Plates and Shells

Course Description:

This course covers the mechanics of plates using classical theory (cylindrical bending, rectangular plates, and circular plates) and plate theory with shear deformation. Includes combined effects of bending and in-plane forces, membrane theory of shells, analysis of thin cylindrical shells of revolution, and general theory of thin elastic shells.

Topics Covered:

1. Introduction

THIN ELASTIC PLATES

2. Governing Equations of Small Deflection Plate Theory
3. Analytical Solutions for Rectangular Plates
4. Analytical Solutions for Circular Plates
5. Large Deflection Theory of Plates

THIN ELASTIC SHELLS

7. Introduction
8. Geometry of the middle surface
9. General Theory of Shells
10. Membrane theory of shells
11. Bending Theory of Shells

References

-Theory of Plates and Shells, S.P. Timoshenko and S. Woinkowsky-Krieger, McGraw-Hill Book Company, NY. 2nd edition 1959, reissued 1987 (QA931 .T56 1959)

-Stresses in Shells. W. Flügge, Springer-Verlag, Berlin, 2nd Edition, 1960, 2nd printing, 1990 (TA660.S5 F58 1973).

-Beams, Plates and Shells, L.H. Donnell, McGraw-Hill Book Company, NY., 1976 (TA660.B4 D66)

-Shell Theory, F. I. Niordson, North-Holland, Amsterdam, 1985 (TA660.S5 N56 1985)

-Stresses in Plates and Shells, A. Ugural, McGraw Hill, 1999. (TA660.P6 U39 1999)

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-Structural mechanics: the behavior of plates and shells, J. R. Vinson, Wiley, New York, 1974, (TA660.P6 V55 1974)

-The Buckling of Plates and Shells, H.L. Cox, Macmillan, NY, 1962 (TA460 .C6x)

-Introduction to the Theory of Shells, C.L. Dym, Hempshire Publishing Corp., 1990. (QA935 .D89 1990)

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Lecture 1:

Introduction to Plate Bending Problems

Introduction

A plate is a planer structure with a very small thickness in comparison to the planer dimensions. The forces applied on a plate are perpendicular to the plane of the plate. Therefore, plate resists the applied load by means of bending in two directions and twisting moment. A plate theory takes advantage of this disparity in length scale to reduce the full three-dimensional solid mechanics problem to a two dimensional problem.

The aim of plate theory is to calculate the deformation and stresses in a plate Subjected to loads. A flat plate, like a straight beam carries lateral load by bending. The analyses of plates are categorized into two types based on thickness to breadth ratio:

1. thick plate
2. thin plate analysis.

If the thickness to width ratio of the plate is less than 0.1 and the maximum deflection is less than one tenth of thickness, then the plate is classified as thin plate. ***The well known as Kirchhoff plate theory is used for the analysis of such thin plates***. On the other hand, ***Mindlin plate theory is used for thick plate where the effect of shear deformation is included.***

بسم الله الرحمن الرحيم

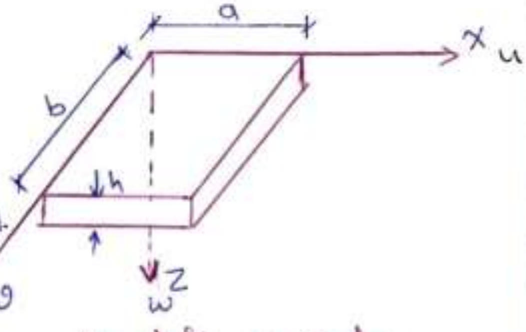
Theory of plates & shells :-

I- Plates :- Theory of plates :-

1- Thin plate with small defl.

2- " " " Large "

3- Thick plate



Assuming that $b < a$

$\therefore$ if $h \leqslant \frac{b}{20} \Rightarrow$ thin plats

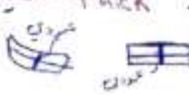
$w < h \Rightarrow$ Thin plate with small defl.

