# CHAPTER 4

# WELL HYDRAULICS

$$T\{\frac{\partial^2 h}{\partial x^2}+\frac{\partial^2 h}{\partial y^2}+\frac{\partial^2 h}{\partial z^2}\}=S\,\frac{\partial h}{\partial t}$$

# STEADY ONE-DIRECTIONAL FLOW:

## A. Confined Aquifer

$$\frac{\partial^2 h}{\partial x^2} = 0$$

$$\frac{\partial h}{\partial x} = C_1 = -\frac{V}{K} \quad \text{(from Darcy's Law)}$$

$$h = C_1 X + C_2$$

let $h = 0$ at $X = 0$ ; $C_2 = 0$

$$h = -\frac{VX}{K}$$

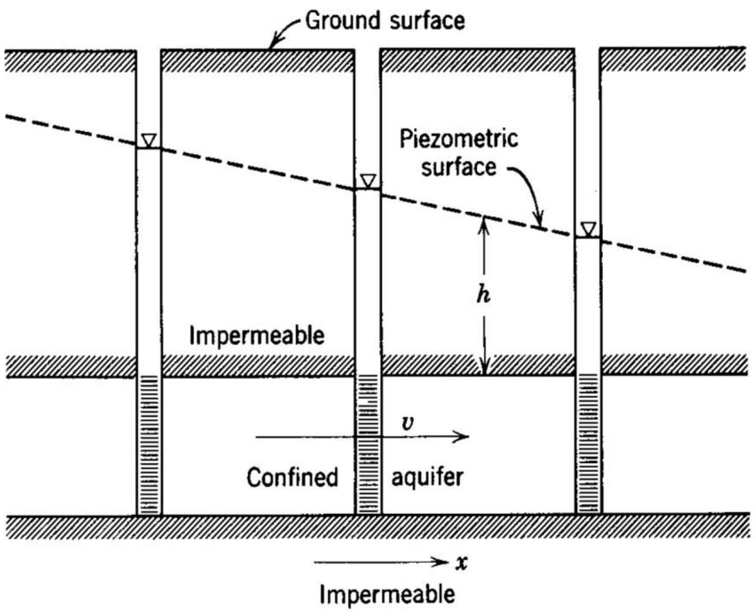



**Fig. 4.1** Steady unidirectional flow in a confined aquifer of uniform thickness.

**This states the *h* decreases linearly, with flow in X direction**

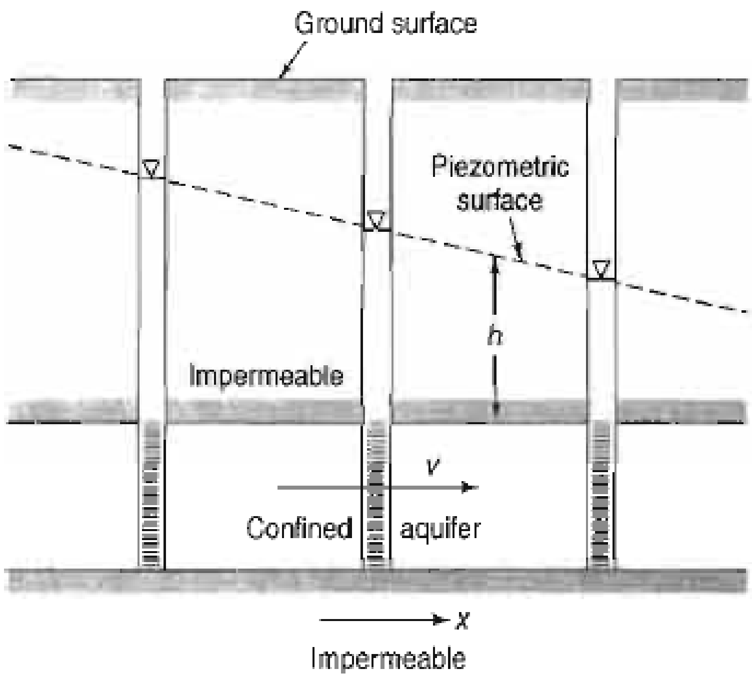



**Figure 4.1.1.** Steady unidirectional flow in a confined aquifer of uniform thickness.

***EXAMPLE 4.1.1***

Referring to Figure 4.1.1, if the distance and the observed piezometric surface drop between two adjacent wells are 1,000 m and 3 m, respectively, find an estimate of the time it takes for a molecule of water to move from one well to the other. Assume steady unidirectional flow in a homogeneous silty sand confined aquifer with a hydraulic conductivity $K = 3.5$ m/day and an effective porosity of 0.35.

