CHAPTER 3
GROUNDWATER MOVEMENT
Rate of Groundwater movement related to transmission property of porous media. What is Groundwater velocity of flow?
Darcy’s law

Henry Darcy (1856) investigated water flow thru horiz. bed of sand.

Steady flow

Laminar flow – velocity very very small.
Bernaulli Equation

\[ P_1 + \frac{v_1^2}{2g} + z_1 = P_2 + \frac{v_2^2}{2g} + z_2 + h_L \]

\[ \gamma 2g \quad \gamma 2g \]

P – pressure
\( \gamma \) – specific weight of water
v – velocity of flow
g – acceleration of gravity
z – elevation
h – head loss
Since $v$ is very small in linear groundwater flow,

$$h_L = \left[\left(P_1/\gamma\right) + z_1\right] - \left[\left(P_2/\gamma\right) + z_2\right]$$

$h_L$ defined as potential loss within sand column.

This energy lost by frictional resistance dissipated as heat energy.

$h_L$ independent of slope of cylinder or column.
Darcy’s experiment showed:

\[ Q \approx h_L \quad Q \propto \left(\frac{h_L}{L}\right) A \quad Q \text{ – flux} \]

\[ \approx \frac{1}{L} \quad K \text{ – constant} \]

\[ \approx A \quad A \text{ – area of cross section} \]

\[ Q = -KA \frac{h_L}{L} \quad L \text{ – column length} \]

\[ Q = -KA \frac{dh}{dL} \quad i = \frac{dh}{dL} = \frac{h_L}{L} = \text{hyd. gradient} \]

\[ V = Q = -\frac{Kdh}{A} \quad \frac{dL}{A} \quad \text{Hence,} \quad V = -Ki \]
Darcy’s law states that flow velocity,

\[ V = \text{Product of Constant } K, \text{ Coefficient of Permeability, and Hydraulic Gradient} \]

Coefficient of Permeability is also known as Permeability or Hydraulic Conductivity.
(1) Darcy velocity or apparent velocity –
\[ V = \frac{Q}{A} \]; Assumes water moving thru solids and pores.

(2) Pore velocity or seepage velocity –
Since water moves thru pores only, actual vel > Darcy vel.

Pore velocity = \( \frac{Q}{\alpha A} \) = \frac{v}{\alpha} = - \frac{k i}{\alpha} \\
where:
Available area of flow = \( \alpha A \); and \( \alpha = \) porosity
(3) Actual velocity –
variation due to pore geometry
more velocity at constriction.
Validity of Darcy’s Law:

Darcy’s law valid in laminar flow, not turbulent flow.

In **laminar flow**, flow vel. relatively small; water molecules travel in smooth path II to solid boundaries of pores by viscous forces of fluid.

Head loss  \( i = av \)
In **turbulent flow**, inertial forces due to increased flow vel. dominant.

Water molecules travel in irregular paths forming eddies.

Head loss  \( i = av + bv^2 \)

\( a \) & \( b \) = constant
Criteria between laminar and turbulent flow –

Reynolds number

\[ R = \frac{\rho v D}{\mu} = \frac{\text{inertial forces}}{\text{viscous forces}} = vD \]

\( v \) – flow velocity

\( \rho \) – fluid density

\( D \) – diameter - (pipe dia. in pipes; grain size or pore dia. in porous med. - grain dia. more convenient and used)

\( \mu \) – fluid dynamic viscosity

\( \nu \) – kinematic viscosity =
Fig. 3.2. Relation of Fanning friction factor to Reynolds number for flow through granular porous media (after Rose).
Fanning factor

\[ f = \frac{d \Delta P}{2 L v^2} = \frac{d \Delta pg}{2v^2 L_g} = \frac{d \Delta h g}{2v^2 L} = \frac{d \Delta h}{4(v^2/2g) L} = \frac{d \Delta h}{4(v^2/2g)} \]

where:

\[ \Delta p \text{ – pressure diff. over } L \]
\[ d \text{ – grains size} \]
Plot $f$ vs. $R$ or $N_R$ for porous media

