River Mechanics
CH-6
Hydrodynamics of Fluid-Particle Systems

What we know already…

- Water flow over mobile bed entrains sediment
  - Movement of sediment modifies the flow, and also the channel bed:
    - Elevation, Roughness, and Slope
  - Two phases:
    - Liquid Phase (Mixture)
    - Solid Phase
  - Coupled flow and transport problem!
    - Need to review some properties of sediment important in sediment transport…

Sediment Grade Scale

Table 2.1 Sediment Grade Scale

Class Name (1)	Size Range: Millimeters (2)	Size Range: Millimeters (3)	Size Range: Microns (4)	Size Range: Inches (5)	Approximate Sieve Mesh Openings Per Inch: Tyler (6)	Approximate Sieve Mesh Openings Per Inch: United States Standard (7)
Very large boulders		4,096–2,048		160–80		
Large boulders		2,048–1,024		80–40		
Medium boulders		1,024–512		40–20		
Small boulders		512–256		20–10		
Large cobbles		256–128		10–5		
Small cobbles		128–64		5–2.5		
Very coarse gravel		64–32		2.5–1.3		
Coarse gravel		32–16		1.3–0.6		
Medium gravel		16–8		0.6–0.3	2–1/2	
Fine gravel		8–4		0.3–0.16	5	5
Very fine gravel		4–2		0.16–0.08	9	10
Very coarse sand	2–1	2.000–1.000	2,000–1,000		16	18
Coarse sand	1–1/2	1.000–0.500	1,000–500		32	35
Medium sand	1/2–1/4	0.500–0.250	500–250		60	60
Fine sand	1/4–1/8	0.250–0.125	250–125		115	120
Very fine sand	1/8–1/16	0.125–0.062	125–62		250	230
Coarse silt	1/16–1/32	0.062–0.031	62–31			
Medium silt	1/32–1/64	0.031–0.016	31–16			
Fine silt	1/64–1/128	0.016–0.008	16–8			
Very fine silt	1/128–1/256	0.008–0.004	8–4			
Coarse clay	1/256–1/512	0.004–0.0020	4–2			
Medium clay	1/512–1/1,024	0.0020–0.0010	2–1			
Fine clay	1/1,024–1/2,048	0.0010–0.0005	1–0.5			
Very fine clay	1/2,048–1/4,096	0.0005–0.00024	0.5–0.24			

Size Frequency Distribution

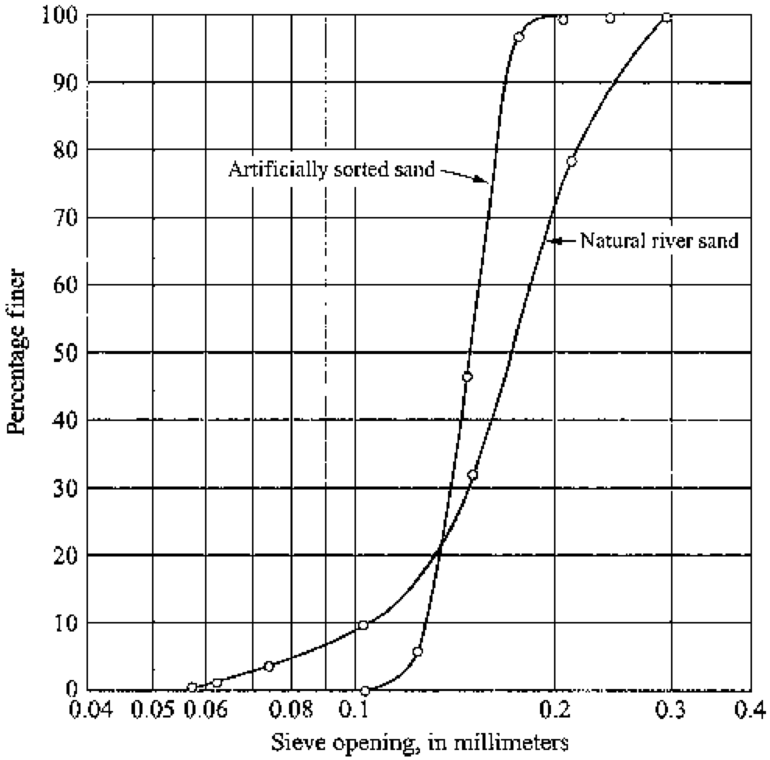


Figure 2.12 Cumulative semilogarithmic size-frequency graphs for two sands.

$$d_{50}$$

$$Geometric\,Mean = d_g = \sqrt{d_{84} d_{16}}$$

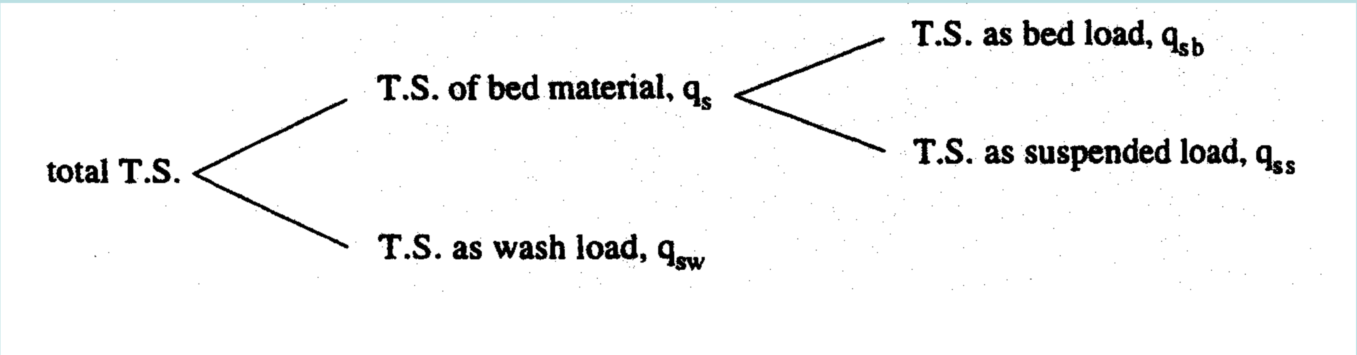
$$Geometric\,Deviation = \sigma_g = \sqrt{d_{84} / d_{16}}$$

Sediment Motion

- At velocities just above critical, grains will begin to roll and slide intermittently along the bed:
  - Material being moved called **"contact load"**
- At higher velocity, grains will make short jumps:
  - Leaves the bed for a short instant of time and returning either to come to rest or to continue in motion on bed
  - Material being moved in this manner called **"saltation load"**
- Further increases in flow velocity lead to more frequent jumps and some grains will be swept into flow and kept in suspension for appreciable lengths
  - Material being moved in this manner called **"suspended load"**

Classification of Sediment Load

- Three modes of transport: Suspension, saltation, and rolling/sliding on bed
  - Occur simultaneously
  - Difficult to separate them, such as the difference between saltation vs. contact load and saltation vs. suspension
  - Difficulties are avoided by introducing the following terms:
    - **Bed Load** – material moving on or near the bed
    - **Total Bed-Material Load** or **Bed Sediment Load (q_s)** – **bed load** (q_{sb}) + **suspended load (q_{ss})**
    - **Wash Load** – sediment that never comes into contact with the bed



Classification of Sediment Load

- Imprecise but estimates of mode of transport (Graf, 1971):
  - **Bed Load** begins at $u_*/v_{ss} > 0.10$
  - **Suspended load** begins at $u_*/v_{ss} > 0.40$

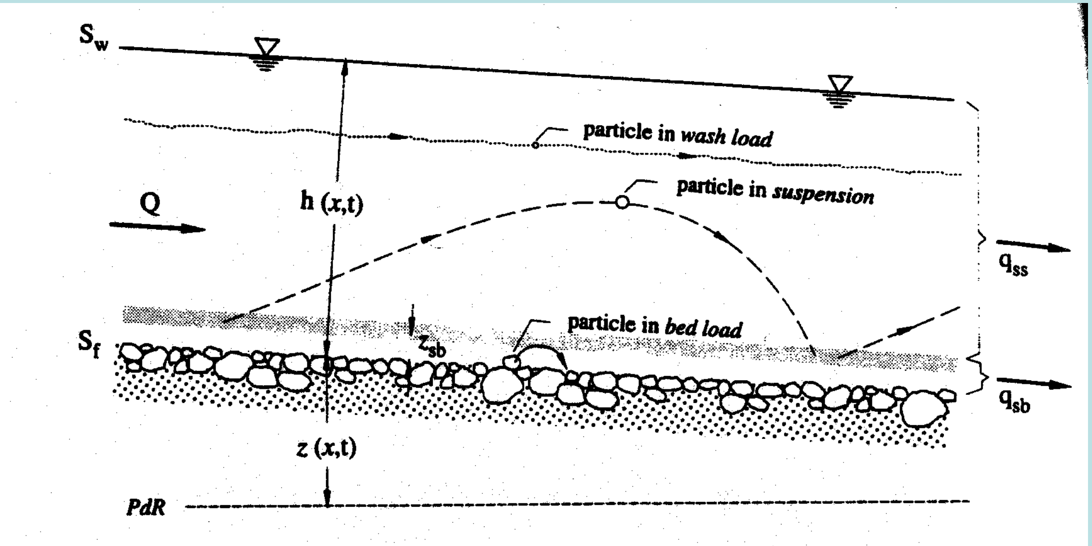


Fig. 6.1 Scheme of the modes of transport.

Particle Motion with Linear Resistance – Low Re Flow

- Based on Newton's 2nd Law
- Resulting equation known as BBO equation (Basset, Boussinesq, and Oseen) – Actually derived by Tchen (1947):
  - Velocity field of infinite extent, no mutual interaction between particles, no particle rotation

$$\frac{4\pi a^3}{3}\rho_s \frac{dv_s}{dt} = \frac{4\pi a^3}{3}\rho \frac{dv}{dt} - \frac{2\pi a^3}{3}\rho\left(\frac{dv_s}{dt} - \frac{dv}{dt}\right)$$

$$-6\pi\mu a\left[\left(v_s - v\right) + \frac{a}{\sqrt{\pi\mu/\rho}}\int_{t_o}^{t} dt_1 \frac{\frac{dv_s(t_1)}{dt} - \frac{dv(t_1)}{dt}}{\sqrt{t - t_1}}\right]$$

$$-\frac{4\pi a^3}{3}\left(\rho_s - \rho\right)g$$

Particle Motion with Linear Resistance – Larger Re Flow

- BBO equation no longer valid as resistance becomes proportional to square of velocity:
  - No theoretically sound approach for derivation of equation at high Re
  - Modification of the slow-motion equation

$$\frac{4\pi a^3}{3}\rho_s\frac{dv_s}{dt}=\frac{4\pi a^3}{3}\rho\frac{dv}{dt}-k\frac{4\pi a^3}{3}\rho\left(\frac{dv_s}{dt}-\frac{dv}{dt}\right)$$

$$-C_D a^2\pi\frac{\rho\left(v_s-v\right)^2}{2}-\frac{6\pi\mu a^2}{\sqrt{\pi\mu/\rho}}\int_{t_o}^{t}dt_1\frac{\frac{dv_s(t_1)}{dt}-\frac{dv(t_1)}{dt}}{\sqrt{t-t_1}}$$

$$-\frac{4\pi a^3}{3}\left(\rho_s-\rho\right)g$$

Usually consider case of steady-state motion…

- Both equations from before are significantly simplified…

$$0 = 0 - 0 - C_D a^2 \pi \frac{\rho (v_s - v)^2}{2} - 0 - \frac{4\pi a^3}{3} (\rho_s - \rho) g$$

$$C_D a^2 \pi \frac{\rho (v_s - v)^2}{2} = \frac{4\pi a^3}{3} (\rho_s - \rho) g$$

  - Need expressions of the drag coefficient to solve this equation…
  - Difficulty is that the drag coefficient is a function of the Re:

$$\mathrm{Re}_p = \frac{2a(v_s - v)}{\mu / \rho}$$

Usually consider case of steady-state motion…

- For $Re_p<0.1$ → $R = 3\pi\mu d v_s$ and $C_D = 24/Re_p$

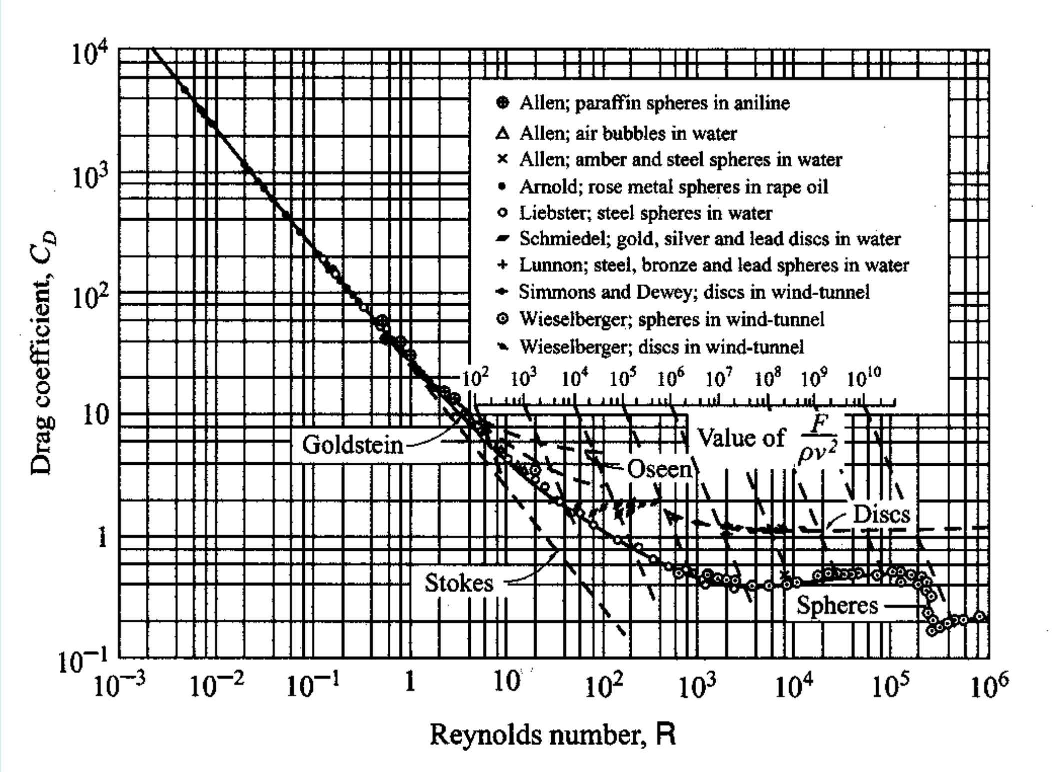
$$v_{ss} = \frac{gd^2(\rho_s - \rho)}{18\mu}$$