***SOLUTION***

First compute the discharge velocity:

$$v = -\frac{hK}{x} = -\frac{(-3\text{m})(3.5\text{ m/d})}{(1000\text{m})} = 0.0105\text{ m/d}$$

The pore (seepage) velocity is computed using the velocity:

$$v_p = v/n_e = (0.0105\text{ m/d})/(0.35) = 0.03\text{ m/d}$$

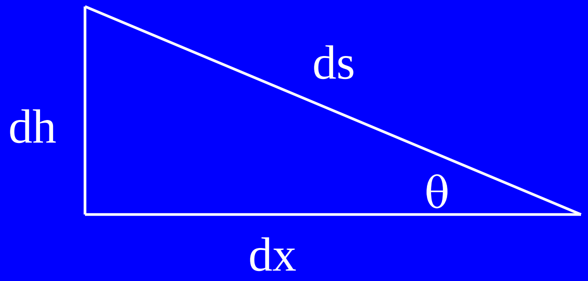
It would take 1000 m/(0.03 m/d × 365) ≈ 91.3 years. ■

# B. Unconfined Aquifer

- Sol. of Laplace equation for unconfined aquifer not possible.
- WT. in 2D flow represents a flow line
- Shape of WT determines the flow distribution, but at the same time flow distribution governs WT shape.

# To obtain the solution, Dupuit Assumptions --

1. **Velocity of flow is proportional to the tangent of hyd. grad.**




$$\frac{dh}{ds} \approx \frac{dh}{dx} \quad \text{or} \quad \sin\theta = \tan\theta$$

| | | |
|---|---|---|
| sin 5° | 0.0872 | 0.3% |
| tan 5° | 0.0875 | |
| sin 10° | 0.1737 | 1.6% |
| tan 10° | 0.1763 | |
| sin 20° | 0.3420 | 6.4% |
| tan 20° | 0.3640 | |

2. **Flow is horizontal and uniform in a vertical section. Flux per unit width at a section**