Laminar range –

$R - 1 \text{ to } 10 \ (<1)$

Darcy’s Law valid

Turbulent flow – occurs near pumped well casing; porous formations as basalt and limestone.
Permeability
Hydraulic Conductivity (Permeability), K:

\[ K = - \frac{Q}{A_i} = - \frac{V A_i}{i A} = - \frac{V}{i} \]

Dimensions –

\[ K = - \frac{V}{(dh/dL)} \text{, m/d or gpd/ft}^2 \text{ or ft/d} \]

\[ K = 1 \text{ if Vol = 1 in time = 1} \]

\[ \text{Area} = 1 \]

\[ i = 1 \]

A porous medium has unit K if it transmits a unit vol. of water in unit time thru unit area of cross section normal to flows under unit i at prevailing temperature.
Standard (Laboratory) Perm., $K_s$ –
flow of water at $60^0\ F$ in gpd thru a porous media having an area of $1\ ft^2$
perpendicular to flow under a hyd grad. of $1\ ft/ft$

$K_s - 10 - 5000\ gpd/ft^2$
$K_s - 2000\ gpd/ft^2 \Rightarrow \text{good aquifer}$
Field perm., $K_f$ –

Flow of water in gpd thru an aquifer of 1 ft thickness by 1 mile width perpendicular to flow under a grad. of 1 ft/mile at field temp.

$$K_s = K_f$$
$$K_s = \mu_f$$
$$K_f \quad \mu_{60}$$
Transmissivity; \( T \):

\[
T = K b \text{ gpd/ft or } m^2/d \quad b = \text{thickness of aquifer}
\]

\[
Q = K i A = K i (b \times 1) = T x i x 1
\]
Intrinsic Perm, \( k \):

\[
K = \frac{k\gamma}{\mu}
\]

\( K \) – hydraulic constant
\( \mu \) – dynamic viscosity
\( \gamma \) – sp. weight of water

\[
K = f \left( \text{P.M., Fluid} \right)
\]

\( k \) is property of porous medium

\[
k = cd^2, \text{ cm}^2 \text{ or ft}^2
\]

\( d \) – grain size
\( c \) – constant

\[
c = f(\text{ porosity, packing, grain size distribution, shape})
\]
Range of Groundwater velocity:

Low velocity – clay average

\[ K = 10 \text{ gpd/ft}^2 \]

\[ i = 10 \text{ ft/mile} \]

\[ v = 10 \left( \frac{10}{5280} \right) = 2.5 \times 10^{-3} \text{ ft/d} \]

High velocity – Alluvial average

\[ K = 5000 \text{ gpd/ft}^2 \]

\[ i = 100 \text{ ft/mile} \]

\[ v = 12.7 \text{ ft/d} \]

Natural velocity – 5 ft/d to 5 ft/yr
A field sample of an unconfined aquifer is packed in a test cylinder (see Figure 3.1.1). The length and the diameter of the cylinder are 50 cm and 6 cm, respectively. The field sample is tested for a period of 3 min under a constant head difference of 16.3 cm. As a result, 45.2 cm³ of water is collected at the outlet. Determine the hydraulic conductivity of the aquifer sample.

**SOLUTION**

The cross-sectional area of the sample is

\[ A = \frac{\pi D^2}{4} = \frac{\pi (0.06 \text{ m})^2}{4} = 0.00283 \text{ m}^2 \]

The hydraulic gradient, \( dh/dl \), is given by

\[ \frac{dh}{dl} = \frac{(-16.3 \text{ cm})}{50 \text{ cm}} = -0.326 \]

and the average flow rate is

\[ Q = \frac{45.2 \text{ cm}^3}{3 \text{ min}} = 15.07 \text{ cm}^3/\text{min} = 0.0217 \text{ m}^3/\text{day} \]

Apply Darcy's law, Equation 3.1.4, to obtain the hydraulic conductivity as

\[ Q = -KA \frac{dh}{dl} \rightarrow K = \frac{Q}{A(dh/dl)} = \frac{0.0217 \text{ m}^3/\text{day}}{(0.00283 \text{ m}^2)(-0.326)} = 23.5 \text{ m/day} \]
A confined aquifer with a horizontal bed has a varying thickness as shown in Figure 3.1.2. The aquifer is inhomogeneous with $K = 12 + 0.006x$, where $x = 0$ at section (1), and the piezometric heads at sections (1) and (2) are 14.2 m and 18.8 m, respectively measured above the upper confining layer. Assuming the flow in the aquifer is essentially horizontal, determine the flow rate per unit width.