- Over the entire range of Re_p:

$$v_{ss}^{\ 2} = \frac{4}{3}\frac{gd}{C_D}\left(\frac{\gamma_s - \gamma}{\gamma}\right)$$

Drag Coefficient vs. Re_p

– Applies to smooth, non-rotating spheres moving in a fluid free of disturbances with constant relative velocity…



Drag Coefficient vs. Re_p

– To use this chart with the C_D-Re_p axes requires trial and error

- Instead you can use the submerged weight of the sphere:

$$F = \frac{\pi d^3}{6}\left(\gamma_s - \gamma\right)$$

- Calculate $F/\rho\nu^2$ (ν = kinematic viscosity)
- Locate ratio on auxiliary scale, move parallel to the sloping lines to the C_D-Re_p curve and read Re_p

Drag Coefficient vs. Re_p

– There are also approximate results generally valid up to $Re_p = 2$…

- Oseen (1927):

$$C_D = \frac{24}{\mathrm{Re}_p}\left(1 + \frac{3}{16}\mathrm{Re}_p\right)$$

- Goldstein (1929):

$$C_D = \frac{24}{\mathrm{Re}_p}\left(1 + \frac{3}{16}\mathrm{Re}_p - \frac{19}{1{,}280}\mathrm{Re}_p^{\,2} + \frac{71}{20{,}480}\mathrm{Re}_p^{\,3}\right)$$

Drag Coefficient vs. Re_p

– Empirical formulas for $Re_p > 2$…

- Schiller et al. (1933):

$$C_D = \frac{24}{\mathrm{Re}_p}\left(1 + 0.150\,\mathrm{Re}_p^{\,0.687}\right)$$

- Olson (1961) for $Re_p < 100$:

$$C_D = \frac{24}{\mathrm{Re}_p}\left(1 + \frac{3}{16}\mathrm{Re}_p\right)^{1/2}$$

Extending to More Difficult Conditions…

- Influence of various effects which complicate the problem (particle shape, boundary effects, multiparticle influences, particle rotation and roughness, turbulence)
- McNown (1951) proposed use of a Stokes number (*K*) to quantify departure from earlier case for $Re_p < 0.1$:

$$F = \frac{\pi d_n^{\,3}}{6}\left(\gamma_s - \gamma\right) = K\left(3\pi\mu v_s d_n\right) = K\left(6\pi\mu v_s a\right)$$

- For higher Re_p, usually simply assume the C_D accounts for complicating effects

Particle Shape

- Up to this point we have only considered spherical particles (not irregular shapes)
- Analytical solutions only exist for low-Reynolds number flow
- McNown et al. (1950) suggested shape factor using a, b, and c lengths of perpendicular axes (“b” is the maximum length, sediment falls in the direction of “a”):

$$SF = \frac{a}{\sqrt{bc}}$$

- For low Reynolds numbers (<0.1), coefficient K is equal to the ratio of the fall velocity of a sphere with the same volume and weight as the particle to the fall velocity of the particle

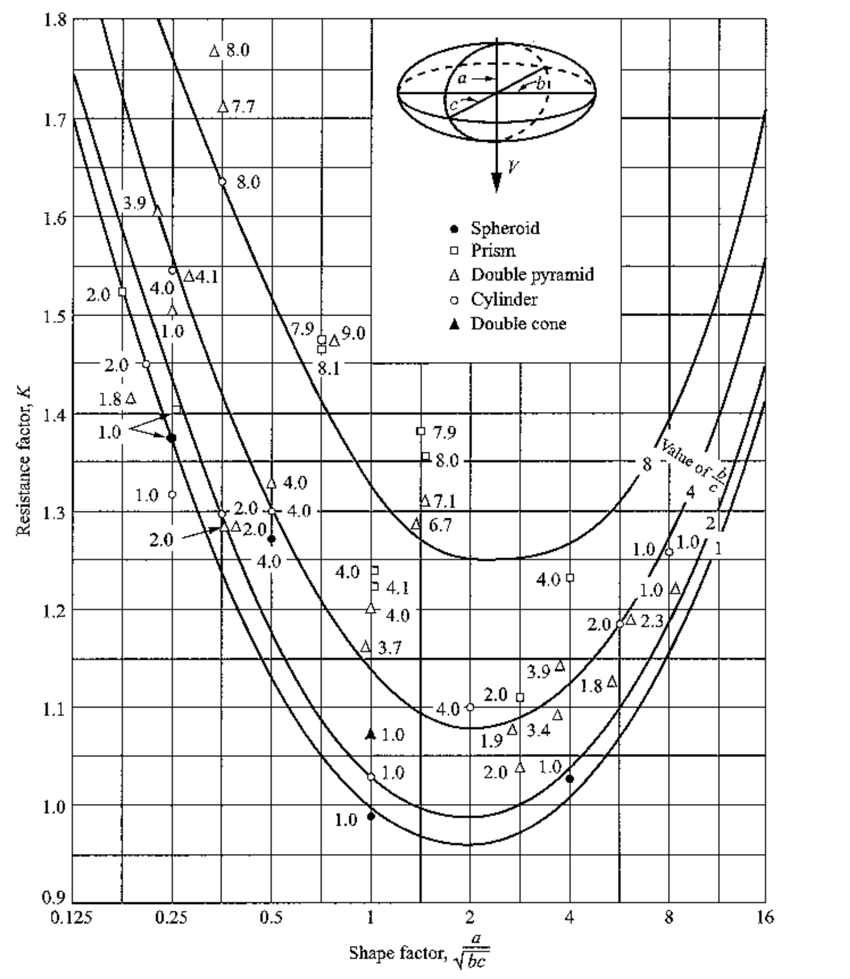


Figure 2.3 Comparison of theoretical values of *K* for ellipsoids and observed values for ellipsoids and several other shapes for Reynolds numbers less than 0.1 (McNown, et al., 1951).

Particle Shape

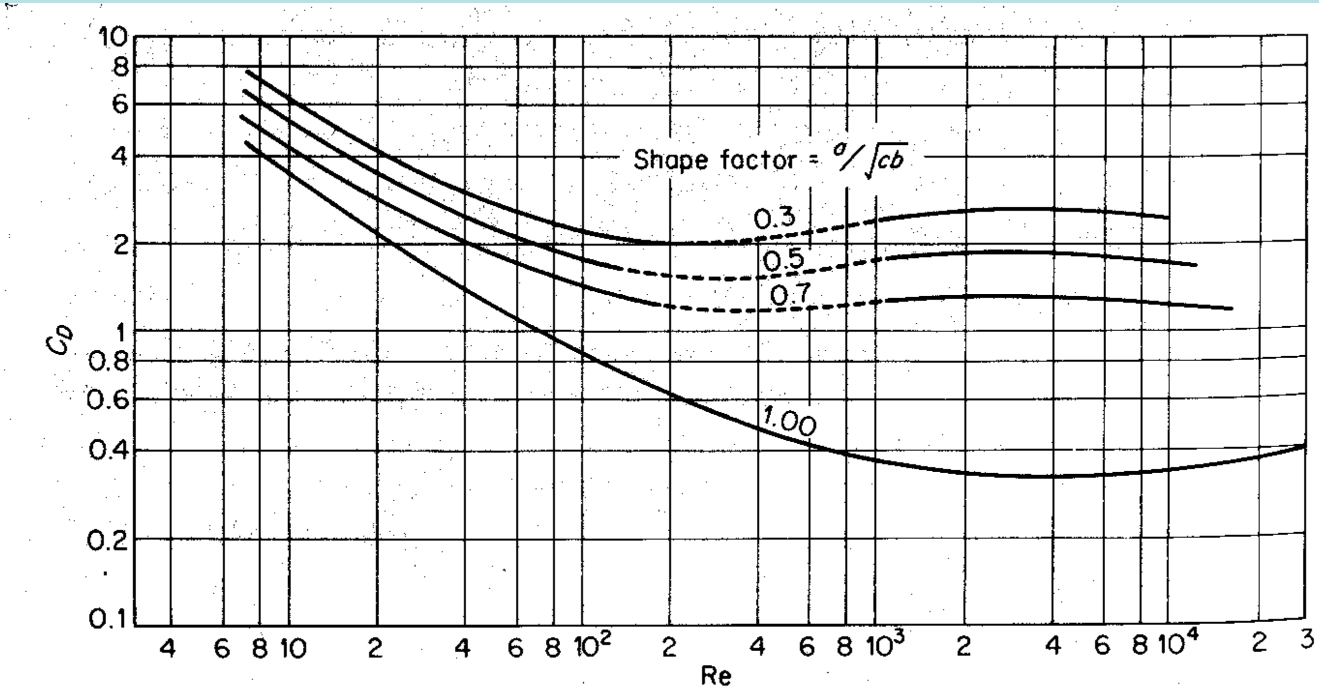


Fig. 4.9 Drag coefficient vs. Reynolds number for different shape factors. [*After* ALBERTSON (*1953*).]

Particle Concentration

- Settling velocity will differ due to mutual interference of particles
  - For a few closely spaced particles, it has been observed that the settling velocity increases (less drag)
  - For particles dispersed throughout the fluid, interference will reduce settling velocity – called **hindered settling**
  - McNown and Lin (1952): uniform quartz sphere (no flocculation) experiments for $Re_p < 2.0$
  - Even for moderate concentrations, the correction in the settling velocity becomes significant

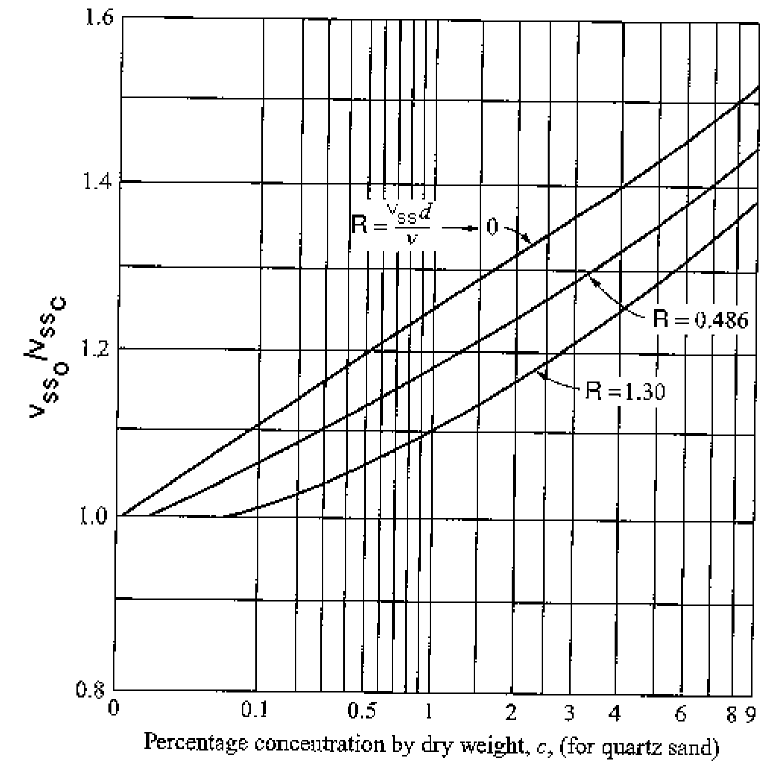


Figure 2.5 Effect of concentration on fall velocity of uniform quartz spheres (McNown and Lin, 1952).

Particle Concentration

- Maude et al. (1958) – empirical equation proposed to be valid regardless of the Reynolds number

$$K = (1 - C_s)^{-m}$$

$$C_s = concentration$$

$$For\ \mathrm{Re}_p < 1.0 \rightarrow m = 4.5$$

$$For\ \mathrm{Re}_p > 10^4 \rightarrow m = 2.2$$

Flow of a Mixture

- Newtonian – if volumetric concentration of the particles is very small, C_s<<1%
  - Difference between density of mixture and fluid, $\Delta\rho$ << 16 kg/m^3
  - Transport as bed load and suspended load
  - Most often encountered (or assumed) in rivers and streams

Flow of a Mixture

- Quasi-Newtonian – if volumetric concentration of the particles remains small, $C_s<8\%$
  - Difference between density of mixture and fluid important, $\Delta\rho < 130$ kg/m^3
  - Transport of sediments as a concentrated suspension

Flow of a Mixture

- Non-Newtonian – if volumetric concentration of the particles exceeds C_s>8%
  - Difference between density of mixture and fluid important, $\Delta\rho > 130$ kg/m^3
  - Need to modify all concepts of Newtonian hydraulics (resistance to flow, distribution of velocity, settling velocity)
  - Sometimes called hyperconcentrated suspensions that occur when enormous quantities of sediment enter small sloped channels due to extensive rainfall events

Fluid Turbulence

- Experimental evidence that spherical particles settle more slowly in a fluid with vertical turbulence
  - Reduction in fall velocity due to nonlinear interaction relation between drag and velocity relative to fluid

Example

Assuming the criterion for suspended load transport is $u_*/v_{ss} > 0.40$ and bed load transport is $u_*/v_{ss} > 0.10$, determine the flow depth at which bed load and suspended load transport commences in a rectangular channel (B = 5 m, $S_f = 0.001$, n = 0.02). Assume uniform, steady flow. Also assume spherical particles with d_{50} = 2.0 mm and neglect bed form roughness.