$w \geqslant h \Rightarrow$ Thin plate with Large "

u = defl. in x-dir.

- if both conditions are not satisified v = defl. in y-dir.
it will become thick plate w = defl. in z-dir.

* في حالة الـ thin plate يهمل الـ σ_z لأنه يكون صغير بالمقارنة ولكن في حالة
الـ Thick لا يمكن اهماله للتشوهات ...



Thin plate with small def. :-

1. $w < h$ $\left(h \leqslant \frac{b}{20}\right.$ where $\left.b < a\right)$

2- Theory is based on Kirchhoff's hypotheses, ie classical Theory (line ⊥ middle surface before deformation will remain ⊥ after deformation ⇒ negligible shear deformations)

3- No middle plane strains are created when bent to a developable surface

4- very small middle plane strain are created when bent to a non-developable surface

5- Bending and membrane actions are not coupled * ...

... $\epsilon_x = \frac{\partial u}{\partial x}$ في حالة الـ plate أما في حالة الـ shells ...

$\epsilon_x = \frac{\partial u}{\partial x} + \frac{w}{r_x}$

6. since deformations are small, the strain-displacement relations are linear
علاقات التشوه تبقى العلاقات خطية ومتوازنة

7. Equilibrium equations are derived in the <u>undeformet</u> state, hence are linear.
* لأن معادلات الاتزان تشتق من الشكل الأصلي غير المشوه لان التشوهات الحاصلة قليلة

Lagrange اسم العالم

<u>NoTe</u> This Theory will not give satisfactory stress in case of plates with holes and highly concentrated loads as shear effects are predominating in those cases
* في حالة وجود فتحات او حمولات مركزة عالية تتكون الـ Concentrated stress والنتائج غير دقيقة.

Thin Plate with Large deflection :-

1- $w \gg h$ ($h \leq \frac{b}{20}$, $b < a$) عادة السمك تبقى على ابعاد نظرية Kerm ونقل الشد والتشوه

2. Theory is again based on Kirchhoff's hypothesis

3- No middle plane strains are created when bent to a developable surface

4- Middle plane strains are created when bent to a non-developable surface (بسبب التشوه العالي بالتعرض لأكبر من الـ sheet وتكون هناك علاقتين الـ Bending والـ membrane)

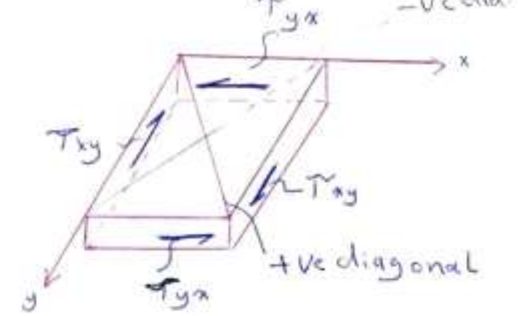
5- Bending and membrane actions are coupled

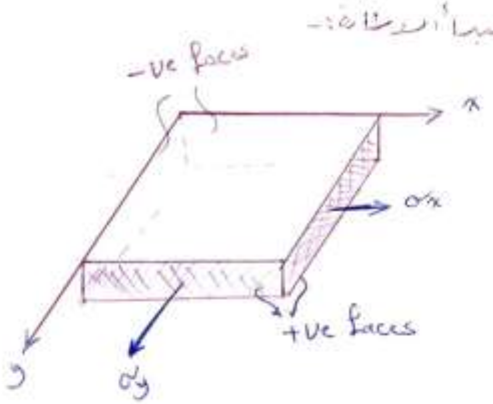
6- since deflections are not small (Geometrical non-linearity) strain-displacement relations are non-linear

7. Equilibrium equations are derived in the deformed state and hence are a non-linear (اي تكون الاعتماد على الشكل الاصلي بالاشتقاقات لحيث اعتماد الشكل المشوه)