**SOLUTION**

Darcy's law for a constant thickness aquifer is given by Equation 3.1.4,

$$Q = -KA \frac{dh}{dl}$$

![Diagram showing the aquifer for Example 3.1.2](image)
Since the aquifer thickness is variable in this problem, we must also write the cross-sectional area and the hydraulic gradient as a function of the distance \( x \). Assuming a unit width, \( A = b_1 + \frac{(b_2 - b_1) x}{L} \), where \( b_1 = 30 \) m, \( b_2 = 75 \) m, and \( L = 3,600 \) m, then we have

\[
A = 30 + \frac{(75 - 30)x}{3,600} = 30 + 0.0125x
\]

Substituting the expressions for \( A \) and \( K \) into Darcy's equation yields the expression for \( Q \) in following form:

\[
Q = -(12 + 0.006x)(30 + 0.0125x) \frac{dh}{dx}
\]

Rearranging this equation and integrating from section (1) to section (2) yields

\[
\int_{0}^{3600} \frac{1}{(12 + 0.006x)(30 + 0.0125x)} dx = \int_{14.2}^{18.8} \frac{-1}{Q} dh
\]

This equation is integrated using partial fraction decomposition to obtain

\[
\int_{0}^{3600} \left[ \frac{0.2}{(12 + 0.006x)(30 + 0.0125x)} - \frac{0.416}{(30 + 0.0125x)} \right] dx = \int_{14.2}^{18.8} \frac{-1}{Q} dh
\]

\[
\left[ 33.333 \ln (12 + 0.006x) - 33.28 \ln (30 + 0.0125x) \right]_{x=0}^{x=3600} = -\frac{1}{Q} h_{h=18.8} - h_{h=14.2}
\]

\[
-26.54 - (-30.36) = -\frac{1}{Q} (18.8 - 14.2)
\]

\[
Q = -1.20 \text{ (m$^3$/day/m)}
\]

The minus sign implies that the flow is from section (2) to (1).
The following additional information is given for the aquifer sample in Example 3.1.1. The sample has a median grain size of 0.037 cm and a porosity of 0.30. The test is conducted using pure water at 20°C. Determine the Darcy velocity, average interstitial velocity, and assess the validity of Darcy’s law.

**SOLUTION**

Darcy velocity is computed using Equation 3.1.5:

\[ v = -K \frac{dh}{dt} = -(23.54 \text{ m/day})(-0.326) = 7.67 \text{ m/day} \]

The average linear velocity is computed using Equation 3.1.6:

\[ v_a = \frac{Q}{\alpha A} = \frac{v}{\alpha} = \frac{7.67 \text{ m/day}}{0.30} = 25.6 \text{ m/day} \]

In order to assess the validity of Darcy’s Law we must determine the greatest velocity for which Darcy’s law is valid using Equation 3.1.7, \( N_R = \frac{\rho v D}{\mu} \). Knowing Darcy’s law is valid for \( N_R < 1 \). For water at

\[ v_{\text{max}} = \frac{\mu}{\rho D} = \frac{1.005 \times 10^{-3} \text{ kg/ms}}{(998.2 \text{ kg/m}^3)(0.00037 \text{ m})} = 0.00272 \text{ m/s} = 235 \text{ m/day} \]

Then Darcy’s law will be valid for Darcy velocities equal to or less than 235 m/day for this sample. Thus, the answer we have found in Example 3.1.1 is valid since \( v = 7.67 \text{ m/day} < 235 \text{ m/day} \).
A leaky confined aquifer is overlain by an aquitard that is also overlain by an unconfined aquifer. The estimated recharge rate from the unconfined aquifer into the confined aquifer is 0.085 m/year. Piezometric head measurements in the confined aquifer show that the average piezometric head in the confined aquifer is 6.8 m below the water table of the unconfined aquifer. If the average thickness of the aquitard is 4.30 m, find the vertical hydraulic conductivity, \( K_v \), of the aquitard. What type of material could this possibly be?