Graf's Classification of Sediment Transport Problems

- Determination of sedimentological rating curve, $q_s = f(q)$ for given cross-section
- Determination of stability of bed in a given cross-section
- Determination of the stability of the channel slope (aggradation and degradation) in a given reach

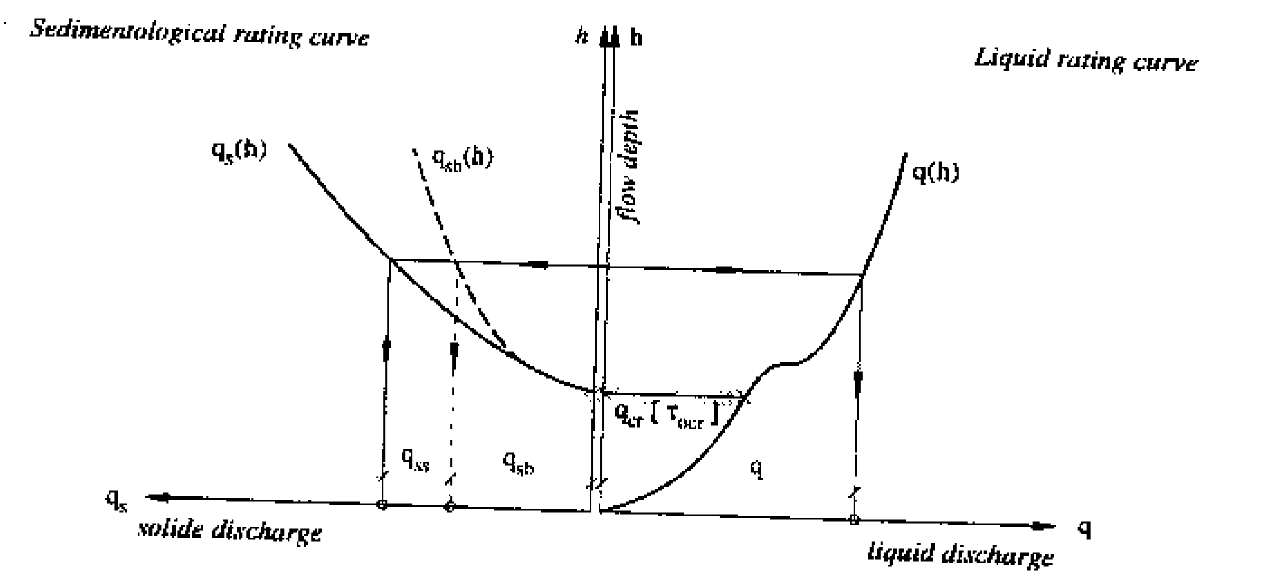


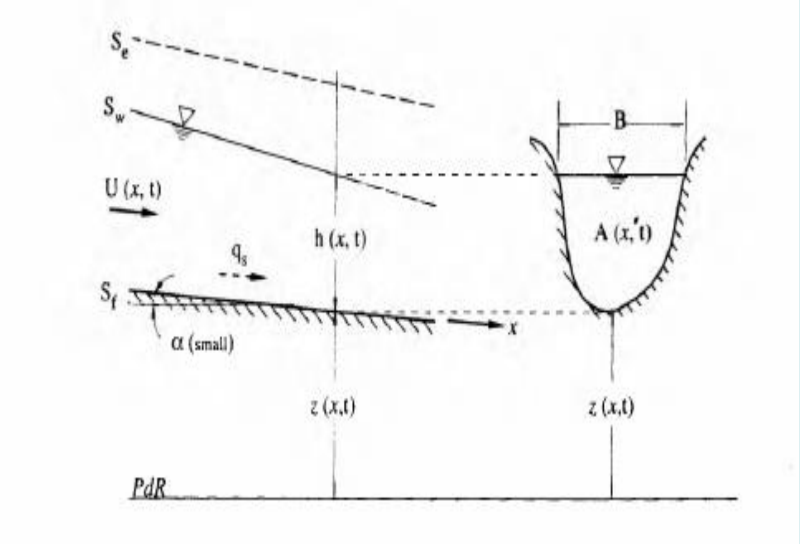
Fig. 6.2 Rating curves for the liquid discharge and the solid discharge.

Graf’s Classification of Sediment Transport Problems

- Formulas used to calculate q_s predict the “capacity” for sediment transport
  - If capacity is larger than supply – net erosion and transport occurs
  - If supply is larger than capacity – deposition and transport occurs
  - If supply = capacity, then transport without erosion or deposition
  - If bed is armoured, capacity may not be satisified

What we know already…

- Saint-Venant equations for unsteady and non-uniform flow over a fixed bed in a prismatic channel with a small slope…



$$\frac{\partial h}{\partial t}+h\frac{\partial U}{\partial x}+U\frac{\partial h}{\partial x}=0 \qquad B=constant$$

$$\frac{\partial U}{\partial t}+U\frac{\partial U}{\partial x}+g\frac{\partial h}{\partial x}+g\frac{\partial z}{\partial x}=-gS_e$$

  - For mobile bed:
    - Se becomes a function of the friction coefficient for a mobile bed (*f*' and *f*'')

$$S_e=f\left(f',f'',U,h\right)$$

Extending this to mobile bed…

– For mobile bed:

- Elevation over a mobile bed may vary, z(x,t)
- Exner expressed this change in bed elevation using the following relationship: $$\frac{\partial z}{\partial t} = -a_E \frac{\partial U}{\partial x}$$
- This relationship is usually expanded using the continuity equation for the solid (particle) phase:

$$\frac{\partial z}{\partial t} + \left(\frac{1}{1-p}\right)\frac{\partial q_s}{\partial x} = 0$$

$$p = porosity = \frac{V_w}{V_T}$$

$$q_s = C_s U h = solid\ discharge\ per\ unit\ width\ (volume)$$

$$C_s = volume\ concentration\ of\ solid\ phase = \frac{V_s}{V_m}$$

Extending this to mobile bed…

– For mobile bed:

- Note that the solid discharge (q_s) is a function of the liquid discharge – sedimentological rating curve

$$\frac{\partial h}{\partial t} + h\frac{\partial U}{\partial x} + U\frac{\partial h}{\partial x} = 0 \qquad B = constant$$

$$\frac{\partial U}{\partial t} + U\frac{\partial U}{\partial x} + g\frac{\partial h}{\partial x} + g\frac{\partial z}{\partial x} = -gS_e$$

$$\frac{\partial z}{\partial t} + \left(\frac{1}{1-p}\right)\frac{\partial q_s}{\partial x} = 0 \leftarrow \quad q_s = f\left(U, h, sediment\right)$$

$$3\,Unknowns: \quad U(x,t),\ h(x,t),\ z(x,t)$$

$$2\,Unknowns\,(Semi-Empirical): \ S_e,\ q_s$$

$$Independent\ Variables: \ x,t$$

Equations of Saint-Venant-Exner

Flow over Mobile Bed

$$\frac{\partial h}{\partial t} + h\frac{\partial U}{\partial x} + U\frac{\partial h}{\partial x} = 0 \qquad B = constant$$

$$\frac{\partial U}{\partial t} + U\frac{\partial U}{\partial x} + g\frac{\partial h}{\partial x} + g\frac{\partial z}{\partial x} = -gS_e$$

$$S_e = f\left(f', f'', U, h\right)$$

$$\frac{\partial z}{\partial t} + \left(\frac{1}{1-p}\right)\frac{\partial q_s}{\partial x} = 0$$

$$q_s = f\left(U, h, sediment\right)$$

Transport: Erosion and Deposition

Analytical Solutions

- Saint-Venant-Exner equations are non-linear and hyperbolic – impossible to derive analytical solutions:
  - Solutions are possible if you assume:
    1. Quasi-Steady Flow – flow at small Froude numbers (Fr<0.6):
       * Variation in liquid discharge is **short-term**
       * Variation in bed elevation is **long-term**
       * Bed changes occur after variation in discharge
       * Flow can be assumed constant
       * Solutions for this assumption allow us to analyze for long-term bed elevation changes
    2. Quasi-uniform flow:

$$\frac{\partial U}{\partial x} = 0$$

Quasi-Steady/Uniform Flow

- Simplified Saint-Venant-Exner equations:

$$g\frac{\partial z}{\partial x} = -gS_e = -g\frac{U^2}{C^2 h} = -g\frac{U^3}{C^2 q} \qquad q = Uh$$

$$g\frac{\partial^2 z}{\partial x^2} = -g\frac{\partial}{\partial x}\left(\frac{U^3}{C^2 q}\right)$$

$$\frac{\partial^2 z}{\partial x^2} = -3\left(\frac{U^2}{C^2 q}\right)\frac{\partial U}{\partial x}$$

$$\frac{\partial U}{\partial x} = -\frac{1}{3}\left(\frac{C^2 h}{U}\right)\frac{\partial^2 z}{\partial x^2}$$

Quasi-Steady/Uniform Flow

- Simplified Saint-Venant-Exner equations:

$$(1-p)\frac{\partial z}{\partial t}-\frac{\partial q_s}{\partial U}\frac{1}{3}\left(\frac{C^2 h}{U}\right)\frac{\partial^2 z}{\partial x^2}=0$$

$$\frac{\partial z}{\partial t}-K(t)\frac{\partial^2 z}{\partial x^2}=0 \quad \leftarrow K(t)=\frac{1}{3}\frac{\partial q_s}{\partial U}\frac{1}{(1-p)}\frac{C^2 h}{U}$$

  – Model is limited to large values of x and t, $x>3h/S_e$, and $Fr < 0.6$:

$$K=K_o=\frac{1}{3}\frac{\partial q_s}{\partial U}\frac{1}{(1-p)}\frac{U_o}{S_{e_o}}$$

Where *K* is the coefficient of diffusion

Quasi-Steady/Uniform Flow

- For now we can use a power-law expression for the solid discharge:

$$q_s = a_s U^{b_s}$$

$$K = K_o = \frac{1}{3} b_s q_s \frac{1}{(1-p)} \frac{1}{S_{e_o}}$$

Quasi-Steady/Uniform Flow

- We can use these solutions to explain the long-term evolution of the channel bed:
  - Degradation
    - Supply of solid discharge is reduced at the upstream
    - Liquid discharge is increased
    - Lowering of a fixed point on the channel bed at the downstream
  - Aggradation:
    - Supply of solid discharge is increased at the upstream
    - Liquid discharge is decreased
    - Raising a fixed point on the channel bed at the downstream

Quasi-Steady/Uniform Flow

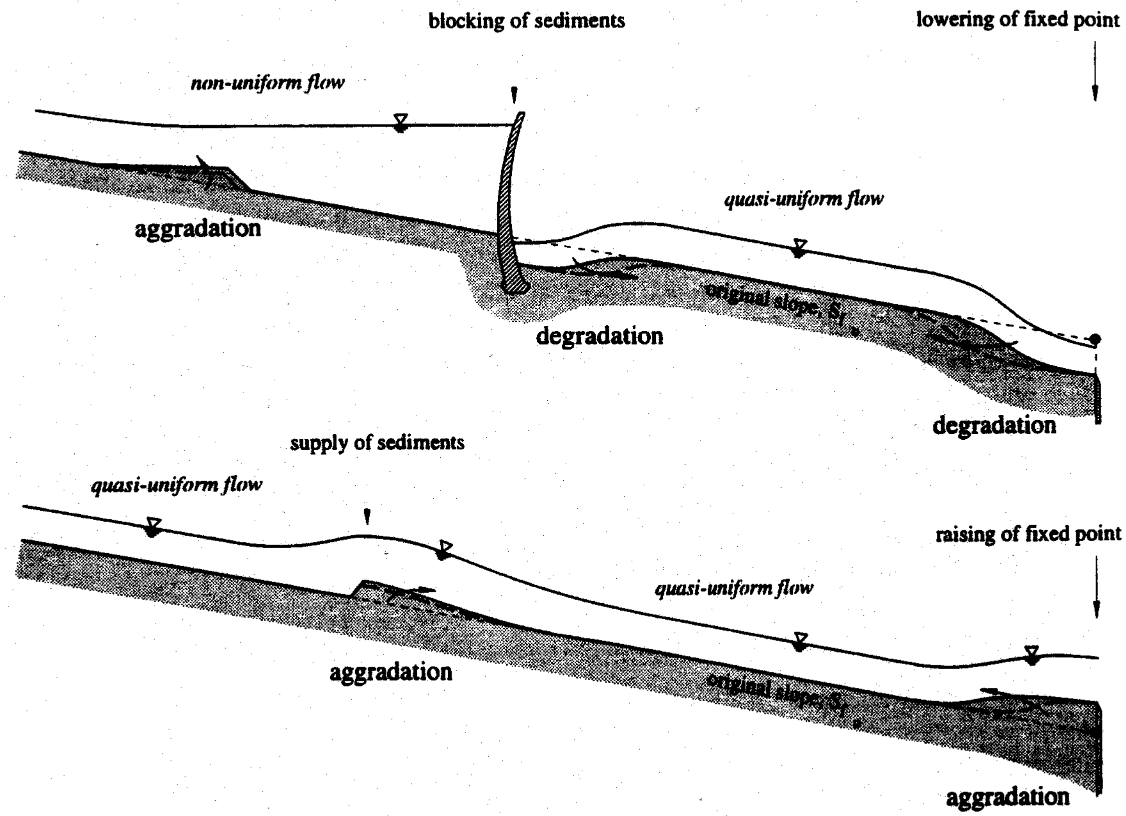


Fig. 6.4 Scheme of a degradation or an aggradation.

Degrading Channel Example

- Consider the following scenario:
  - Channel with mobile bed with uniform flow at depth, $h = h^o$
  - Discharge enters into a reservoir whose water level is lowered by Δh_w
  - Causes a lowering of the fixed bed of Δh
  - Degradation of the bed is initiated
  - After long time, flow depth will return to h^o
  - During degradation, flow depth and discharge remain quasi-constant

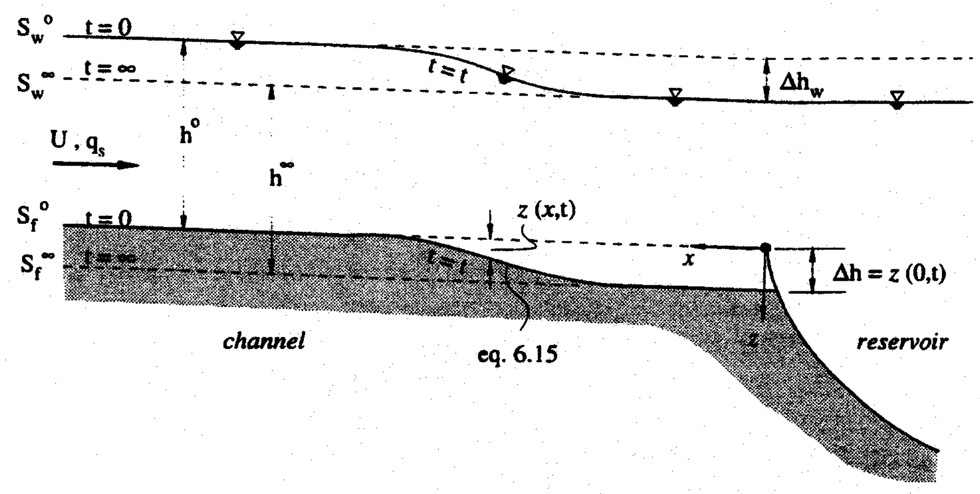


Fig. 6.5 Degradation due to lowering, Δh, of the fixed point on the bed.

Degrading Channel Example

- Note position of x and z axes
- Initial and boundary conditions:

$z(x,0)=0$, $z(0,t)=\Delta h$, lim (x→infinity) $z(x,t)=0$

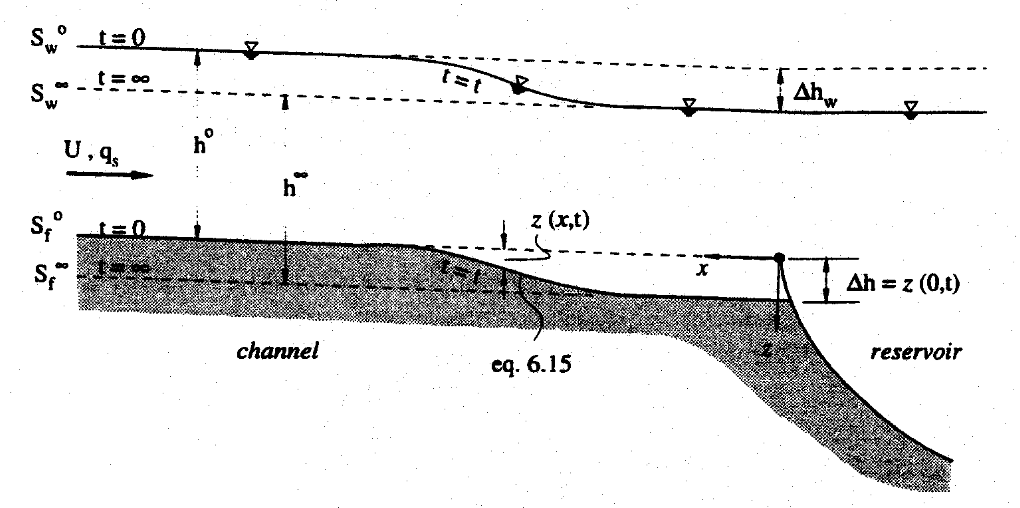


Fig. 6.5 Degradation due to lowering, Δh, of the fixed point on the bed.