* Karman - developed this theory

مبادئ أساسية:-





Bending , shear هتشوف تأثير كل نوع

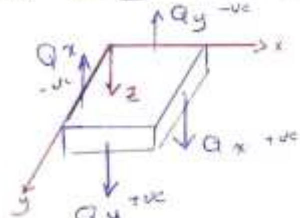
Shear stress +ve if The produce tension along the +ve diagonal

forces are :- moments M_x + M_y

Twisting mom. M_{xy} + M_{yx}

shear forces Q_x + Q_y

+ve if in direction of +ve Z



Differential Equation of Plates

Shape of the plate is adequately by describing The geometry of its middle surface, which is a surface that bisects the plate thickness (h) at each point

منتصف سمك الـ plate

Assumptions :-

1- The material of the plate is elastic homogeneous and isotropic

2- The plate is initially flat

3 The thickness (h) is small campared to other dimensions

4- The deflections are small compared to plate thickness

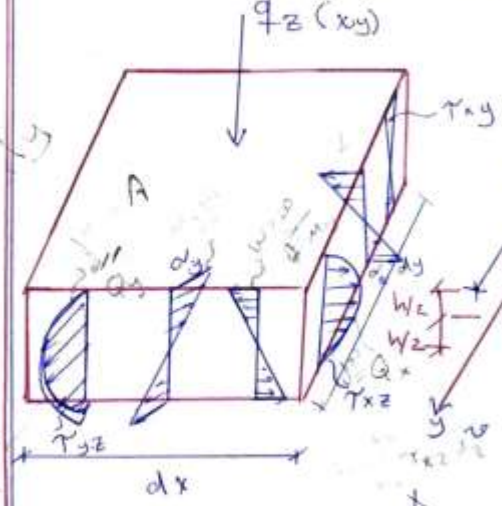
5- The deformations are such that a straight line ~~...~~ initially ⊥ to middle surface remain straight line + ⊥ after deformation.

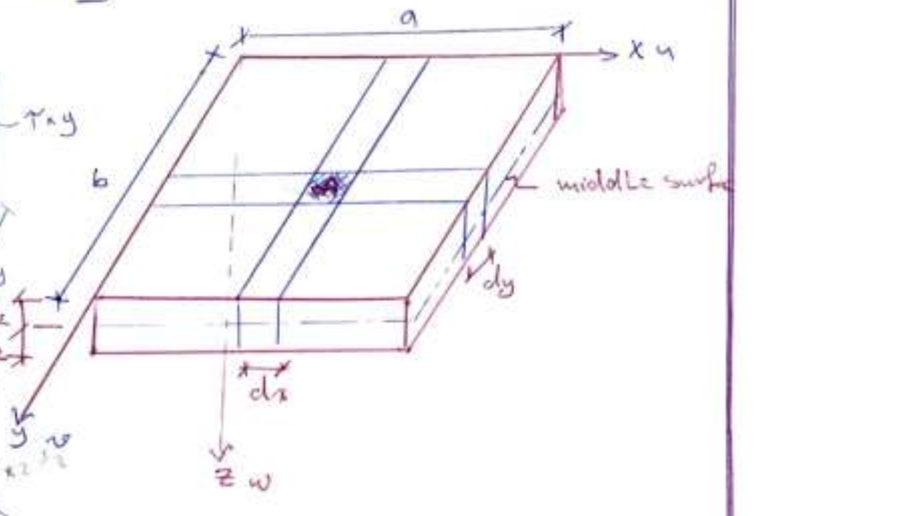
* الخطوط التي كانت عمودية على خط الوسط تظل عمودية بعد التشوه طالما خلصنا من الـ shear