$$q = -Kh\frac{dh}{dx}$$

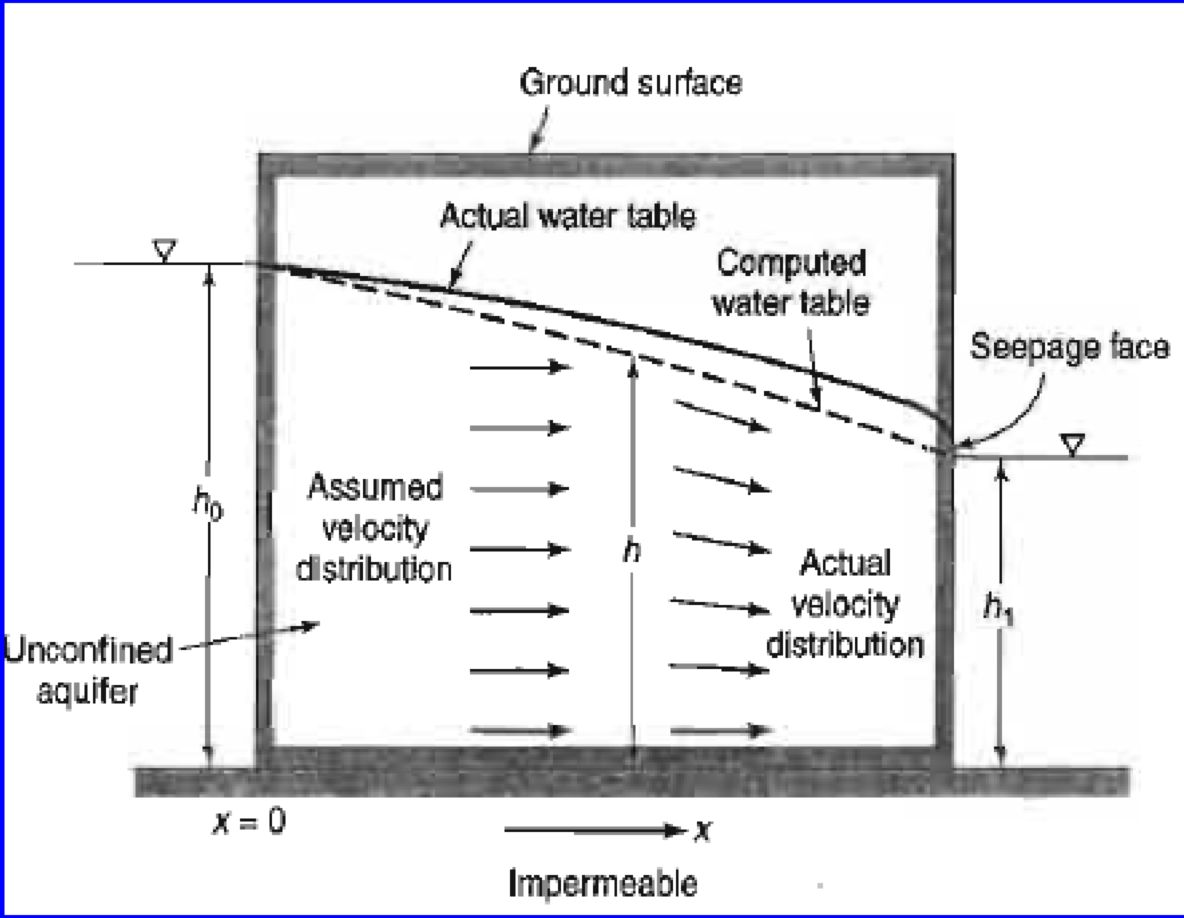
$$q = -\frac{Kh^2}{2X} + C$$

If $h = h_0$ at $X = 0$

$$C = +K\frac{h_0^2}{2}$$

$$q = -\frac{K}{2x}(h^2 - h_0^2)$$

$$q = \frac{K}{2x}(h_0^2 - h^2)$$



- **This indicates W.T. of parabolic form.**
- **Dupuit assumptions become increasingly poor approximations to actual flow.**
- **Actual W.T. deviates more and more from computed W.T. in the flow direction.**
- **W.T. actually approaches the boundary tangentially above water surface and forms a seepage face.**

**This indicates that W.T. is not of parabolic form; however, for flat slopes, where**

$$\sin\theta = \tan\theta$$

**It closely predicts W.T. position except near the outflow.**

**EXAMPLE 4.1.2**

A stratum of clean sand and gravel between two channels (see Figure 4.1.2) has a hydraulic conductivity $K = 10^{-1}$ cm/sec, and is supplied with water from a ditch ($h_0 = 6.5$ m deep) that penetrates to the bottom of the stratum. If the water surface in the second channel is 4 m above the bottom of the stratum and its distance to the ditch is $x = 150$ m (which is also the thickness of the stratum), estimate the unit flow rate into the gallery.

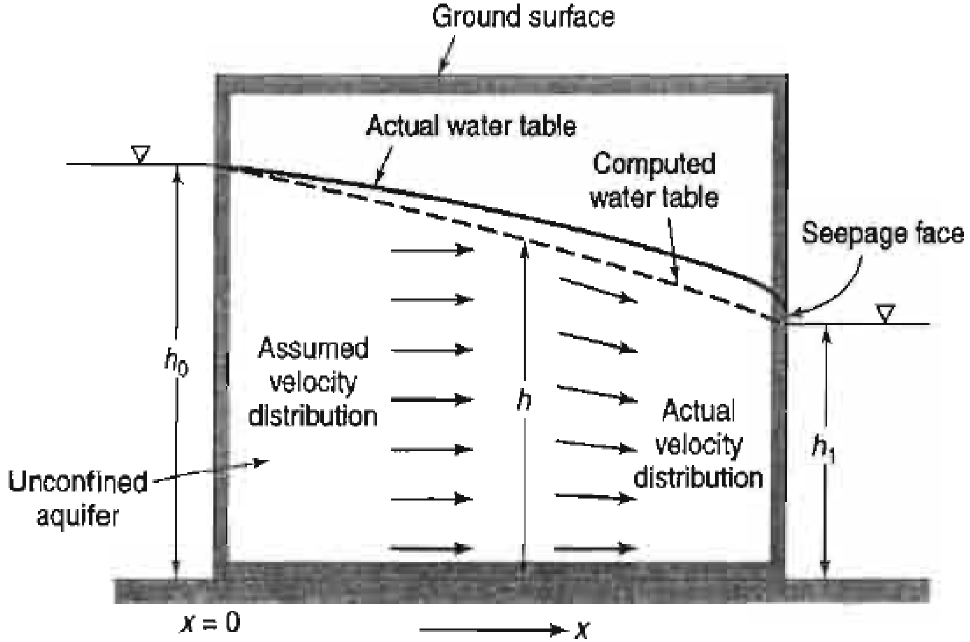



**Figure 4.1.2.** Steady flow in an unconfined aquifer between two water bodies with vertical

*SOLUTION*

The flow is computed using the Dupuit equation (4.1.6) for unit flow, where

$$K = 10^{-1} \text{ cm/sec} = 86.4 \text{ m/day}$$

$$q = \frac{K}{2x}\left(h_0^2 - h^2\right) = \frac{86.4 \text{ m/day}}{2(150 \text{ m})}\left(6.5^2 - 4^2\right)\text{m}^2 = 7.56 \text{ m}^2/\text{day}$$