Degrading Channel Example

- For these boundary conditions!!!!
  - Solution: $$z(x,t) = \Delta h \, erfc\left(\frac{x}{2\sqrt{Kt}} \right)$$

$$erfc(Y) = \frac{2}{\pi} \int_{Y}^{\infty} e^{-\xi^2} d\xi$$

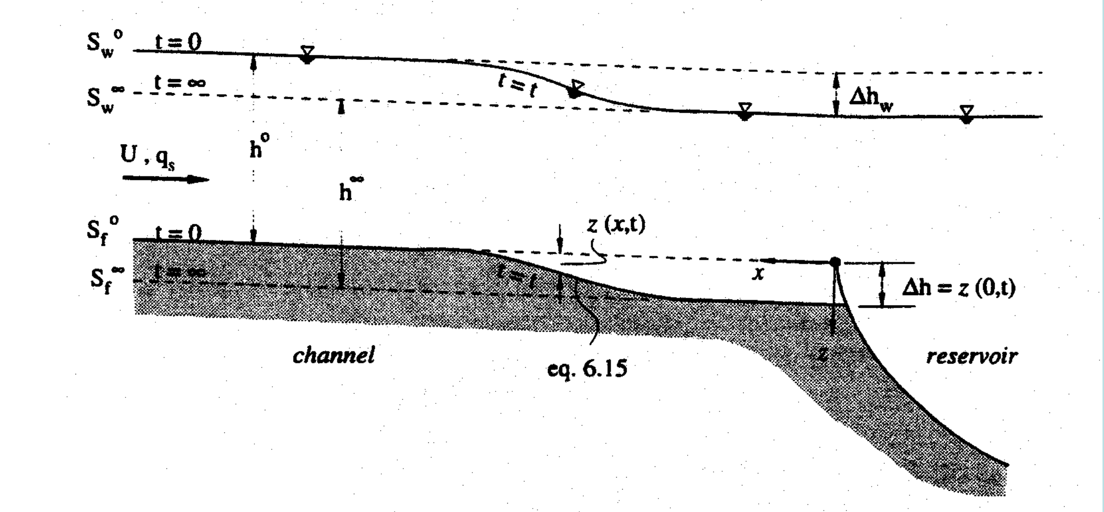


Fig. 6.5 Degradation due to lowering, Δh, of the fixed point on the bed.

Table for the complementary error function

(see *Handbook of Mathematical Functions*, 1964, National Bureau of Standards, pp. 310-311, formula 7.1.28)

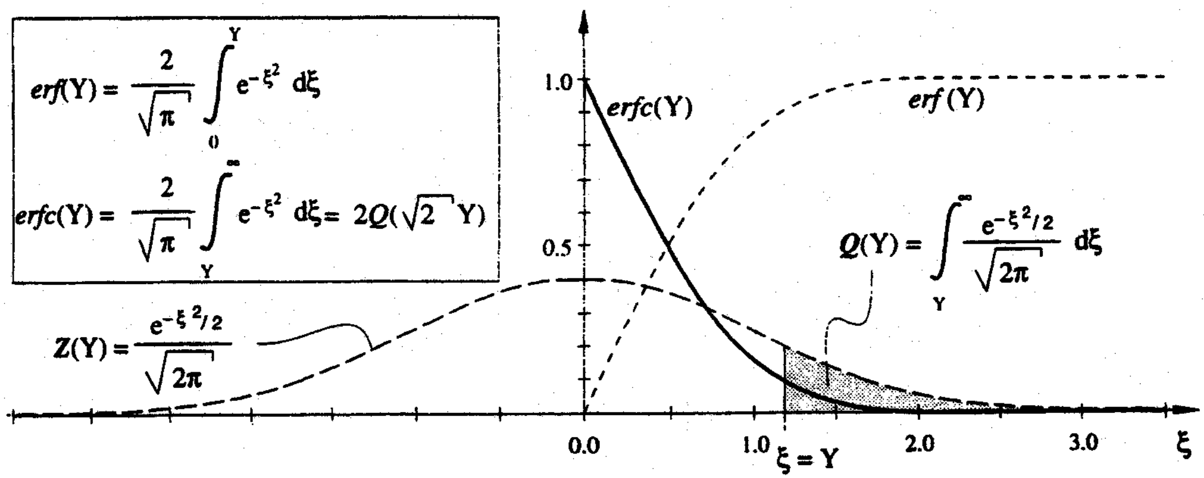
Y	*erfc*(Y)
0.00	*1.00000*
0.10	*0.88754*
0.20	*0.77730*
0.30	*0.67137*
0.40	*0.57161*
0.50	*0.47950*
0.60	*0.39614*
0.70	*0.32220*
0.80	*0.25790*
0.90	*0.20309*
1.00	*0.15730*
1.10	*0.11979*

Y	*erfc*(Y)
1.20	*0.08969*
1.30	*0.06599*
1.40	*0.04772*
1.50	*0.03390*
1.60	*0.02365*
1.70	*0.01621*
1.80	*0.01091*
1.90	*0.00721*
2.00	*0.00468*
2.10	*0.00298*
2.20	*0.00186*

Y	*erfc*(Y)
2.30	*0.00114*
2.40	*0.00069*
2.50	*0.00041*
2.60	*0.00024*
2.70	*0.00013*
2.80	*0.00008*
2.90	*0.00004*
3.00	*0.00002*
3.10	*0.00001*
3.20	*0.00001*
3.30	*0.00000*

$$erf(Y) = \frac{2}{\sqrt{\pi}} \int_0^Y e^{-\xi^2}\, d\xi$$

$$erfc(Y) = \frac{2}{\sqrt{\pi}} \int_Y^\infty e^{-\xi^2}\, d\xi = 2Q(\sqrt{2}\,Y)$$



The complementary error function can be calculated approximately using the following expression :

$$erfc(Y) = 1 / (1 + a_1 Y + a_2 Y^2 + a_3 Y^3 + a_4 Y^4 + a_5 Y^5 + a_6 Y^6)^{16} + \varepsilon(Y) \quad \text{where} \quad |\varepsilon(Y)| \le 3 \cdot 10^{-7}$$

$a_1 = 0.0705230784$; $a_2 = 0.0422820123$; $a_3 = 0.0092705272$

$a_4 = 0.0001520143$; $a_5 = 0.0002765672$; $a_6 = 0.0000430638$

Degrading Channel Example

- For these boundary conditions!!!!
  - Graf also discusses the time ($t_{50\%}$) when the bed elevation is lowered 50% with respect to the final elevation:

$$\frac{z(x,t)}{\Delta h} = 0.5 = erfc\left(\frac{x_{50\%}}{2\sqrt{Kt_{50\%}}}\right)$$

$$erfc(0.48) = 0.5 \rightarrow \frac{x_{50\%}}{2\sqrt{Kt_{50\%}}} = 0.48$$

$$x_{50\%} = 0.48\left(2\sqrt{Kt_{50\%}}\right)$$

$$t_{50\%} \approx \frac{{x_{50\%}}^2}{0.96^2 K}$$

Aggrading Channel Example

- Consider the following scenario:
  - Channel with a mobile bed having uniform flow
  - Particular cross section is overloaded with sediment: Δq_s is increased
  - Aggradation of the channel bed will occur
  - After some time Δt, elevation of the bed and water surface will increase by Δh
  - During aggradation, the discharge remains quasi-steady

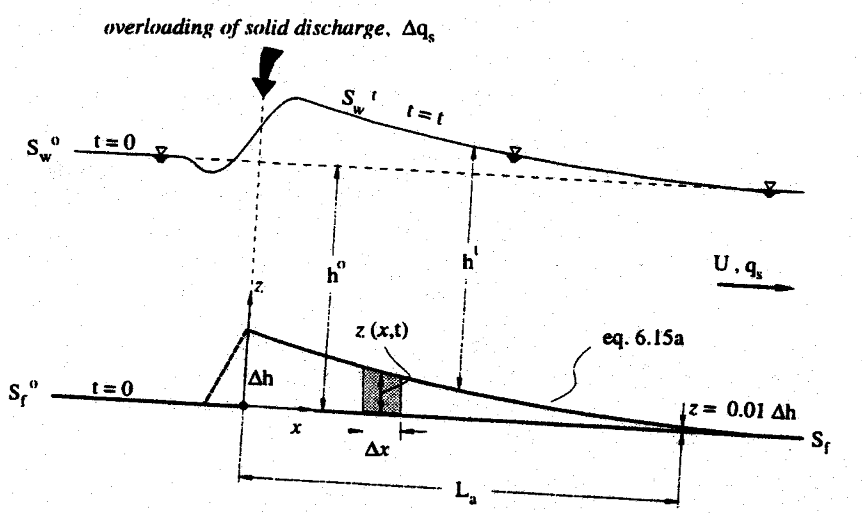


Fig. 6.6 Aggradation by overloading the supply of solid discharge, Δq_s .

Aggrading Channel Example

- Note position of x and z axes
- Initial and boundary conditions:

$z(x,0)=0$, $z(0,t)=\Delta h(t)$, $\lim (x \rightarrow \text{infinity})\ z(x,t)=0$

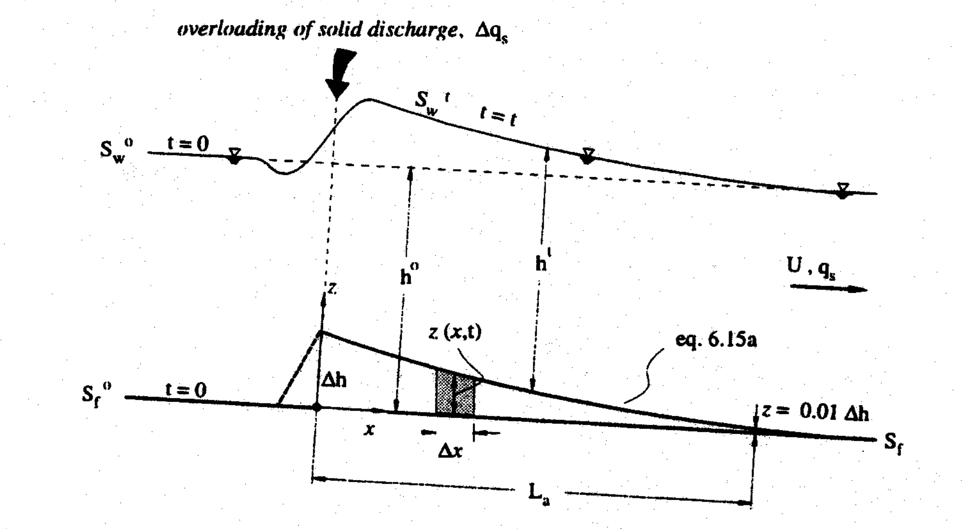


Fig. 6.6 Aggradation by overloading the supply of solid discharge, Δq_s .

Aggrading Channel Example

- For these boundary conditions!!!!
  - Solution:

$$z(x,t) = \Delta h(t)\, erfc\left(\frac{x}{2\sqrt{Kt}}\right)$$

$$K = K_o \left(before\, overload\right)$$

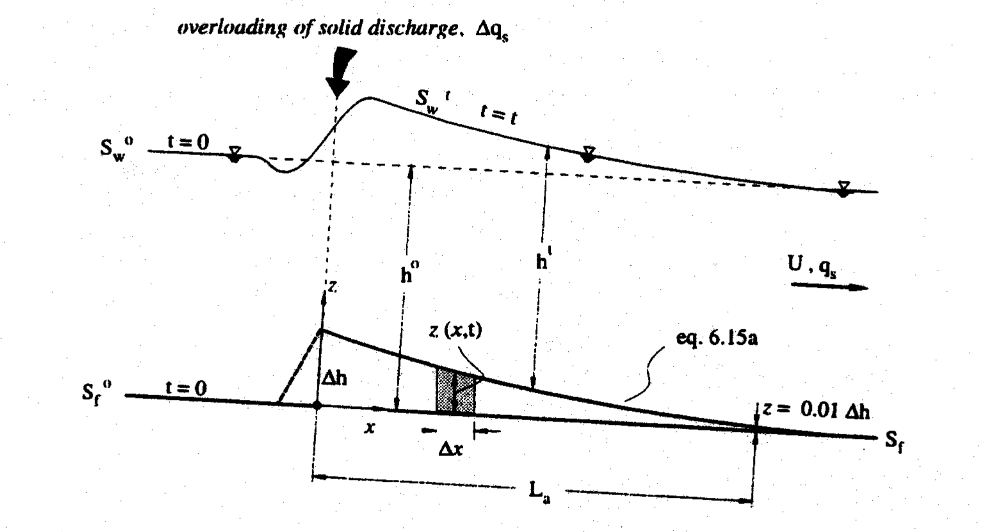


Fig. 6.6 Aggradation by overloading the supply of solid discharge, Δq_s .