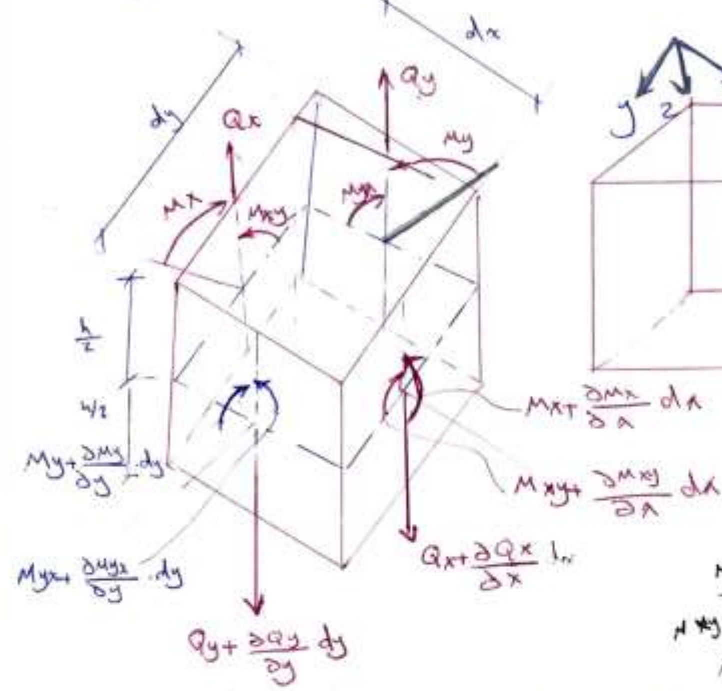
6. The stress ⊥ to middle surface are of negligible order of magnitude

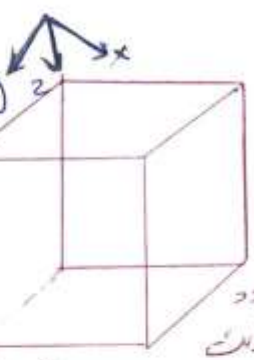
" ولسوف يعطيك ربك فترضى "

7- The strains in the middle surface produced by inplane -forces can usually be neglected in camparision with the strains due to bending.

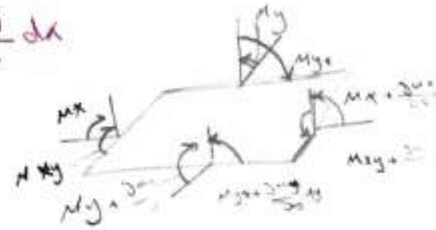








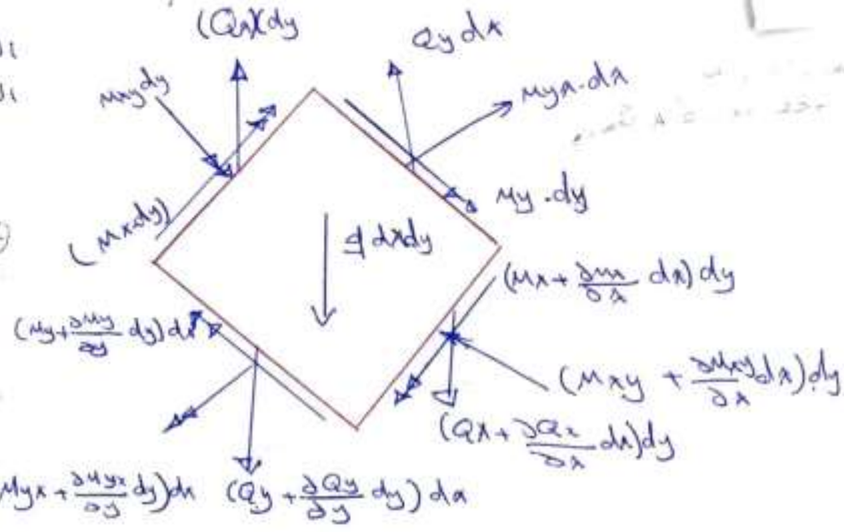
* تحت اثرات المقابل تحمل القوى من الطرف الخلفي عندما $\frac{\partial Q_y}{\partial y} dy$ لوجود المسافة dy في لذلك سوف نهمل الـ stress



