$$m\frac{\mu E}{\tau} = eE$$

$$m\frac{\mu}{\tau} = e$$

$$\frac{\mu m}{e} = \tau$$

$$\mu = \frac{el'}{mv_D}$$

$$\sigma = ne\mu \qquad \because \mu = \frac{\sigma}{ne}$$

$$\frac{\sigma}{ne} = \frac{el'}{mv_D}$$

$$\sigma = \frac{nel'}{mv_D}$$

Equilibrium

- No other external excitation other than temperature
- No net motion of charge or energy

Energy Band Model

I- Allowed states

- Only certain energy states are allowed.
- Electrons occupy lowest energy states available.
- No two electrons occupy the same energy state.

II- Distribution of allowed states over energy N(E) [density of states].

III- At any T, under equilibrium only fraction of states are occupied $f(E,T)$, $f(E,T)$:-Fermi-Dirac function

(i.e. Allowed states 4×5×10^{22} in Si).

Distribution of allowed states over energy N(E) [density of states].

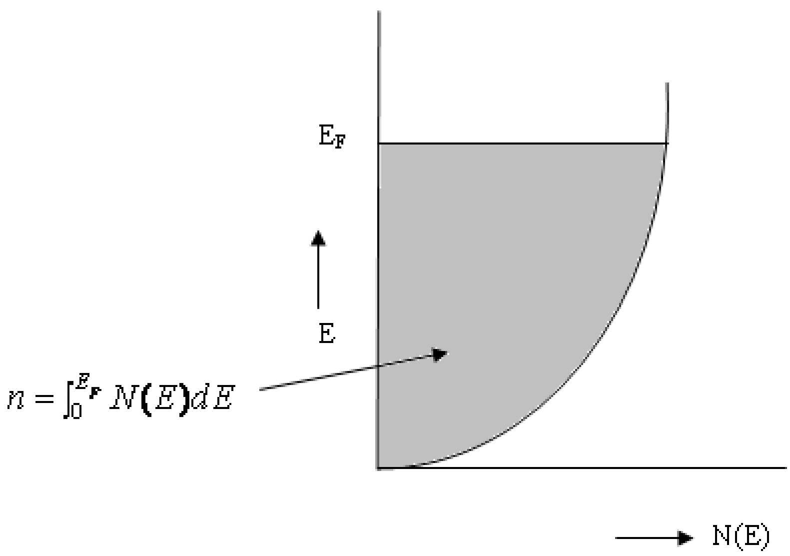


Fig. 3.1. Density of states verses energy in a conductor

At the absolute zero of temperature the electrons have the lowest possible energy so will fill the energy band up to the value Fermi level E_F

N(E) is the density of states (number of state per electron volt per cubic meter) and *where E_F is the Fermi level*

$$N(E) = \gamma E^{\frac{1}{2}}$$

where γ is a constant defined by

$$\gamma = \frac{4\pi}{h^3}(2m)^{\frac{3}{2}}\left(1.6\times10^{-19}\right)^{\frac{3}{2}}$$

The area under the curve represents the total number of free electron (per cubic meter)

$$n = \int_0^{E_F} N(E)dE = \int_0^{E_F} \gamma E^{\frac{1}{2}} dE = \frac{2}{3}\gamma E_F^{\frac{3}{2}}$$

$$E_F = \left(\frac{3n}{2\gamma}\right)^{\frac{2}{3}}$$

$$E_F = 3.64\times10^{-19}(n)^{\frac{2}{3}} \quad \text{eV}$$

Example

For copper conductor having l=10m, A=0.5mm^2 and R=0.34Ω, calculate (i) conductivity, (ii) mobility, (iii) relaxation time, and (iv) Fermi level. If density=8.46×10^{3}kg/m^3, and atomic weight=63.5.

Solution

(i)

$$R = \rho \frac{l}{A} = \frac{l}{\sigma A}$$

$$\sigma = \frac{l}{RA} = 5.9 \times 10^{7}\, Sm^{-1}$$

$$n = N = \frac{N_{Avo} \times Density}{Atomicweight} = 8.5 \times 10^{28} / m^{3}$$

(ii)

$$\mu = \frac{\sigma}{ne} \quad = 4.33 \times 10^{-3} m^{2} / V.s$$

(iii)

$$\tau = \frac{m\mu}{e} = 2.5 \times 10^{-14} s$$

(iv)

$$E_F = 3.64 \times 10^{-19} (n)^{\frac{2}{3}} = 7.05 \quad \text{eV}$$

Fraction of available states occupied at any E, T.