Aggrading Channel Example

- For these boundary conditions!!!!
  - Beneficial to define length of the zone of aggradation:

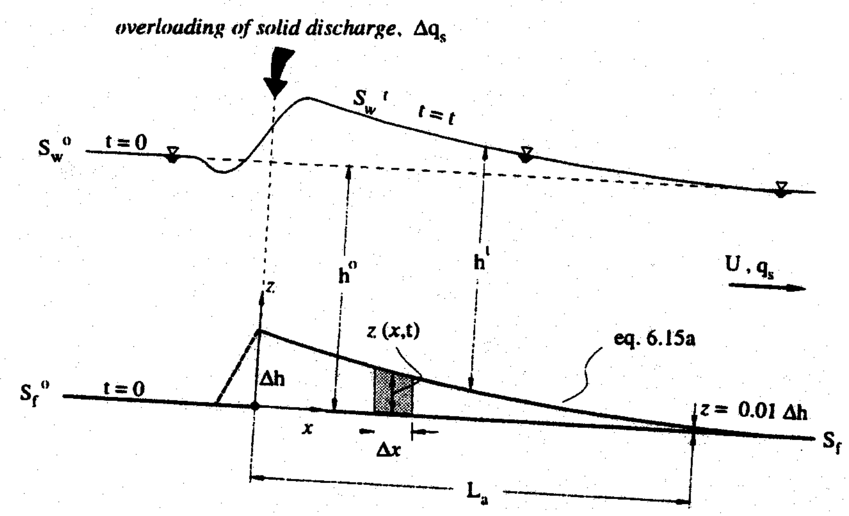


Fig. 6.6 Aggradation by overloading the supply of solid discharge, Δq_s .

$$L_a = x_{1\%}$$

$$\frac{z}{\Delta h} = 0.01 \rightarrow Y = 1.80$$

$$L_a = 3.65\sqrt{Kt_{1\%}}$$

Aggrading Channel Example

- For these boundary conditions!!!!
  - Volume of the supply of sediment, Δq_s, during time, Δt, is given by $\Delta q_s \Delta t$ and this quantity is distributed over the bed of the channel:

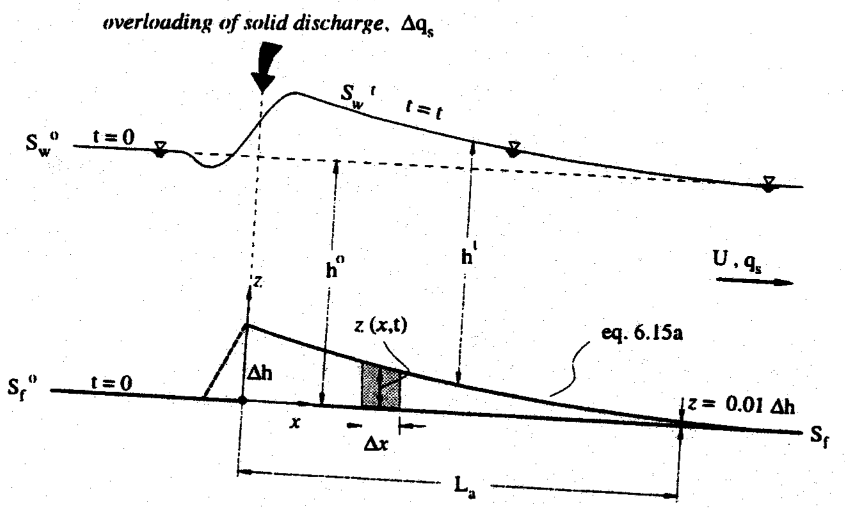


Fig. 6.6 Aggradation by overloading the supply of solid discharge, Δq_s .

$$\Delta q_s \Delta t = (1-p)\int_0^{L_a} z\,dx$$

$$\Delta h(t) = \frac{\Delta q_s \Delta t}{1.13(1-p)\sqrt{K\Delta t}}$$

Example - Graf 6.A

A rectangular channel has a width of 5 m. At some point, the bed of the channel changes from a fixed bed to a mobile bed with a d_{50} = 1 mm, p = 0.3, and s_s = 2.6. The discharge of Q = 15 m^3/s remains constant and the water depth is 2.2 m.

A degradation of the channel starts at the junction between the fixed bed and the mobile bed. Determine the time it will take to lower the bed level down to z = 0.4Δh at a station located at L = $6R_h/S_f$ downstream from the junction and draw the bed profile. Also, what is the resulting bed profile if the length of the mobile bed is limited to 90 km?

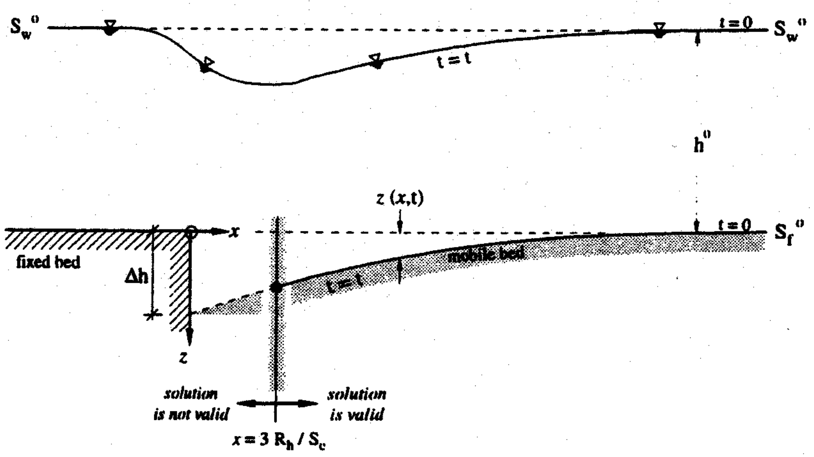


Fig. Ex.6.A.1 Scheme of the degradation.

Example - Graf 6.A

- Note: To solve this problem, we need to be able to calculate the solid-discharge
  - We will use the Graf et al. (1968) formula for total-load (we will learn more about this formula later):

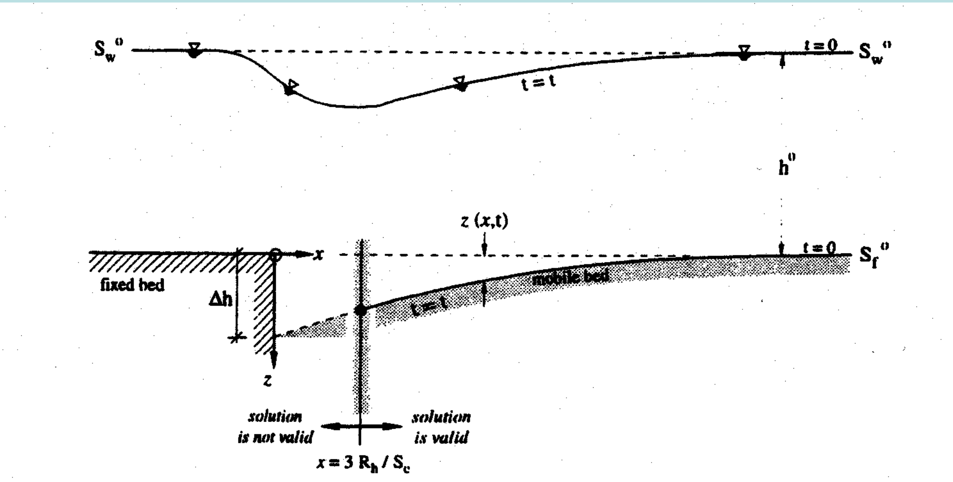


Fig. Ex.6.A.1 Scheme of the degradation.

$$q_s = C_s U h$$

$$q_s = a_s U^{b_s}$$

$$\Phi = \alpha (\tau_*)^{\beta}$$

$$\Phi_A = transport\ \ parameter = \frac{C_s U R_h}{\sqrt{(s_s - 1) g d^3}}$$

$$\Phi_A = 10.39 (\Psi_A)^{-2.52}$$

$$\Psi_A = shear\ intensity\ parameter = \frac{(s_s - 1) d}{S_e R_h}$$

$$b_s \approx 5$$

Example - Graf 6.A

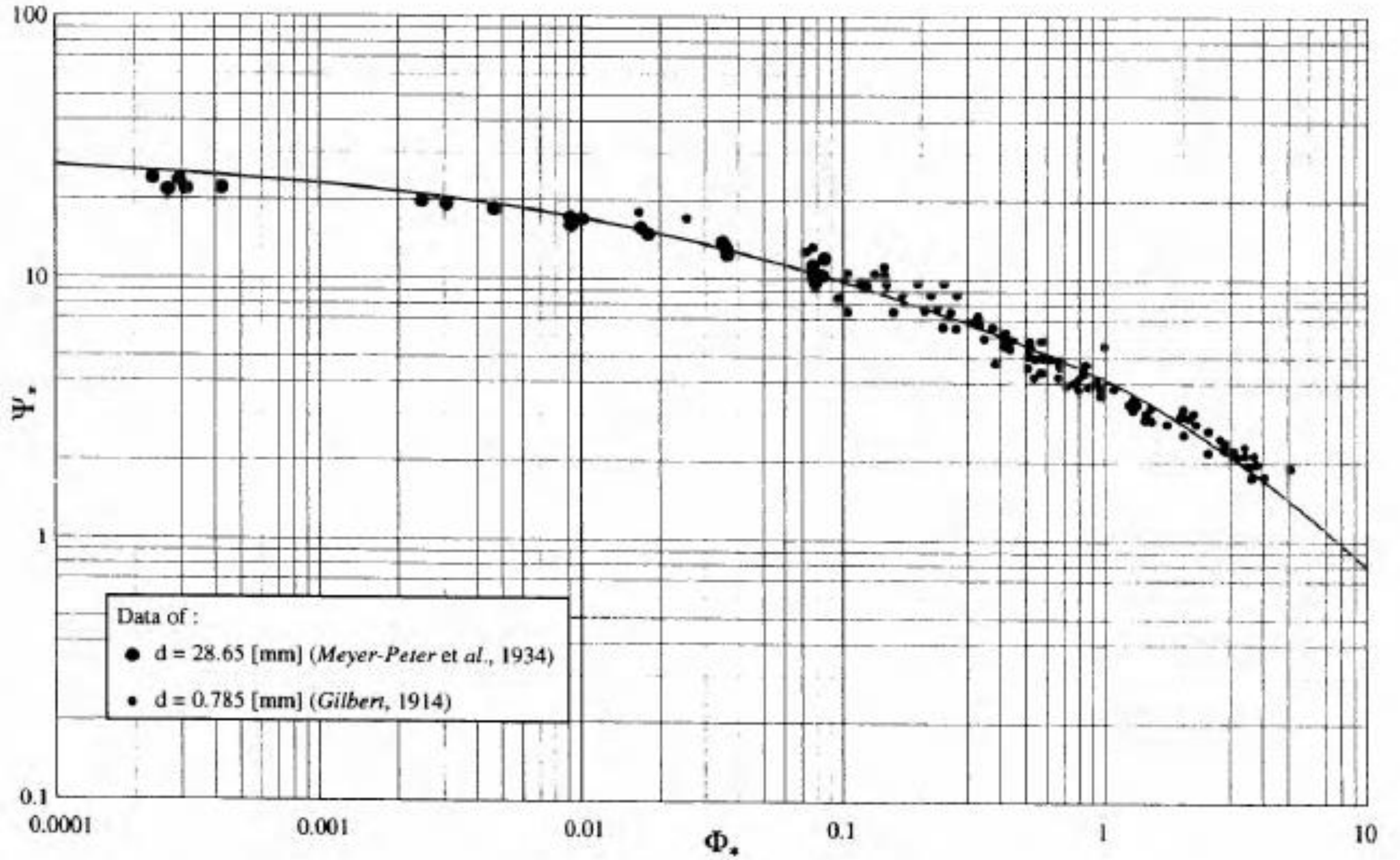


Fig. 6.8 Equation of bed load, $\Phi_* = f(\Psi_*)$, of Einstein (see *Graf*, 1971, p. 148).

Example - Graf 6.A

- To solve for the solid discharge, we need several parameters: S_e, R_h
  - Manning-Strickler equation for Se based on Q (= UBh)
  - Use Graf total load equation to get C_sUR_h
  - Multiply C_sUR_h by h/R_h to get q_s
  - Calculate K assuming $K = K_o$

$$K = K_o = \frac{1}{3} b_s q_s \frac{1}{(1-p)} \frac{1}{S_{e_o}}$$

Example - Graf 6.A

$$K = K_o = \frac{1}{3} b_s q_s \frac{1}{(1 - p)} \frac{1}{S_{e_o}}$$

$$K = K_o = \frac{1}{3}(5)\left(7.3 \times 10^{-5} \frac{m^2}{s}\right)\frac{1}{(1 - 0.3)}\frac{1}{0.00034}$$

$$K = K_o = 0.511 \frac{m^2}{s}$$

Example - Graf 6.A

- We need to solve for the time it takes to lower the bed down to z = 0.4Δh:

$$\frac{z(x,t)}{\Delta h} = \frac{0.4\Delta h}{\Delta h} = 0.4$$

$$erfc(Y) = 0.4 \rightarrow Y = 0.6 = \frac{x}{2\sqrt{Kt}}$$

$$x = L = 6R_h / S_e = 20.65\, km$$

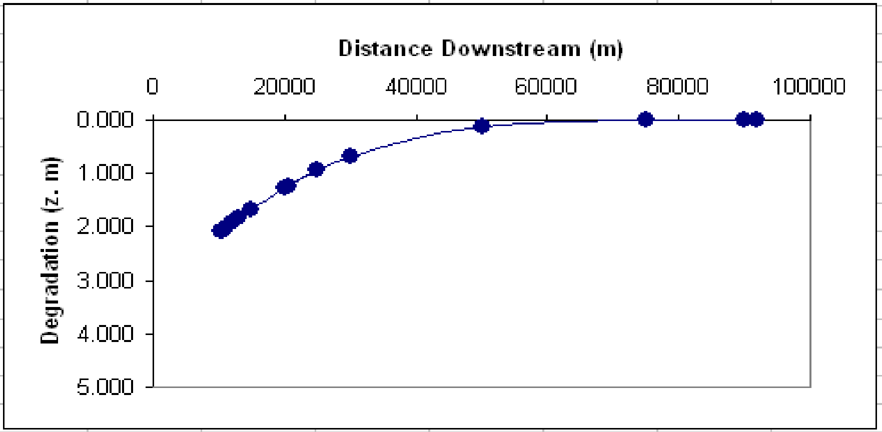
$$t = 18.5\, yrs$$

Example - Graf 6.A

R_h	1.17	m
S_f	0.00034	
K	0.511	m^2/s
Δh	3.107	m
t	579339310	s
x_{min}	10323.529	

L = 20647.06 m

x	Y	erfc(Y)	z	z/Δh
10324	0.300	0.671	2.086	0.671
10400	0.302	0.669	2.079	0.669
10500	0.305	0.666	2.070	0.666
10750	0.312	0.659	2.047	0.659
11000	0.320	0.651	2.024	0.651
12000	0.349	0.622	1.933	0.622
13000	0.378	0.593	1.843	0.593
15000	0.436	0.538	1.671	0.538
20000	0.581	0.411	1.278	0.411
25000	0.726	0.304	0.945	0.304
20647.06	0.600	0.396	1.231	0.396
30000	0.872	0.218	0.676	0.218
50000	1.453	0.040	0.124	0.040
75000	2.179	0.002	0.006	0.002
90000	2.615	0.000	0.001	0.000
91900	2.671	0.000	0.000	0.000



Example - Graf 6.A

- If the mobile bed is limited to 90 km…

$$z = 0.01\Delta h$$

$$\frac{z(x,t)}{\Delta h} = 0.01 \rightarrow erfc(Y) = 0.01 \rightarrow Y = 1.82$$