**SOLUTION**

Given \( v = 0.085 \text{ m/year} = 2.329 \times 10^{-4} \text{ m/day} \), Equation 3.2.6 is used to compute the vertical hydraulic conductivity of the aquitard:

\[
K = -\frac{v}{\frac{dh}{dl}} = -\frac{2.329 \times 10^{-4} \text{ m/day}}{(6.8 \text{ m/4.30 m})} = 1.473 \times 10^{-4} \text{ m/day}
\]

From Table 3.2.1, the aquitard is composed of clay.
Measurement of Permeability:

1. Formulas
2. Lab. Measurement (permeameters)
3. Tracers
4. Pumping tests of wells – CH4
Measurement of Permeability:

1. **Formulas** – derived from analytic or experimental work

   \[ k = f(\alpha, \text{packing, grain size}) \]

   Basically, problem reduces to relating factor \( C \) to media properties
Measurement of Permeability:

Fair and Hatch formula –

$$k = \frac{1}{n \left[\frac{(1-\alpha)/\alpha^3}{\left(\theta/100\right)\sum P/d_m}\right]^2}$$

$k$ = intrinsic perm.  : $K = k \gamma / \mu$
$n$ = packing factor (found experimentally;)
$\theta$ = shape factor (spherical sand 6, angular grains 7.7)
$\alpha$ = porosity
$P$ = % of sand held between adjacent sieves
$d_m$ = geometric mean of the adjacent sieves
$$d_m = (d_1 \cdot d_2 \ldots d_m)^{1/m}$$
2. Lab. Measurement (permeameters)

Method of determining K
(i) constant head
(ii) falling head
(iii) non-discharging

\[ Q = KA \frac{dh}{dL} \quad K = \frac{Q}{A} \]

Criticism:
(i) representative sample
(ii) undisturbed sample
Fig. 3.4 "Permeameters for measuring hydraulic conductivity of geologic samples. (a) Constant head. (b) Falling head."
3. Tracers: Field method of determining $K$

Introduce tracer in U/S well.

Observe the time required for it to appear in D/S well.

Estimate Groundwater velocity
Measurement of Permeability:

3. Tracers cont.

Use this vel. and hyd. grad. to determine $K$. Since flow occurs only in pores,

$Q = (A\alpha) \, V_p$

$V_p = (K/\alpha)(h/L)$

$K = \frac{\alpha V_p L}{h} = \frac{\alpha L^2}{t \, h}$

where: $V_p = L/t$ ;

$t = \text{time of tracer appearance in well B}$
Measurement of Permeability:

3. Tracers cont.

Criticism:

(i) Direction of flow
(ii) Front moves at unequal vel. due to variation of $K$
(iii) As tracers miscible with water, there is diffusion & dispersion.
4. Pumping tests of wells

Anisotropy –

\[ K_x \gg K_z , \text{ anisotropic aquifer} \]

If \( K_x = K_z \) at a point, isotropic aquifer
4. Pumping tests of wells

Heterogeneous (Nonhomogeneous) Aquifer - Layered Aquifer

If $K_x$ or $K_z$ same at various points in aquifer, homogeneous aquifer.

If it varies, nonhomogenous aquifer.

Average $K$ for horizontal and vertical flows.

(Prob. Given: see textbook)
EXAMPLE 3.3.1

A field sample of medium sand with a median grain size of 0.84 mm will be tested to determine the hydraulic conductivity using a constant-head permeameter. The sample has a length of 30 cm and a diameter of 5 cm. For pure water at 20° C, estimate the range of piezometric head differences to be used in the test.