Under equilibrium only fraction of states are occupied is given by Fermi-Dirac probability function *f*(E,T),

f(E,T):-Fermi-Dirac function

$$f(E,T) = \frac{1}{1 + \exp\left(\frac{E - E_F}{kT}\right)}$$

where *k* is Boltzmann constant (eV/°K or J/°K), T is temperature °K, E_F is the Fermi level eV or J.

At any temperature Fermi level represents the energy states with 50% probability of being filled if no forbidden band exists. Because

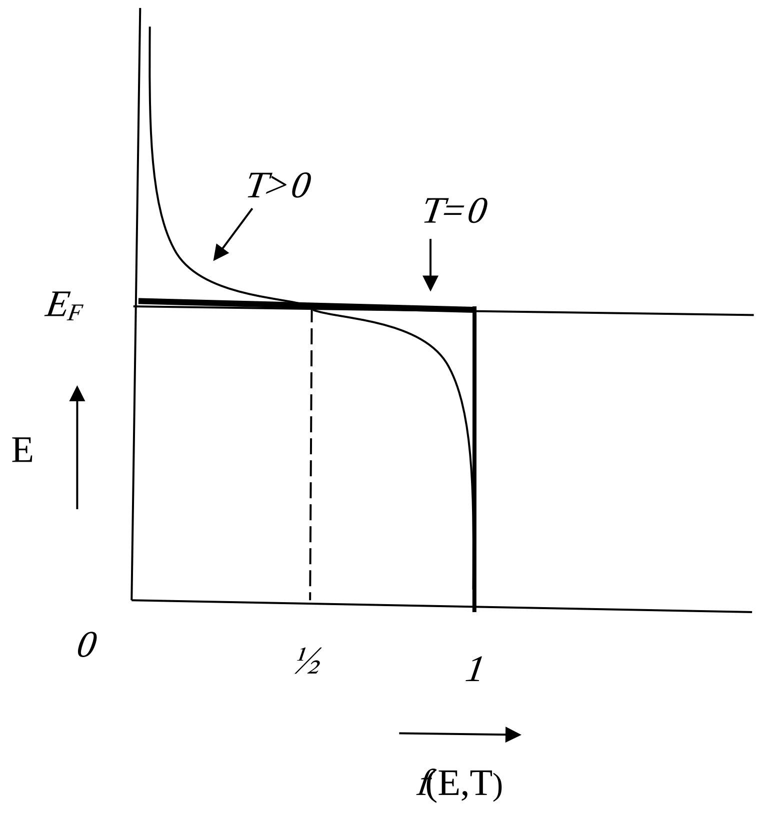
- *For* $E=E_F$ *then* $f(E,T)=½$

When T=0°K , two possible condition exist

1. $E>E_F$ $(+X/0)=\infty$, $\exp(\infty)=\infty$, $1/\infty=0$ and $f(E,T)=0$. Consequently, there is no probability of finding an occupied quantum state of energy greater than E_F at absolute zero.

2. $E<E_F$ $(-X/0)=-\infty$, $\exp(-\infty)=0$, $1/1=1$ and $f(E,T)=1$. All quantum levels with energies less than E_F will be occupied at T=0°K.

There are no electrons at 0°K which have energies in excess of E_F. That is, the Fermi energy is the maximum energy that any electron may possess at absolute zero.