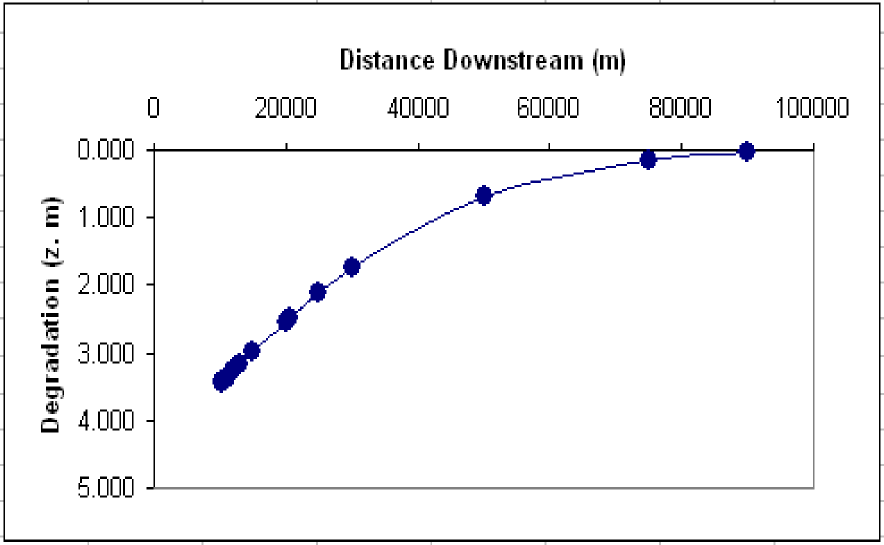
$$Y = \frac{x}{2\sqrt{Kt}}$$

$$t = \frac{x^2}{4Y^2K} = 1.2\times 10^9\ s = 37.93\ yrs$$

Example - Graf 6.A

R_h	1.17	m			
S_f	0.00034			L	20647.06 m
K	0.511	m^2/s			
Δh	4.472	m			
t	1.2E+09	s			
x_{min}	10323.529				

x	Y	erfc(Y)	z	z/Δh
10324	0.208	0.768	3.435	0.768
10400	0.210	0.766	3.428	0.766
10500	0.212	0.764	3.418	0.764
10750	0.217	0.759	3.394	0.759
11000	0.222	0.753	3.370	0.753
12000	0.242	0.732	3.273	0.732
13000	0.262	0.710	3.177	0.710
15000	0.303	0.668	2.989	0.668
20000	0.404	0.568	2.540	0.568
25000	0.505	0.475	2.126	0.475
20647.06	0.417	0.555	2.484	0.555
30000	0.606	0.392	1.751	0.392
50000	1.010	0.153	0.686	0.153
75000	1.514	0.032	0.144	0.032
90000	1.817	0.010	0.045	0.010



SOLUTION :

i) The steady flow will be considered to be quasi-uniform during the phase
degradation (see Fig. Ex. 6.A.1); therefore the *parabolic model* can be used :

$$\frac{\partial z}{\partial t} - K\frac{\partial^2 z}{\partial x^2} = 0 \qquad (6.1$$

where x is positive towards the downstream and follows the initial bed profil
z represents the bed-level variation with respect to the initial bed, $S_f^{\,o}$. Note that th
use of the parabolic model is limited to : $Fr < 0.6$ and $x > 3R_h/S_e$.

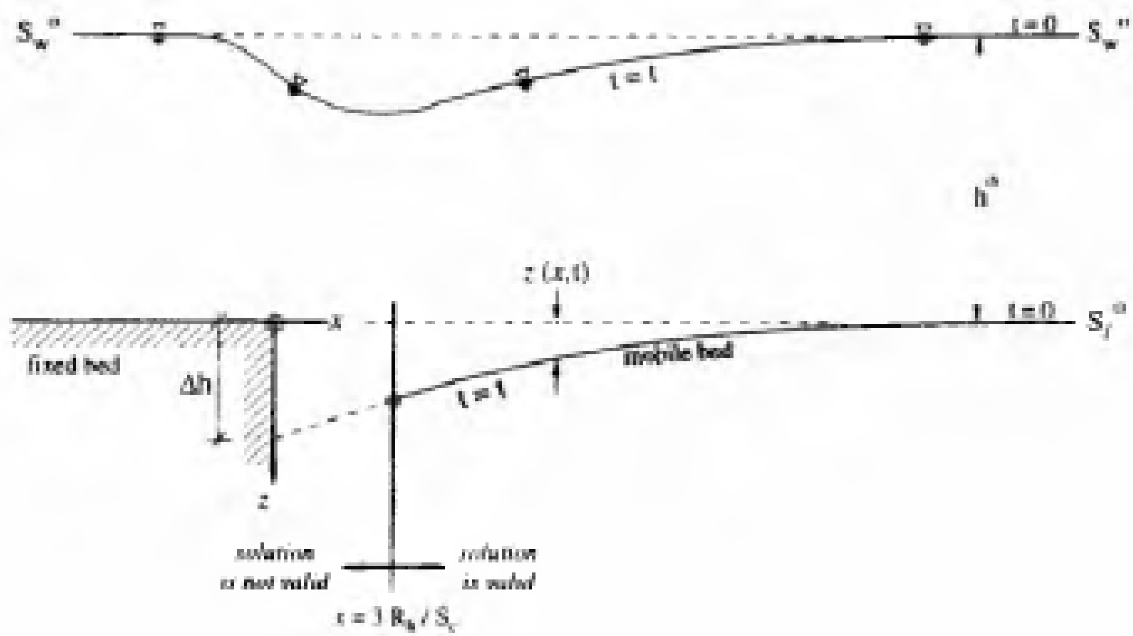


Fig. Ex.6.A.1 Scheme of the degradation.

The initial and boundary conditions are given as :

$$z(x,0) = 0 \quad ; \quad \lim_{x \to \infty} z(x,t) = 0$$

$$z(0,t) = \Delta h(t)$$

The solution to eq. 6.11 is given by :

$$z(x,t) = \Delta h \; erfc \left(\frac{x}{2\sqrt{Kt}} \right) \tag{6.15}$$

ii) Calculation of the quasi-uniform *flow* in the mobile-bed channel.

The slope of the energy line, S_e , is calculated using the Manning-Strickler formula:

$$U = \frac{Q}{Bh} = K_s \; R_h^{2/3} \; S_e^{1/2} \tag{3.16}$$

with $K_s = 21.1/d_{50}^{1/6} = 66.7 \; [m^{1/3}/s]$ (3.18)

$h = 2.2 \; [m]$, $B = 5.0 \; [m]$, $R_h = 1.17 \; [m]$

$Q = 15.0 \; [m^3/s]$, $q = Q / B = 3 \; [m^2/s]$

$U = q / h = 1.36 \; [m/s]$

The slope of the energy line : $S_e = \mathit{0.00034}$ [-]

The Froud number is : $Fr = \dfrac{U}{\sqrt{gh}} = 0.29$ [-]

It should be emphasized that the Froude number has to be small, $Fr < 0.6$, being one of the conditions (see sect. 6.2.3) for the validity of the parabolic model, eq. 6.11.

iii) Calculation of the *solid discharge* in the mobile-bed channel.

The solid discharge, $q_s = C_s U h$, is calculated using the *Graf* et *al.* (1968) formula :

$$\frac{C_s\, U R_h}{\sqrt{[(\rho_s-\rho)/\rho]\, g\, d_{50}^3}} = 10.39 \left\{ \frac{[(\rho_s-\rho)/\rho]\, d_{50}}{S_f\, R_h} \right\}^{-2.52} \qquad (6.63)$$

with
$$(\rho_s-\rho)/\rho = 1.6 \text{ [-]}$$
$$d_{50} = 1 \text{ [mm]}$$
$$S_f \equiv S_e = 0.00034 \text{ [-]}$$

$C_s\, U R_h = 3.9 \cdot 10^{-5}$ [m²/s]

The solid discharge is : $q_s = C_s\, U\, h\, \frac{R_h}{R_h} = 3.9 \cdot 10^{-5}\, \frac{2.2}{1.17} = \mathit{7.3 \cdot 10^{-5}}$ [m²/s] *iv)* The *coefficient*, K, in the parabolic model, eq. 6.11, is approximately given by :

$$K_o \equiv K = \frac{1}{3}\, b_s q_s \, \frac{1}{(1-p)} \, \frac{1}{S_e^{\,o}} \qquad (6.12$$

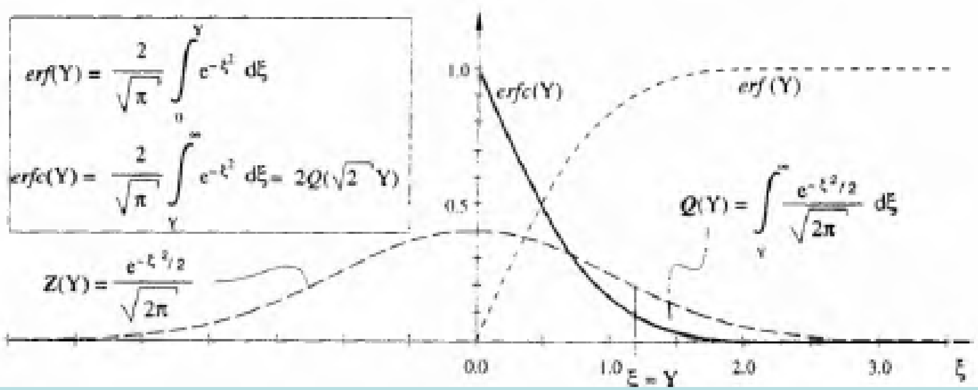
with $S_e^{\,o} = 0.00034$ [-]
$(1-p) = 0.7$ [-]
$b_s = 2\,(2.52) \equiv 5$ (where $\beta = 2.52$ is the exponent in eq. 6.6 according to eq. 6.5a and eq. 6.30)

The coefficient is : $K = 0.511$ [m²/s]

Table for the complementary error function

(see *Handbook of Mathematical Functions*, 1964, National Bureau of Standards, pp. 310-311, formula 7.1.2

Y	*erfc*(Y)	Y	*erfc*(Y)	Y	*erfc*(Y)
0.00	*1.00000*	1.20	*0.08969*	2.30	*0.00114*
0.10	*0.88754*	1.30	*0.06599*	2.40	*0.00069*
0.20	*0.77730*	1.40	*0.04772*	2.50	*0.00041*
0.30	*0.67137*	1.50	*0.03390*	2.60	*0.00024*
0.40	*0.57161*	1.60	*0.02365*	2.70	*0.00013*
0.50	*0.47950*	1.70	*0.01621*	2.80	*0.00008*
0.60	*0.39614*	1.80	*0.01091*	2.90	*0.00004*
0.70	*0.32220*	1.90	*0.00721*	3.00	*0.00002*
0.80	*0.25790*	2.00	*0.00468*	3.10	*0.00001*
0.90	*0.20309*	2.10	*0.00298*	3.20	*0.00001*
1.00	*0.15730*	2.20	*0.00186*	3.30	*0.00000*
1.10	*0.11979*				



v) In the present problem, it is asked to determine the time it takes to lower the bed level down to $z = 0.4\Delta h$, thus :

$$\frac{z(x,t)}{\Delta h} = \frac{0.4\Delta h}{\Delta h} = 0.4$$

The eq. 6.15 is now written as :

$$0.4 = erfc\left(\frac{x}{2\sqrt{Kt}}\right) = erfc\,(Y)$$

Using the table of the complementary error function yields :

$$Y \cong 0.6 = \left(\frac{x}{2\sqrt{Kt}}\right) \qquad \Rightarrow \qquad t \cong \frac{x^2}{4\,Y^2\,K} \cong \frac{x^2}{1.44\,K}$$

At the station $x \equiv L = 6R_h/S_e = 20.73$ [km], the lowering of the bed down to a level of $z = 0.4\Delta h$ occurs at the time :

$$t = \frac{(20.73 \cdot 10^3)^2}{(1.44)\,(0.511)} = 5.84 \cdot 10^8\ [s] = \mathit{1.62 \cdot 10^5}\ [h] \cong \mathit{18.52}\ [years]$$

To draw the bed profile for the entire channel at this particular moment, $t = 5.84 \cdot 10^8$ [s], the calculations are repeated for different values for the distance x (see following table).

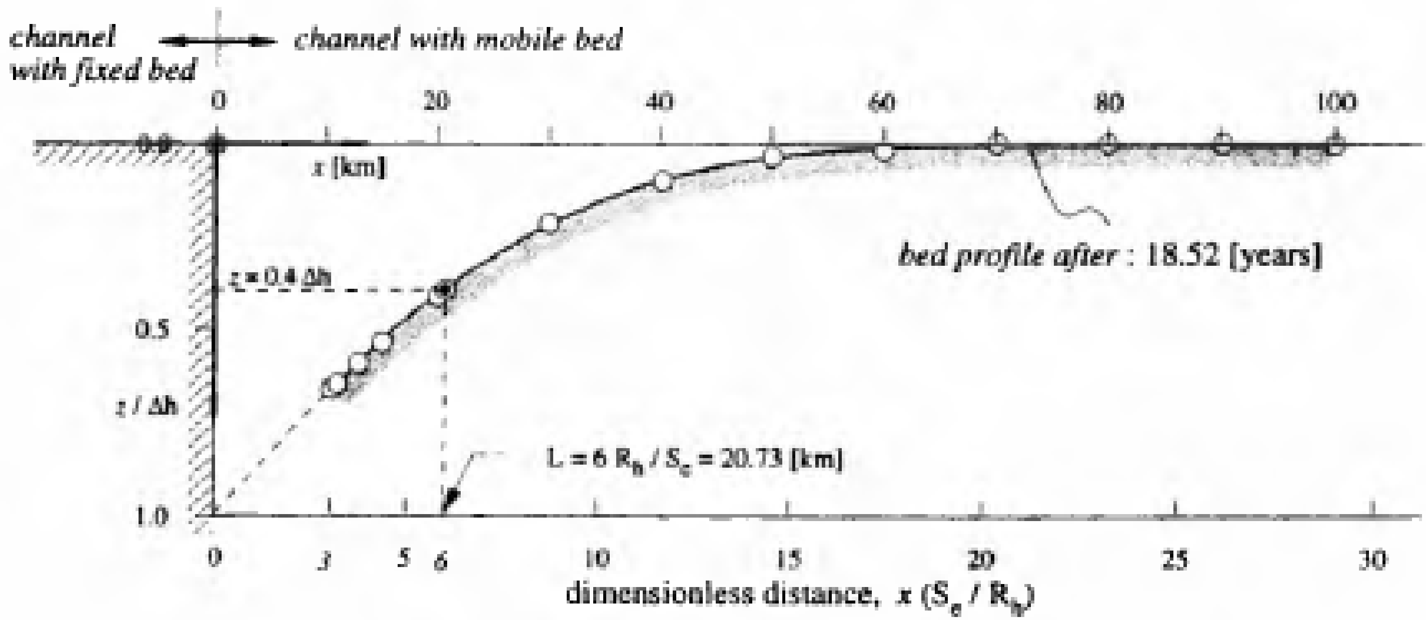
Calculation of the bed profile				
$R_h = 1.17$ [m] ; $S_f = 0.00034$ [-] ; $K = 0.511$ [m^2/s]				
$\Delta h =$ **3.11** [m] ; $t = 5.84 \cdot 10^8$ [s]				
x [m]	$x\ (S_e / R_h)$ [-]	$Y = x/(2\sqrt{Kt})$ [-]	$z/\Delta h = erfc(Y)$ [-]	z [m]
10500	3.04	0.30	0.66735	2.073
11000	3.18	0.32	0.65253	2.027
13000	3.76	0.38	0.59465	1.847
15000	4.34	0.43	0.53923	1.675
20000	5.79	0.58	0.41299	1.283
20730	*6.00*	*0.60*	*0.39615*	*1.231*
30000	8.68	0.87	0.21946	0.682
40000	11.58	1.16	0.10157	0.316
50000	14.47	1.45	0.04070	0.126
60000	17.37	1.74	0.01405	0.044
70000	20.26	2.03	0.00417	0.013
80000	23.15	2.32	0.00106	0.003
90000	26.05	2.60	0.00023	0.001
100000	28.94	2.89	0.00004	0.000

The depth of degradation of the channel bed due to a solid discharge $q_s = 7.3 \cdot 10^{-5}$ [m^2/s] , during a time period of $t = 5.84 \cdot 10^8$ [s] is given by :

$$\Delta h = \frac{q_s \cdot \Delta t}{1.13\,(1-p)\sqrt{K\ \Delta t}} = \frac{(7.3 \cdot 10^{-5})\sqrt{5.84 \cdot 10^8}}{(1.13)\,(0.7)\,\sqrt{0.511}} = \mathbf{3.11}\ [m] \qquad (6.$$

and $z = 0.4\Delta h = 1.23$ [m] .