■

# STEADY RADIAL FLOW TO A WELL:

## A. Confined Aquifer

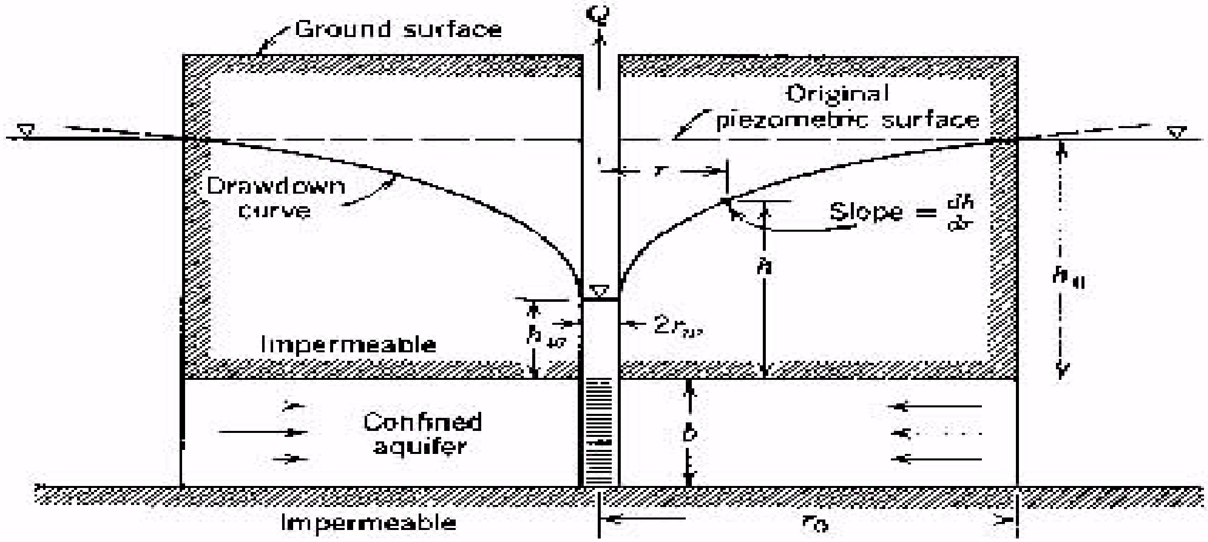
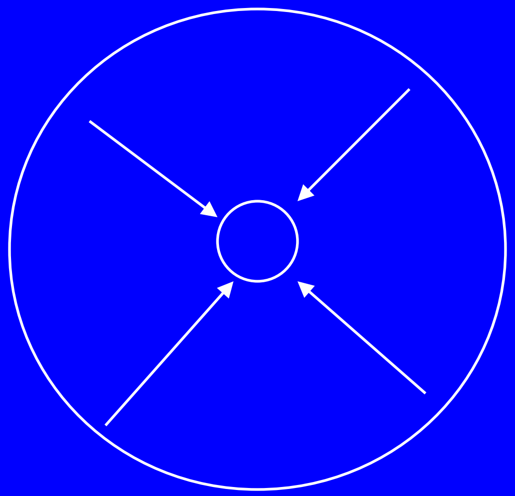



**Fig. 4.4** Steady radial flow to a well penetrating a confined aquifer on an island.

**When well is pumped, water is removed from aquifer surrounding the well and W.T. or P.S. lowered depending upon the type of aquifer.**

Drawdown - Distance the water level is lowered.



Cone of Depression - 3D

Area of Influence - 2D

Radius of Influence - 1D

# Assumptions for Well Flow Equations

1. Const. Discharge
2. Fully Penetrating Well
3. Homogeneous, isotropic, horz. aquifer with infinite horz. extent
4. Water released immediately from aquifer storage due to W.T. or P.S. decline