SOLUTION

The maximum allowable Darcy velocity (assuming \( N_R = 1 \)) for \( d = 0.84 \) mm is

\[
\nu_{\text{max}} = \frac{\mu}{\rho D} = \frac{1.005 \times 10^{-3} \text{ kg/m s}}{\left(998.2 \text{ kg/m}^3 \right) \left(0.00084 \text{ m} \right)} = 0.0012 \text{ m/s} = 103.6 \text{ m/day}
\]

Thus, the Darcy velocity in the test must be equal to or less than 103.6 m/day so that Darcy's law will be valid, so that

\[
\nu = -K \frac{dh}{dl} \leq 103.6 \text{ m/day} \implies |dh| \leq \frac{(103.6 \text{ m/day})(0.30 \text{ m})}{K}
\]

For the representative value of hydraulic conductivity for medium sand given in Table 3.2.1,

\[
K = 12 \text{ m/day}, \text{ then } |dh| \leq \frac{(103.6 \text{ m/day})(0.30 \text{ m})}{12 \text{ m/day}} \approx 2.6 \text{ m} = 260 \text{ cm}
\]

It should be noted that the \( K \) value for clean sand ranges approximately from 0.1 m/day to 4,320 m/day. See Figure 3.2.1. Therefore, the early series of tests must be conducted with relatively low piezometric head differences if possible. After analyzing the results of early test data, a better estimate of the maximum allowable piezometric head difference can be made using the above inequality.
If the field sample in Example 3.3.1 is tested with a head difference of 5.0 cm and 200 ml of water is collected at the outlet in 15 min, determine the hydraulic conductivity of the sample. What should the maximum allowable piezometric head difference be for a series of tests?

**SOLUTION**

Equation 3.3.3 is used to compute the hydraulic conductivity in a constant-head permeameter test:

$$K = \frac{V L}{A h} = \frac{(200 \text{ cm}^3)(30 \text{ cm})}{\left(\frac{\pi (5 \text{ cm})^2}{4}\right)(15 \text{ min} \times 60 \frac{\text{s}}{\text{min}})(5.0 \text{ cm})} = 0.0679 \text{ cm/s} = 58.7 \text{ m/day}$$

Based upon this estimate and referring to Example 3.3.1, the maximum allowable piezometric head difference for tests should be approximately

$$|d h| \leq \frac{(103.6 \text{ m/day})(0.30 \text{ m})}{58.7 \text{ m/day}} \approx 0.53 \text{ m} = 53 \text{ cm}$$
A 20-cm long field sample of silty, fine sand with a diameter of 10 cm is tested using a falling-head permeameter. The falling-head tube has a diameter of 3.0 cm and the initial head is 8.0 cm. Over a period of 8 hr, the head in the tube falls to 1.0 cm. Estimate the hydraulic conductivity of the sample.

**SOLUTION**

Equation 3.3.6 is used to compute the hydraulic conductivity in a falling-head permeameter test:

\[
K = \frac{r^2L}{r_c^2t} \ln \frac{h_1}{h_2} = \frac{(1.5 \text{ cm})^2(20 \text{ cm})}{(5.0 \text{ cm})^2(8 \times 3600 \text{ sec})} \ln \frac{8.0 \text{ cm}}{1.0 \text{ cm}} = 1.3 \times 10^{-4} \text{ cm/s} = 0.112 \text{ m/day}
\]

**Figure 3.3.2.** Cross section of an unconfined aquifer illustrating a tracer test for determining hydraulic conductivity.
Dupuit – Forchheimer Assumption:

Darcy’s law –

\[ V = K \left( \frac{dh}{dL} \right) \approx K \tan \theta \]

\[ K \sin \theta \approx K \tan \theta \]

Applicable to 1-D horizontal or vertical flow.

In vertical flow, horizontal flow component is neglected and in horizontal flow, vertical flow is neglected.