The bed profile, $z(x)$, for $t = 5.84 \cdot 10^8$ [s] = 18.52 [years], is plotted Fig. Ex. 6.A.2. This solution is valid only if $x > 3R_h/S_e$. For distances $x < 3R_h/S_e$, the solution is only an indicative one.



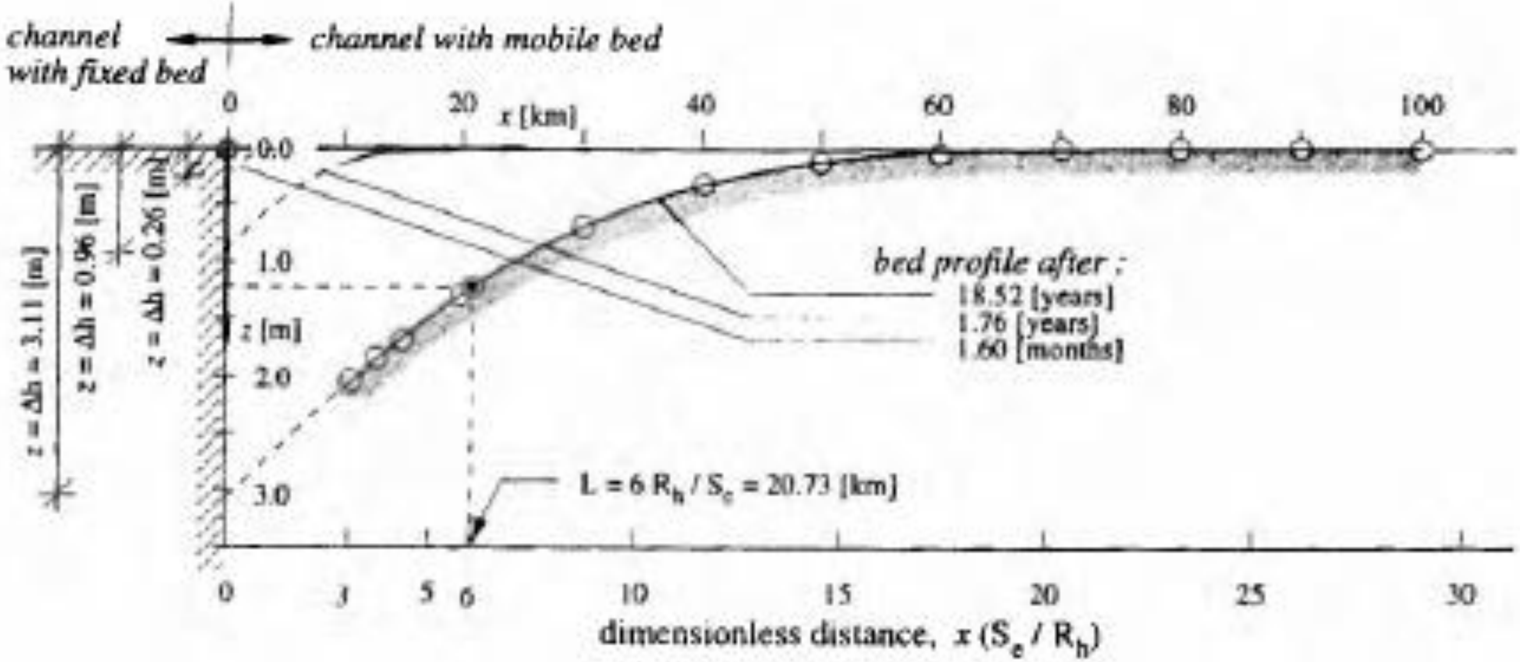


Fig. Ex.6.A.2 Bed profile after *18.52* [years] of degradation.

For sake of comparison, the bed profiles, $z(x)$, for t = 1.76 [year] and f
t = 1.6 [month] are also plotted (without giving the calculations) in Fig. Ex. 6.A.

vi) The *temporal evolution of the degradation* at the station located at $x \equiv L = 6R_h/S_e = 20.73$ [km] is given by :

$$z(t) = \Delta h \; erfc\left(\frac{x}{2\sqrt{K\,\Delta t}}\right) = \Delta h \; erfc\left(\frac{20730}{2\sqrt{0.511\,\Delta t}}\right) \tag{6.15}$$

where, $\Delta h(t)$ can be evaluated by :

$$\Delta h = \frac{q_s \cdot \Delta t}{1.13\,(1-p)\sqrt{K\,\Delta t}} \tag{6.20}$$

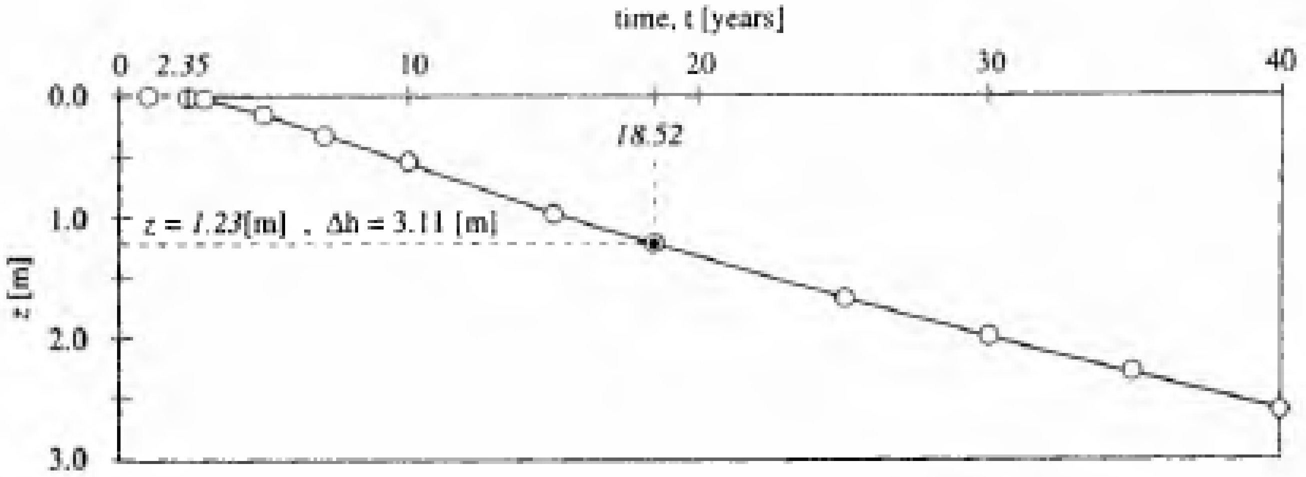


Fig. Ex.6.A.3 Evolution of the degradation at the station $x \equiv L = 6R_h/S_e = 20.73$ [km].

Calculation of the evolution of the degradation					
$R_h = 1.17$ [m] ; $S_f = 0.00034$ [-] ; $K = 0.511$ [m^2/s]					
$x \equiv L = 6R_h/S_{f_0} = 20730$ [m]					
t [years]	t [s]	$Y = x/(2\sqrt{Kt})$ [-]	$z/\Delta h = erfc(Y)$ [-]	Δh [m]	z [m]
1	3.15E+07	2.58	0.00026	0.72	0.0002
3	9.46E+07	1.49	0.03502	1.25	0.0438
5	1.58E+08	1.15	0.10248	1.61	0.1654
7	2.21E+08	0.98	0.16756	1.91	0.3201
10	3.15E+08	0.82	0.24823	2.28	0.5667
15	4.73E+08	0.67	0.34579	2.80	0.9669
18.52	*5.84E+08*	*0.60*	*0.39618*	*3.11*	*1.2309*
25	7.88E+08	0.52	0.46522	3.61	1.6794
30	9.46E+08	0.47	0.50500	3.95	1.9970
35	1.10E+09	0.44	0.53711	4.27	2.2941
40	1.26E+09	0.41	0.56371	4.57	2.5740
45	1.42E+09	0.38	0.58622	4.84	2.8391
50	1.58E+09	0.37	0.60559	5.11	3.0916

The evolution of the bed degradation can now be calculated by assuming diff values for $\Delta t \equiv t$. By using the approximate formula for the complementary function (see before), the calculation can easily be programmed on a spreads The table above summarizes these calculations; Fig. Ex. 6.A.3 shows the evol of the erosion, $z(t)$, at the station, $x \equiv L$.

This solution is however only valid (see *Ribberink* et *Sande*, 1984, p. 30) for :

$$t > \frac{40}{30}\frac{R_h^2}{S_f}\frac{1}{q_s} = \frac{40}{30}\;\frac{1.17^2}{0.00034}\;\frac{1}{7.3 \cdot 10^{-5}} = 7.42 \cdot 10^7 \text{ [s]} \cong 2.35 \text{ [years]}$$

vii) Calculation of the final bed profile if the channel reach with the mobile b *limited* to a length of $x_f = 90$ [km].

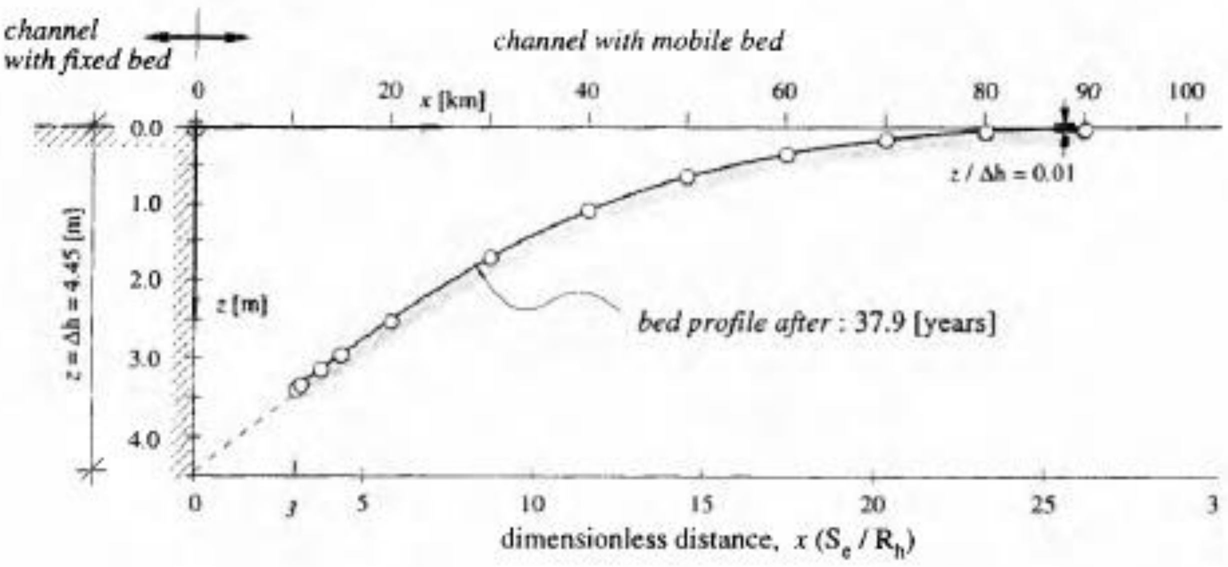


Fig. Ex.6.A.4 The channel-bed profile after *37.9* [years] of degradation

By assuming a very small amount of erosion, such as $z = 0.01\Delta h$, at the st
$x_f = 90$ [km] , one can write :

$$\frac{z(x,t)}{\Delta h} = 0.01 = erfc\left(\frac{x_f}{2\sqrt{Kt}}\right) = erfc(Y)$$

Using the table of the complementary error function yields :

$$Y = 1.82 = \left(\frac{x}{2\sqrt{Kt}}\right) \quad \Rightarrow \quad t = \frac{x^2}{4\ Y^2\ K} \cong \frac{x^2}{13.25\ K}$$

and with $K = 0.511$ [m²/s], one calculates :

$$t = \frac{(90 \cdot 10^3)^2}{(13.25)\ (0.511)} = 1.2 \cdot 10^9\ [s] = 3.3 \cdot 10^5\ [h] \cong 37.93\ [years]$$

To obtain the bed profile for the entire channel at this moment, $t = 1.2 \cdot 10^9$ [s], the calculations for the degradation are repeated using different values for x (see the following table). The final bed profile, calculated in this way, is plotted in Fig. Ex. 6.A.4.

This solution is valid only if $x > 3R_h / S_e$.