$$Q = AV$$

$$Q = (2\pi r b)K\frac{dh}{dr}$$

$$Q\int_{r_2}^{r_1}\frac{dr}{r} = 2\pi b\int_{h_2}^{h_1} dh$$

$$Q = \frac{2\pi K b(h_2 - h_1)}{\ln(\frac{r_2}{r_1})}$$

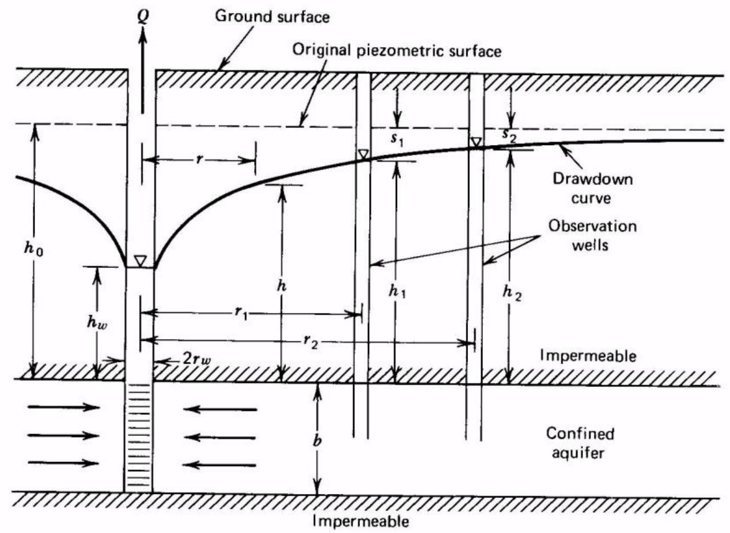



**Fig. 4.5** Radial flow to a well penetrating an extensive confined aquifer.

*h* – Piezometric head above aquifer bottom

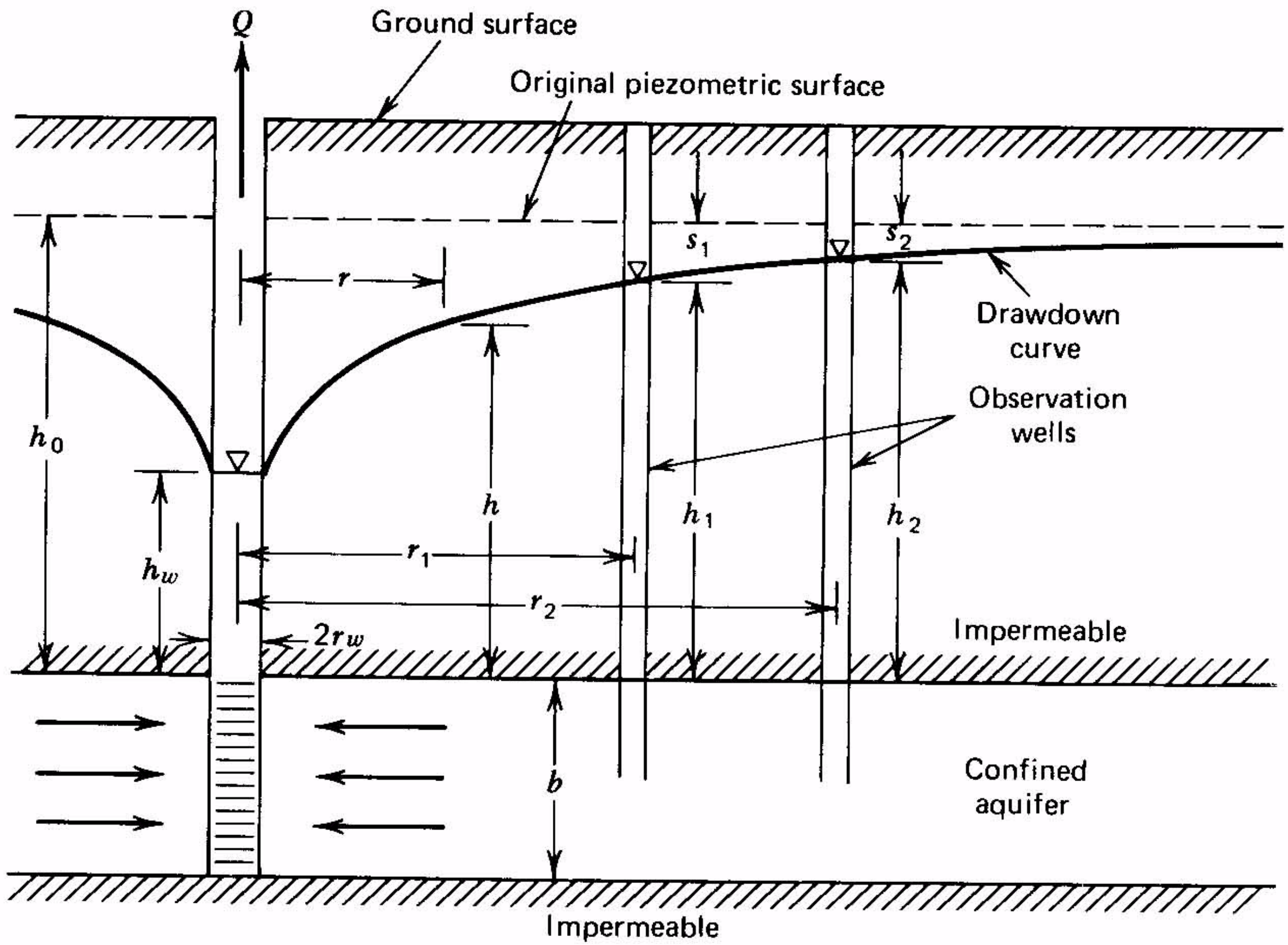



**Fig. 4.5** Radial flow to a well penetrating an extensive confined aquifer.

# For Infinite Aquifer

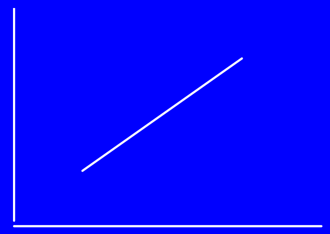
$$\boldsymbol{h_1 \rightarrow h_w}$$

$$\boldsymbol{h_2 \rightarrow h_0} \quad \textbf{(Original P.S.)}$$

$$\boldsymbol{Q = 2\pi T \frac{(h_o - h_w)}{\ln(r_o / r_w)}}$$

Equilibrium Equation (Thiem Equation) Valid within the radius of influence

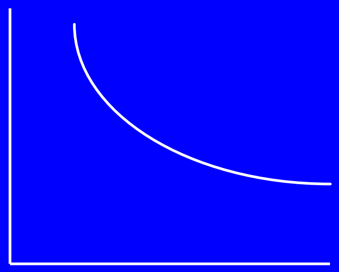
**h**



**ln r**

$$K = \frac{Q \ln(r_2 / r_1)}{2\pi b(h_2 - h_1)}$$

**h**



t

**Value of h must be measured in steady state condition only. Not a very practical method of determining K.**

***EXAMPLE 4.2.1***

A well fully penetrates a 25-m thick confined aquifer. After a long period of pumping at a constant rate of 0.05 $m^3/s$, the drawdowns at distances of 50 and 150 m from the well were observed to be 3 and 1.2 m, respectively. Determine the hydraulic conductivity and the transmissivity. What type of unconsolidated deposit would you expect this to be?