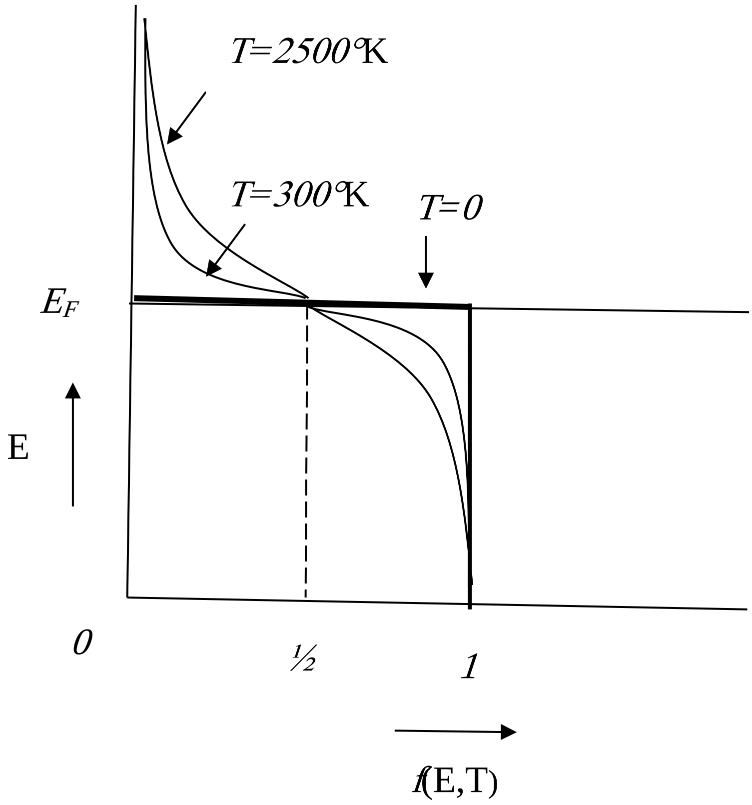


Fig 3.2 Fermi-Dirac distribution function *f*(E) gives a probability that a state of energy E is occupied

A plot of the distribution in energy shown below for metallic tungsten at T=0°K and T=2500°K. The area under each curve is simply the total number of particles per cubic meter of the metal; hence the two areas must be equal. Also, the curves for all temperatures $f(E,T)=½$ for $E=E_F$. Note that the distribution function changes only very slightly with temperature, even though the temperature is 2500°K. Only few electrons have large value of energy.

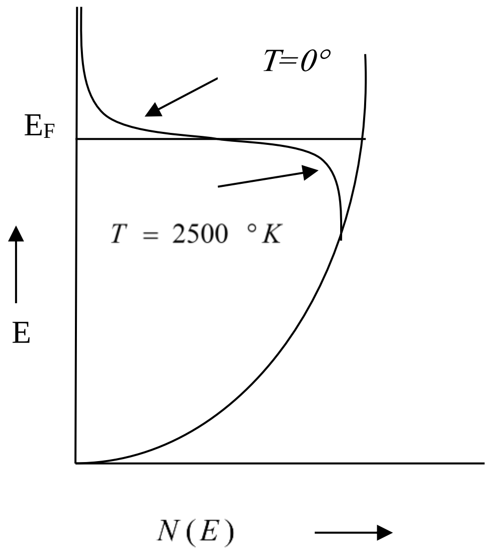


Fig. 3.3 The distribution in energy for metallic tungsten at T=0°K and T=2500°K.

Example

Calculate the probability that an energy state above E_F is occupied by an electron. Let T=300 K. Determine the probability that an energy level 3kT above the Fermi energy is occupied by an electron.

Solution

$$f(E,T)=\frac{1}{1+\exp\left(\frac{E-E_F}{kT}\right)}=\frac{1}{1+\exp\left(\frac{3kT}{kT}\right)}$$

$$f(E,T)=\frac{1}{1+20.09}=0.0474=4.74\%$$

At energies above E_F, the probability of a state being occupied by an electron can become significantly less than unity.

Work function (E_w): Work function of metal may be defined as the minimum amount of energy that must be given to the its fastest moving electron at 0^oK in order to enable it to escape from metal.

Problems

Q1: For copper conductor having l=5m, A=0.1mm^2 and R=0.15Ω, calculate (i) conductivity, (ii) mobility, (iii) relaxation time, (iv) Thermal velocity, (v) drift velocity and (vi) Fermi level. If T=300K, V=5V, density=8.46×10^3kg/m^3, and atomic weight=63.5.

Q2: For tungsten the total number of free electrons (n) and Fermi level. Assume that there are two free electrons per atom.
If density=18.8×10^3kg/m^3, and atomic weight=184.

(Ans. 1.23×10^{29}/m^3, 8.95eV)

Q3: Assume the Fermi energy level is 0.03eV below the conduction band energy. (a) Determine the probability of state being occupied by an electron at E_c.(b) Repeat part (a) for an energy state at E_c+kT. Assume T=300K.