The depth of the bed degradation due to a solid discharge, $q_s = 7.3 \cdot 10^{-5}$ [m²/s] , during a time period of $t = 1.2 \cdot 10^9$ [s] , is given by the eq. 6.20 :

$$\Delta h = \frac{q_s \cdot \Delta t}{1.13\ (1-p)\sqrt{K\ \Delta t}} = \frac{(7.3 \cdot 10^{-5})\sqrt{1.2 \cdot 10^9}}{(1.13)\ (0.7)\ \sqrt{0.511}} = \mathbf{4.45}\ [m]$$

Calculation of the final bed profile				
$R_h = 1.17$ [m] ; $S_f = 0.00034$ [-] ; $K = 0.511$ [m^2/s]				
$\Delta h =$ **4.45** [m] ; $t = 1.2 \cdot 10^9$ [s]				
x [m]	x (S_e / R_h) [-]	$Y = x/(2\sqrt{Kt})$ [-]	$z/\Delta h = erfc(Y)$ [-]	z [m]
10500	3.04	0.21	0.76396	3.397
11000	3.18	0.22	0.75307	3.349
13000	3.76	0.26	0.71005	3.157
15000	4.34	0.30	0.66793	2.970
20000	5.79	0.40	0.56734	2.523
30000	8.68	0.61	0.39091	1.738
40000	11.58	0.81	0.25264	1.123
50000	14.47	1.01	0.15273	0.679
60000	17.37	1.21	0.08617	0.383
70000	20.26	1.42	0.04529	0.201
80000	23.15	1.62	0.02214	0.098
90000	*26.05*	*1.82*	*0.01006*	*0.045*

Example – Graf 6.B

A river on a bed slope of 0.0005 conveys a unit discharge of 1.5 m^2/s. The river bed is made of granular material of uniform size with a d_{50} of 0.00032 with s_s = 2.6; the porosity of the bed is p = 0.4. There exists a weak transport of sediments.

At a certain station on this river, the solid discharge is locally increased by Δq_s = 0.0001 m^2/s for a time period of Δt = 50 hr. Determine the aggradation of the bed.

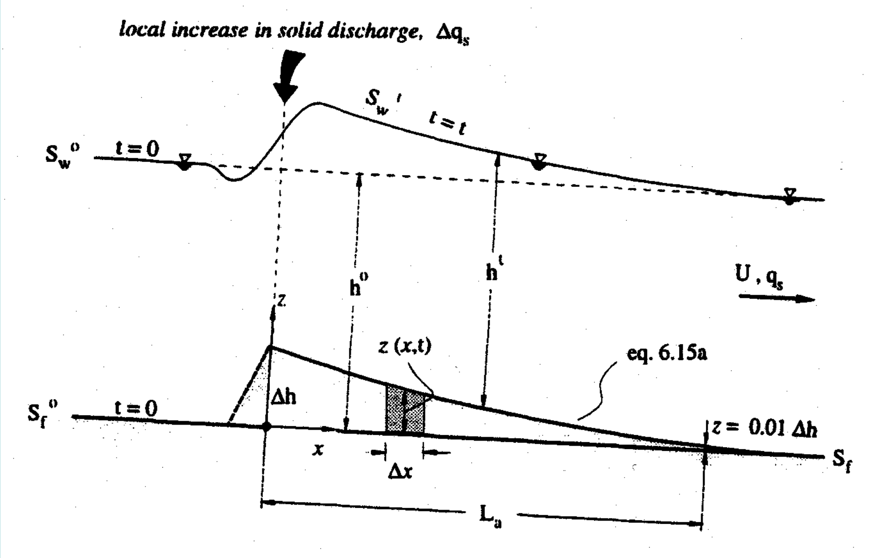


Fig. Ex.6.B.1 Sketch of the aggradation.

SOLUTION :

i) The flow is steady and is considered to be quasi-uniform during the period aggradation (see Fig. Ex. 6.B.1); thus the *parabolic model* can be used :

$$\frac{\partial z}{\partial t} - K\frac{\partial^2 z}{\partial x^2} = 0 \qquad (6.1$$

where x is positive towards the downstream and follows the initial bed profi z represents the bed-level variation with respect to the initial bed, S_{f_0}. Note that t use of the parabolic model is limited to : $Fr < 0.6$ and $x > 3R_h/S_e$.

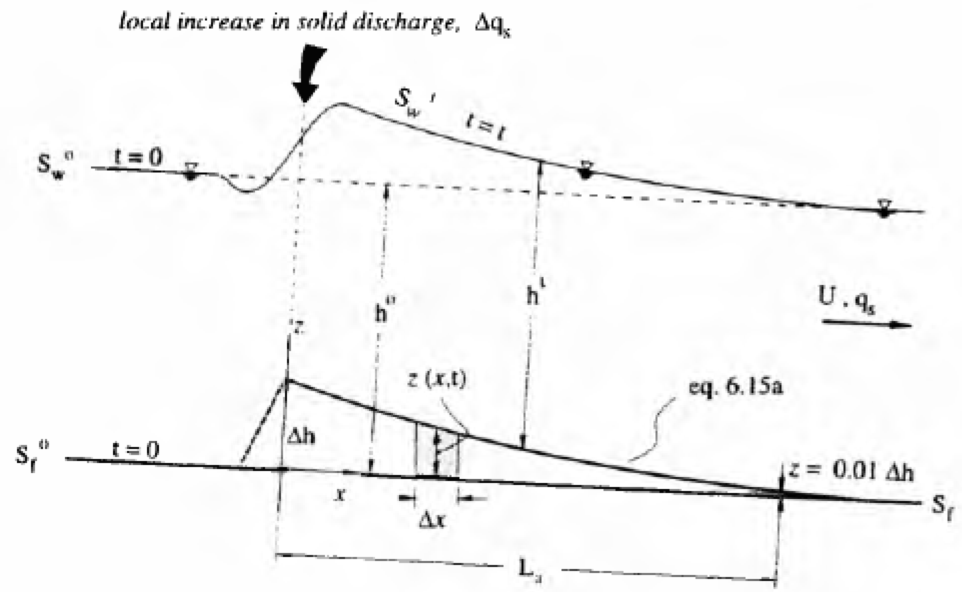


Fig. Ex.6.B.1 Sketch of the aggradation.

The initial and boundary conditions are given as :

$z(x,0) = 0 \quad ; \quad \lim_{x \to \infty} z(x,t) = 0$

$z(0,t) = \Delta h(t)$

The solution to eq. 6.11 is given by :

$$z(x,t) = \Delta h \, erfc\left(\frac{x}{2\sqrt{Kt}}\right) \tag{6.15}$$

ii) Calculation of the quasi-uniform *flow* in the river having a mobile bed.

The normal depth is calculated using the Manning-Strickler formula :

$$U = \frac{q}{h} = K_s \, h^{2/3} \, S_f^{1/2} \tag{3.16}$$

with

$$K_s = 21.1/d_{50}^{1/6} = 80.7 \; [m^{1/3}/s] \tag{3.18}$$

$$q = 1.5 \; [m^2/s]$$

$$S_f = 0.0005 \; [-]$$

The flow depth is : $h = 0.895$ [m]

The average velocity is : $U = 1.676$ [m/s]

The Froude number is : $Fr = \frac{U}{\sqrt{gh}} = 0.566$

It should be remembered that the Froude number has to be small, namely $Fr < 0.6$.

iii) Calculation of the *solid discharge* in the river having a mobile bed.

The solid discharge, $q_s = C_s\, Uh$, is calculated using the relationship given by *Graf* et *al.* (1968) :

$$\frac{C_s\, UR_h}{\sqrt{[(\rho_s-\rho)/\rho]g\, d_{50}^3}} = 10.39 \left\{ \frac{[(\rho_s-\rho)/\rho]\, d_{50}}{S_f\, R_h} \right\}^{-2.52} \qquad (6.63)$$

with $(\rho_s-\rho)/\rho = 1.6$ [-]

$d_{50} = 0.32$ [mm]

$R_h \cong h = 0.895$ [m]

The solid discharge is : $q_s = \mathit{1.678 \cdot 10^{-4}}$ [m²/s]

iv) The *coefficient*, K , in the parabolic model, eq. 6.11, is approximately given by :

$$K_o \equiv K \approx \frac{1}{3}\, b_s q_s\, \frac{1}{(1-p)}\, \frac{1}{S_e{}^o} \qquad (6.12c)$$

with $S_f{}^o \equiv S_e{}^o = 0.0005$ [-]

$(1-p) = 0.6$ [-]

$b_s = 2\,(2.52) \cong 5$ (where $\beta = 2.52$ is the exponent in eq. 6.63, according to eq. 6.5a and eq. 6.30)

The coefficient is : $K = \mathit{0.932}$ [m²/s] *v*) The thickness of the aggradation of the bed (see Fig. Ex. 6.B.1) due to a loc increase in solid discharge, $\Delta q_s = 0.0001$ [m²/s], during a time period ‹ $\Delta t = 50$ [h] $= 1.8 \cdot 10^5$ [s] , is given by eq. 6.20, or :

$$\Delta h(t) = \frac{\Delta q_s \cdot \Delta t}{1.13\,(1-p)\sqrt{K\ \Delta t}} = \frac{(0.0001)\,\sqrt{1.8 \cdot 10^5}}{(1.13)\,(0.6)\,\sqrt{0.932}} = \mathbf{0.065}\ [\mathrm{m}]$$

The length of the zone of aggradation, L_a, can be calculated with eq. 6.15 b assuming, for example, a precision of $z/\Delta h = 0.01$:

$$\frac{z(x,t)}{\Delta h} = \frac{0.01\Delta h}{\Delta h} = 0.01 = erfc\left(\frac{x}{2\sqrt{K\ \Delta t}}\right) = erfc\,(Y)$$

Using the table of the complementary error function (see Ex. 6.A), yields :

$$Y = 1.821 = \left(\frac{x}{2\sqrt{K\ \Delta t}}\right)$$

The length of the zone of aggradation (see eq. 6.19) can now be calculated a follows :

$$L_a \equiv x_{1\%} = 2Y\sqrt{K\,\Delta t} = (2)\,(1.821)\,\sqrt{(0.932)\,(1.8 \cdot 10^5)} = \mathit{1492.3}\ [\mathrm{m}]$$

vi) To plot the bed profile after a time period of $\Delta t = 50$ [h] $= 1.8 \cdot 10^5$ [s], calculation are made using eq. 6.15 for different distances, x. (see the following table).

The resulting bed profile, $z(x)$, is plotted in Fig. Ex. 6.B.2.

The calculations, summarized in the following table, are valid only if $x > 3h/S_e$ In the present case, it can be shown that :

$$x = 3h/S_e = (3)\,(0.895) / (5 \cdot 10^{-4}) = 5370 \text{ [m]} \gg L_a = 1492.3 \text{ [m]}$$

However, experimental data (see *Soni* et *al.*, 1980), have shown that the calculate value is only indicative, but nevertheless acceptable.

Calculation of the bed profile due to aggradation				
$R_h = h = 0.895$ [m] ; $S_f = 0.0005$ [-] ; $K = 0.932$ [m^2/s]				
$\Delta h =$ **0.065** [m] ; $\Delta t = 1.8 \cdot 10^5$ [s]				
x [m]	$x\ (S_e / R_h)$ [-]	$Y = x/(2\sqrt{Kt})$ [-]	$z/\Delta h = erfc(Y)$ [-]	z [m]
10.0	0.01	0.01	0.98623	0.064
50.0	0.03	0.06	0.93123	0.060
100.0	0.06	0.12	0.86296	0.056
300.0	0.17	0.37	0.60459	0.039
500.0	0.28	0.61	0.38813	0.025
700.0	0.39	0.85	0.22696	0.015
900.0	0.50	1.10	0.12032	0.008
1000.0	0.56	1.22	0.08434	0.005
1100.0	0.61	1.34	0.05761	0.004
1300.0	0.73	1.59	0.02484	0.002
1492.3	*0.83*	*1.82*	*0.01000*	*0.001*
1500.0	0.84	1.83	0.00962	0.001
1600.0	0.89	1.95	0.00575	0.000

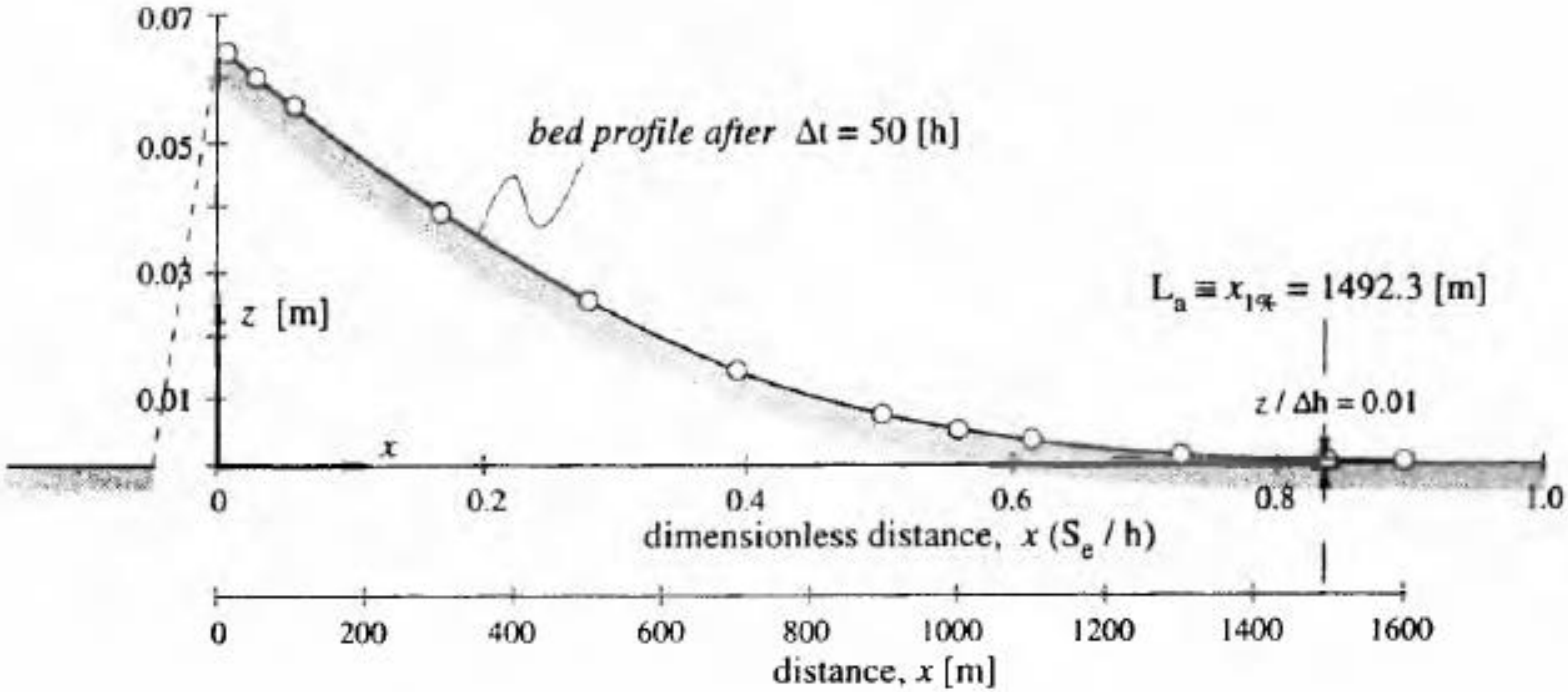


Fig. Ex.6.B.2 Bed profile after 50 [hours] of aggradation.