***SOLUTION***

Use Equation 4.2.5 to compute the hydraulic conductivity with $Q = 0.05\ m^3/s$, $r_1 = 50$ m, $r_2 = 150$ m, $s_1 = h_0 - h_1$, and $s_2 = h_0 - h_2$, so $s_1 - s_2 = h_2 - h_1 = 3 - 1.2 = 1.8$ m. $Q = 0.05\ m^3/s = 4320\ m^3/day$, and

$$K = \frac{Q}{2\pi b(h_2 - h_1)} \ln\left(\frac{r_2}{r_1}\right) = \frac{4320\ m^3/day}{2\pi(25\ m)(1.8\ m)} \ln\left(\frac{150}{50}\right) = 16.8\ m/day$$

The transimissivity is $T = Kb = (16.8\ m/day)(25\ m) = 420\ m^2/day$. Referring to Figure 3.2.2 and Table 3.2.1 with $K = 1.94 \times 10^{-4}$ m/s shows that this aquifer is probably a medium clean sand. ■

**EXAMPLE 4.2.2** A 1-m diameter well penetrates vertically through a confined aquifer 30 m thick. When the well is pumped at 113 $m^3$/hr, the drawdown in a well 15 m away is 1.8 m; in another well 50 m away, it is 0.5 m. What is the approximate head in the pumped well for steady-state conditions and what is the approximate drawdown in the well? Also compute the transmissivity of the aquifer and the radius of influence of the pumping well. Take the initial piezometric level as 40 m above the datum.

**SOLUTION** First determine the hydraulic conductivity using Equation 4.2.5: $Q = 113\ m^3/hr = 2712\ m^3/day$. Then

$$K = \frac{Q}{2\pi b(s_1 - s_2)}\ln\left(\frac{r_2}{r_1}\right) = \frac{2712\ m^3/day}{2\pi(30\ m)(1.8\ m - 0.5\ m)}\ln\left(\frac{50}{15}\right) = 13.3\ m/day$$

The transmissivity is $T = Kb = 13.3\ m/day \times 30\ m = 400\ m^2/day$.