This is called D–F assumption.
Horizontal flow–

Flux to effluent stream

\[ Q = V \times A \]

\[ = K \left( \frac{dh}{dL} \right) (b \times 1) \]

\[ = \text{flux per unit width, (} m^3/d/m \text{ or gpd/ft)} \]
Vertical flow –

Flux from shallow influent stream to aquifer (depth is small and width is large)

Assume vertical leakage

\[ Q = K \frac{dh}{dL} \times (W \times 1) \]

= flux per unit length of stream
Flow Equations:
Darcy’s law

\[ V = -K \]

\( s \) - distance along flow direction
Velocity components in \(x, y, z\) directions –

\[ V_x = \quad ; \quad V_y = \quad ; \quad V_z = \quad \]

\(K_x, K_y, K_z\) – Perm. in \(x, y, \text{ and } z\) directions

Assume homogeneous and isotropic aquifer,

\[ K_x = K_y = K_z = K \]

\[ V_x = \quad ; \quad V_y = \quad ; \quad V_z = \quad \]
In hydrodynamics, vel. potential, $\phi$, defined as a scalar function of space and time, such that

$$V_x = \quad ; \quad V_y = \quad ; \quad V_z =$$

Thus $\phi = Kh$
Steady Flow

Equation of continuity

$$t\frac{\partial V}{\partial t} + \frac{\partial}{\partial x}(Vx) + \frac{\partial}{\partial y}(Vy) + \frac{\partial}{\partial z}(Vz) = 0$$

For steady flow, flow condition unchanged with time.

Water incompressible, so $\rho = \text{constant}$.

Thus

$$0 = \frac{\partial}{\partial x}(Vx) + \frac{\partial}{\partial y}(Vy) + \frac{\partial}{\partial z}(Vz)$$
Substituting $V_x = x\frac{\partial}{\partial x}$ etc. into

$$0 = \frac{\partial^2}{\partial z^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial x^2} \phi$$

gives the Laplace Equation.
• Since

• This is general partial differential equation for steady flow of water in homo. & isotropic medium.
• Unsteady Flow:
  – To derive unsteady flow equation, consider storage coefficient and aquifer compressibility in confined aquifer, and yield in unconfined aquifer, $S$ related to aquifer compressibility,
• \( v \) – volume
• \( p \) – pressure
• \( E \) – elastic modulus
• It is assumed the compressive force act in vertical direction and is negligible in horizontal direction. When p.s. lowered by 1 ft., water released = S

• Thus
• Volume of aquifer column
  – \( v = b \cdot 1 = b \)

• Change in pressure

• is negative because of decline in water level
For elastic material:

\[ V = \rho \cdot m \quad \text{or} \quad V \rho = m \quad \text{vol (ft}^3) \quad \text{m – mass (slug)} \]

or

\[ \rho \Delta \rho \Delta = V \quad \rho \quad \text{density (slug/ft}^3) \]
It is assumed that water is compressible, and that the grains porous media are rigid; however, they may be packed more closely by compressive forces.

\[
\frac{d\rho}{V} = \rho - \delta \rho
\]

\[
\delta S = \beta \delta \rho
\]
Substitute in eqn. of continuity –

\[ \frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x} (\rho V_x) + \frac{\partial}{\partial y} (\rho V_y) + \frac{\partial}{\partial z} (\rho V_z) = 0 \]

This is approx. PDE for unsteady flow of water in a compressible confined aquifer of thickness b.
PDE for unconfined aquifer is nonlinear. The confined aquifer P.D. applied to an unconfined aquifer where variations in sat. thickness is small.
Boundary conditions–

1. Infinite aquifer
2. Impermeable boundary – fault
3. Permeable boundary – wells, water table, surface water body (lakes, etc.)
Method of Solution— (steady & unsteady Equations):

1. Analytic – transformations (Hodographs)
2. Flow nets - steady state flow
3. Hydraulic models
4. Analog models – electric network
5. Digital models
6. Hybrid models
Flow Nets

Flow fields in groundwater

Steady state GW flow

Hyd. Grad.

\[
\text{For square flow nets, } \frac{\text{ds}}{\text{dm}} \approx 1
\]