Note:- All the action (shear force & bending moment are per unit length of the middle surface)

بسم الله الرحمن الرحيم

الاتجاه نحسب حسب قاعدة اليد اليمنى



1- Equilibrium in Z direction :- $\sum F_z = 0$

$$\left(Q_x+\frac{\partial Q_x}{\partial x}dx\right)dy - Q_x\,dy + \left(Q_y+\frac{\partial Q_y}{\partial y}dy\right)dx - Q_y\,dx + q\,dx\,dy = 0$$

$$\frac{\partial Q_x}{\partial x}+\frac{\partial Q_y}{\partial y}+q=0 \quad ---- (1)$$

2- Moments about y-axis :-

$$\left(M_x+\frac{\partial M_x}{\partial x}dx\right)dy - M_x\,dy + \left(M_{yx}+\frac{\partial M_{yx}}{\partial y}dy\right)dx - M_{yx}\,dx - \left[\left(Q_x+\frac{\partial Q_x}{\partial x}dx\right)dy\right]\frac{dx}{2} - (Q_x\,dy)\frac{dx}{2} = 0$$

$$\frac{\partial M_x}{\partial x}+\frac{\partial M_{yx}}{\partial y} = Q_x \quad ---- (2)$$

Note:- term with (dx^2) are eleminated

3- Moment about x-axis :-

$$\left(M_y+\frac{\partial M_y}{\partial y}dy\right)dx - M_y\,dx + \left(M_{xy}+\frac{\partial M_{xy}}{\partial x}dx\right)dy - M_{xy}\,dy - \left[\left(Q_y+\frac{\partial Q_y}{\partial y}dy\right)dx\right]\frac{dy}{2} - (Q_y\,dx)\frac{dy}{2} = 0$$

$$-\frac{\partial M_y}{\partial y}+\frac{\partial M_{xy}}{\partial x} = -Q_y \quad ---- (3)$$

الآن تفاضل ونقلل من عدد المجاهيل بالاشتقاق

Diffrentiating eq (2) with respect to (X) and eq (3) with respect to (y) and substituting in eq. (1) we get

في الـ plate اذا كان الحل منتظم على جميع المحاور في شكل المعادلات

$$\frac{\partial^2 M_x}{\partial x^2} + \frac{\partial^2 M_{yx}}{\partial x \partial y} + \frac{\partial^2 M_{xy}}{\partial y \partial x} + \frac{\partial^2 M_y}{\partial y^2} + q = 0$$

observing that $M_{xy} = -M_{yx}$

$$\therefore \frac{\partial^2 M_x}{\partial x^2} + 2\frac{\partial^2 M_{xy}}{\partial x \partial y} + \frac{\partial^2 M_y}{\partial y^2} + q = 0 \quad \ldots (19)$$

هنا المعادلة لم تساعد في الحل لانها ... لذلك نلجأ الى الـ deformation :-

Compatibility Relations :-

a) strain – displacement relations

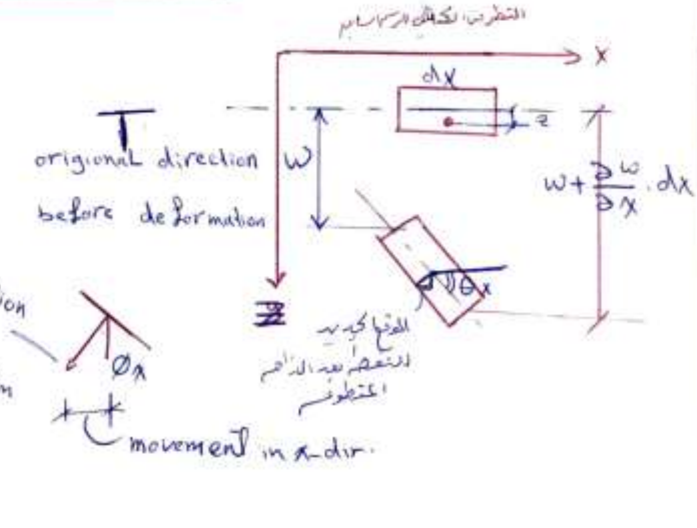
b- strain – stress "

c- stress – force "

d- force – displacement relations.