To compute the approximate head, $h_w$, in the pumped well, rearrange Equation 4.2.5 and use $h_2 = h_0 - s_2 = 40 - 0.5 = 39.5$ m

$$h_w = h_2 - \frac{Q}{2\pi Kb}\ln\left(\frac{r_2}{r_w}\right) = 39.5\ m - \frac{2712\ m^3/day}{2\pi(13.3\ m/day)(30\ m)}\ln\left(\frac{50\ m}{0.5\ m}\right) = 34.5\ m$$

Drawdown is then

$$s_w = h_0 - h_w = 40\ m - 34.5\ m = 5.5\ m$$

The radius of influence ($R$) of pumping well can be found by rearranging Equation 4.2.5 and solving for $r_0$ which is $R$:

$$R = (r_1)\exp\left[\frac{2\pi Kb(h_0 - h_1)}{Q}\right] = (15\ m)\exp\left[\frac{2\pi(13.3\ m/day)(30\ m)(40\ m - 38.2\ m)}{2712\ m^3/day}\right] = 79\ m$$

■

# B. Unconfined Aquifer

$$Q = (2\pi r h) K \frac{dh}{dr}$$

$$Q \int_{r_1}^{r_2} \frac{dr}{r} = 2\pi K \int_{h_1}^{h_2} h\,dh$$

$$Q = \pi K \frac{(h_2^2 - h_1^2)}{\ln(r_2 / r_1)}$$

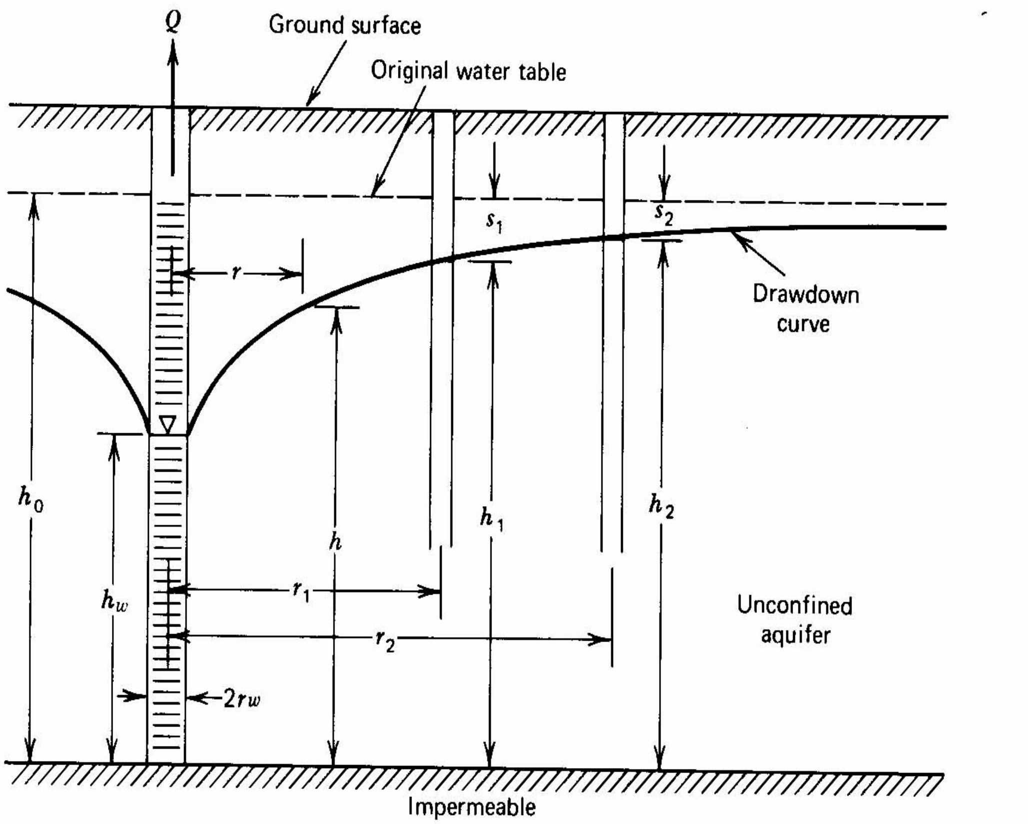



**Fig. 4.6** Radial flow to a well penetrating an unconfined aquifer.

**If aquifer is infinite $h_2 \rightarrow h_0$ (orig. static water level) and $h_1 \rightarrow h_w$**

$$Q = \pi K \frac{(h_0^2 - h_w^2)}{\ln(r_0 / r_w)}$$

**If *h* is constant, i.e., steady state cond.**

$$(h_0^2 - h_w^2) = (h_0 - h_w)(h_0 + h_w)$$

$$= 2(h_0 - h_w)\bar{h}$$

**Av. thickness**

$$\bar{h} = \frac{(h_0 + h_w)}{2}$$

$$\bar{T} = K\bar{h}$$

$$Q = 2\pi\bar{T} \frac{(h_0 - h_0)}{\ln(r_0 / r_w)}$$

**EXAMPLE 4.2.3**

A well penetrates an unconfined aquifer. Prior to pumping the water level (head) is $h_0 = 25$ m. After a long period of pumping at a constant rate of 0.05 $m^3$/s, the drawdowns at distances of 50 and 150 m from the well were observed to be 3 and 1.2 m, respectively. Compute the hydraulic conductivity of the aquifer and the radius of influence of pumping well. What type of deposit is the aquifer material?

***SOLUTION***

Use Equation 4.2.10 to compute $K$ with $Q = 0.05\ \mathrm{m^3/s} = 4320\ \mathrm{m^3/day}$, $r_1 = 50$ m, $r_2 = 150$ m, $h_1 = 25 - 3 = 22$ m, and $h_2 = 25 - 1.2 = 23.8$ m.

$$K = \frac{Q}{\pi\left(h_2^2 - h_1^2\right)} \ln\left(\frac{r_2}{r_1}\right) = \frac{4320\ \mathrm{m^3/day}}{\pi\left(23.8^2 - 22^2\right)} \ln\left(\frac{150\ \mathrm{m}}{50\ \mathrm{m}}\right) = 18.3\ \mathrm{m/day}$$