Flow in each flow line

\[
\frac{dq}{ds} = K \frac{dh}{ds}
\]
Equi-potential lines

Flow lines or streamlines

Stream Tube
If \( n \) squares along a flow line & \( h \) is total head loss

\[
\begin{align*}
n & = \text{no. equipotential tubes} \\
\frac{dh}{n} & = \frac{h}{n} \\
\frac{dh}{n} & = \text{head loss in 1 square} \\
\frac{dq}{m} & = \frac{Kh}{m}
\end{align*}
\]

If total flows \( Q \) divided into \( m \) squares \((m = \text{no. of flow tubes})\)

\[
Q = mdq = \frac{Kmh}{n}
\]
Contour map of W.T or P.S. and flow lines useful for locating new wells. Flow occurs along stream tube, velocity of flow zero across tubes.

\[ q = A_1 V_1 = A_2 V_2 \]

\[ A = \text{flow area perpendicular to flow} \]
Darcy's law

$A_1 K_1 i_1 = A_2 K_2 i_2 - \text{hyd. grad.}$

If aquifer depth uniform and flow lines equispaced

$K_1 \propto \frac{1}{i_1}$

Portions of aquifer having wide contour spacing (flat hyd. grad) will have higher than those with narrow spacing (steep hyd. grad)
EXAMPLE 3.5.1

Determine $h_B$ and the vertical velocity for the situation shown in Figure 3.5.2.

SOLUTION

Assume steady-state conditions. Writing Darcy's law from point $A$ to $B$ with the dimensions indicated in Figure 3.5.2, we have

$$v = K \frac{dh}{dl} = 10 \frac{27 - h_B}{27}$$  \hspace{1cm} (3.5.3)

and from point $B$ to $C$,

$$v = K \frac{dh}{dl} = 0.2 \frac{h_B + 5 - 30}{5}$$

Solving these yields, $h_B = 26.8 \text{ m}$ and $v = 0.07 \text{ m/day}$. 

Figure 3.5.2. Diagram illustrating application of Darcy's law for vertically downward flow.
Three observation wells are installed to determine the direction of groundwater movement and the hydraulic gradient in a regional aquifer. The distance between the wells and the total head at each well are shown in Figure 3.6.7a.

![Diagram of three observation wells](image)
**SOLUTION**

**Step 1:** Identify the well with the intermediate water level—Well 1 in this case.

**Step 2:** Along the straight line between the wells with the highest head and the lowest head, identify the location of the same head of the well from Step 1. Note that this is accomplished by locating the elevation of 32.55 m between Well 2 and Well 3 in the graphical solution.

**Step 3:** Draw a straight line between the intermediate well from Step 1 and the point identified in Step 2. This is a segment of the equipotential line along which the total head is the same as that in the intermediate well (i.e., equipotential line of 32.55 m head in this case).

**Step 4:** Draw a line perpendicular to the equipotential line passing through the well with the lowest head. The hydraulic gradient is the slope of that perpendicular line. Also, the direction of the line indicates the direction of groundwater movement. The graphical procedure above is illustrated in Figure 3.6.7b. The hydraulic gradient is then computed as

\[ i = \frac{32.55 \text{ m} - 32.41 \text{ m}}{115.93 \text{ m}} = 0.0012 \]
The average daily discharge from the Patuxent Formation (see Figure 3.6.8) in the Sparrows Point district of Baltimore, Maryland, in 1945 was estimated as $1 \times 10^6 \text{ ft}^3/\text{day}$. A flow net of the region is constructed using the available contour lines as shown in Figure 3.6.8. (This example is adapted from Lohman.⁶⁶) Compute the transmissivity of the regional aquifer.