a- strain – displacement relations :-

$$\left.\begin{aligned} \epsilon_x &= \frac{\partial u}{\partial x} \\ \epsilon_y &= \frac{\partial v}{\partial y} \\ \gamma_{xy} &= \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} \end{aligned}\right\} - (A)$$



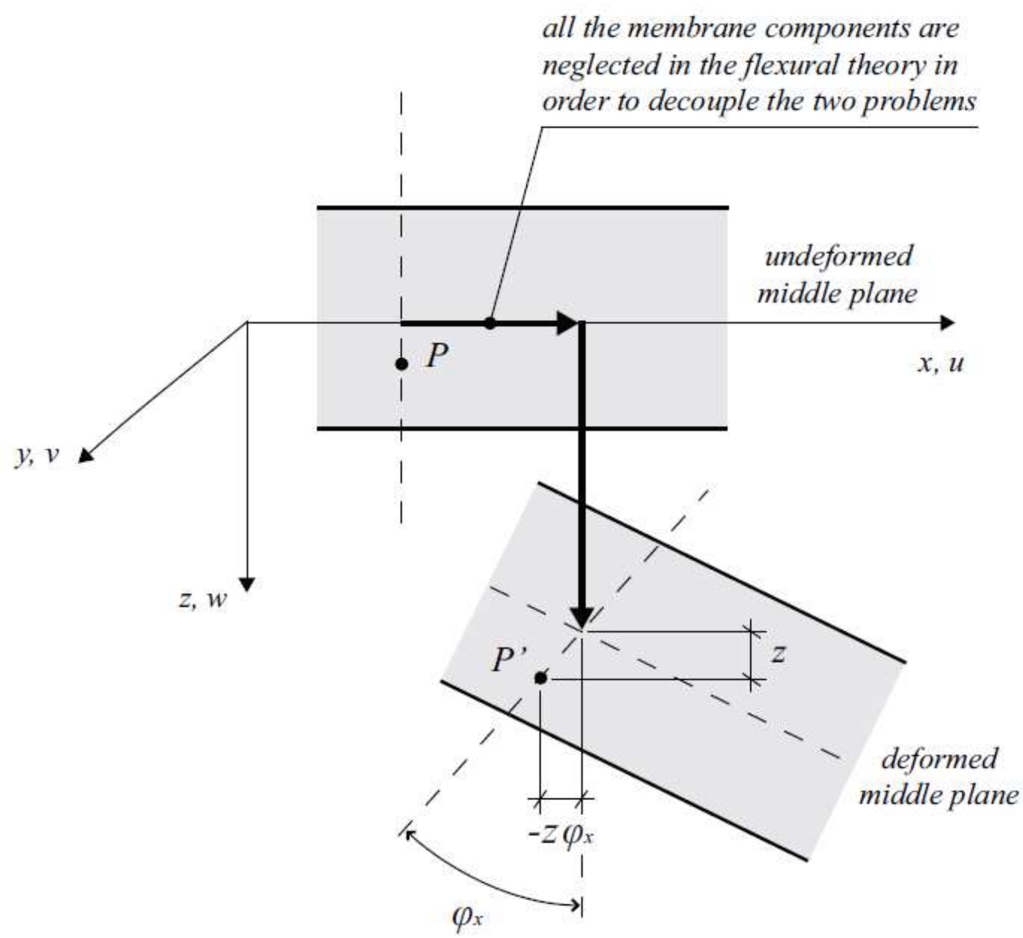


Figure 3: A section of a plate, traced in the x-z plane, before and after the deformation.