**SOLUTION**

As shown in the flow net, there are 15 flow channels, hence $m = 15$. There are four equipotential drops from the 60-ft contour line to the 20-ft contour line, so $h = 40 \text{ ft}$ and $n = 4$. Then the overall transmissivity of the district can be computed using Equation 3.6.13:

$$T = \frac{nQ}{mh} = \frac{(4)(1\times10^6 \text{ ft}^3/\text{day})}{(15)(40 \text{ ft})} = 6700 \text{ ft}^2/\text{day}$$

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![Map of Baltimore industrial area, Maryland, showing potentiometric surface in 1945 and generalized flow lines in the Patuxent Formation. From Bennett and Meyer (as presented in Lohman).](image)
Flow Across a boundary of different perm.-

From Continuity normal compts. of flow approaching & leaving the boundary must be equal
$V_{n1} = V_{n2}$

$V_1 \cos \theta_1 = V_2 \cos \theta_2$

$\theta_1, \theta_2$ – angle w/ normal
must be equal in the two regions

\[ b = \text{distance along boundary for stream tube} \]

\[ dL_1 = dL_2 \]

\[ \theta_1 = \theta_2 \]

\[ \sin \theta_1 = \sin \theta_2 \]
Since $dh_1 = dh_2$ between two equipotential lines

\[ \tan(\theta) = \tan(\alpha) \]

Refraction of flow lines occurs from one region to another region.
\[ \frac{dL_1}{dL_2} = \frac{\sin \Theta_1}{\sin \Theta_2} \]

\[ K_1 \frac{dL_1}{dL_2} \frac{\cos \Theta_1}{\sin \Theta_2} = K_2 \frac{dL_2}{dL_2} \frac{\cos \Theta_2}{\sin \Theta_1} \]

\[ K_1 \tan \Theta_2 = K_2 \tan \Theta_1 \]
EXAMPLE 3.6.4

Consider a case where a leaky confined aquifer with 4.5 m/day horizontal hydraulic conductivity is overlain by an aquitard with 0.052 m/day vertical hydraulic conductivity. If the flow in the aquitard is in the downward direction and makes an angle of $5^\circ$ with the vertical (see Figure 3.6.12), determine $\theta_2$.

SOLUTION

Given $K_1 = 0.052$ m/day, $K_2 = 4.5$ m/day, and $\theta_1 = 5^\circ$, Equation 3.6.25 is used to compute $\theta_2$:

$$\frac{K_1}{K_2} \cdot \frac{\tan \theta_1}{\tan \theta_2} = \frac{0.052 \text{ m/day}}{4.5 \text{ m/day}} = \frac{\tan(5^\circ)}{\tan \theta_2} \rightarrow \theta_2 = 82.5^\circ$$

The flow lines become nearly horizontal as they enter into the confined aquifer. This is a typical case for regional flow systems, as the hydraulic conductivity of a confined aquifer is generally a few orders of magnitude larger than that of the confining layers.

Figure 3.6.12. Example 3.6.4
Perm. in Unsat. Flow –

Darcy's law applicable in unsat. Flow, with a different perm., which is a function of water content.

\[ K_u = K_s (0.6) \]

\[ S_s = \text{degree of saturation} \]
\[ S_0 = \text{threshold sat. (20%)} \]
\[ \alpha = \text{porosity} \]
\[ c_d = \frac{K_u}{\gamma m} - \frac{K_u}{S_s} \]

- that part of voids which is filled with non-moving water.
Using Darcy’s Law –

\[ Q = A \left( \frac{k \gamma}{\mu} \right) \frac{dh}{dL} \]

\[ k = \frac{\mu Q}{A} = \frac{\mu Q}{A} \frac{\gamma (dh/dL)}{(dp/dL)} \]

Value of \( k \) in cm² or ft² is very small; so a large unit darcy used in Petroleum eng. Groundwater hydrology.
1 darcy = \frac{1 \text{ centipoise}}{(1 \text{ cm}^3/\text{s})/1\text{ cm}^2} (1 \text{ atm.}/1 \text{ cm})

1 \text{ centipoise} = 0.01 \text{ poise} = 0.01 \text{ dyne-sec/cm}^2

1 \text{ atm} = 1.10132 \times 10^6 \text{ dynes/cm}^2

thus,

1 \text{ darcy} = 0.987 \times 10^{-8} \text{ cm}^2

= 1.062 \times 10^{-11} \text{ ft}^2