The deposit is probably a medium clean sand. Equation 4.2.10 is used to compute the radius of influence:

$$R = (r_1)\exp\left[\frac{K\pi\left(h_0^2 - h_1^2\right)}{Q}\right] = (50\ \mathrm{m})\exp\left[\frac{(18.3\ \mathrm{m/day})\pi\left(25^2 - 22^2\right)}{4320\ \mathrm{m^3/day}}\right] = 327\ \mathrm{m}$$

■

*EXAMPLE 4.2.4* A well 0.5 m in diameter penetrates 33 m below the static water table. After a long period of pumping at a rate of 80 $m^3$/hr, the drawdowns in wells 18 and 45 m from the pumped well were found to be 1.8 and 1.1 m respectively. (a) What is the transmissivity of the aquifer? (b) What is the approximate drawdown in the pumped well? (c) Determine the radius of influence of the pumping well.

*SOLUTION*

(a) Use Equation 4.2.10 for steady-state radial flow to a well in an unconfined aquifer to compute the hydraulic conductivity, where $Q = 80$ $m^3$/hr = 1920 $m^3$/day; $h_1 = 33 - 1.8 = 31.2$ m.; $h_2 = 33 - 1.1 = 31.9$ m; $r_2 = 45$ m and $r_1 = 18$ m:

$$K = \frac{Q}{\pi\left(h_2^2 - h_1^2\right)} \ln\left(\frac{r_2}{r_1}\right) = \frac{1920 \text{ m}^3/\text{day}}{\pi\left(31.9^2 - 31.2^2\right)} \ln\left(\frac{45}{18}\right) = 12.7 \text{ m/day}$$

The transmissivity is computed as $T = Kb = 12.7$ m/day × 33 m = 418 $m^2$/day.

(b) Next compute the head and drawdown at the well. First rearrange Equation 4.2.10 to solve for the head at the well:

$$h_w = \sqrt{h_2^2 - \frac{Q}{\pi K} \ln\left(\frac{r_2}{r_1}\right)} = \sqrt{31.2^2 - \frac{1920 \text{ m}^3/\text{day}}{\pi(12.68 \text{ m/day})} \ln\left(\frac{18 \text{ m}}{0.25 \text{ m}}\right)} = 27.7 \text{ m}$$

The drawdown is computed as $s_w = 33$ m − 27.7 m = 5.3 m.

(c) The radius of influence of the pumping well is computed by rearranging Equation 4.2.5:

$$R = (r_1)\exp\left(\frac{\pi K\left(h_0^2 - h_1^2\right)}{Q}\right) = (45 \text{ m})\exp\left(\frac{\pi(12.68 \text{ m/day})\left(33^2 - 31.9^2\right)}{1920 \text{ m}^3/\text{day}}\right) = 198 \text{ m}$$

■

# C. Well Flow in Uniform Recharge

**Equilibrium cond. or steady state cond. can be reached in unconfined aquifers due to recharge from rainfall or irrigation.**

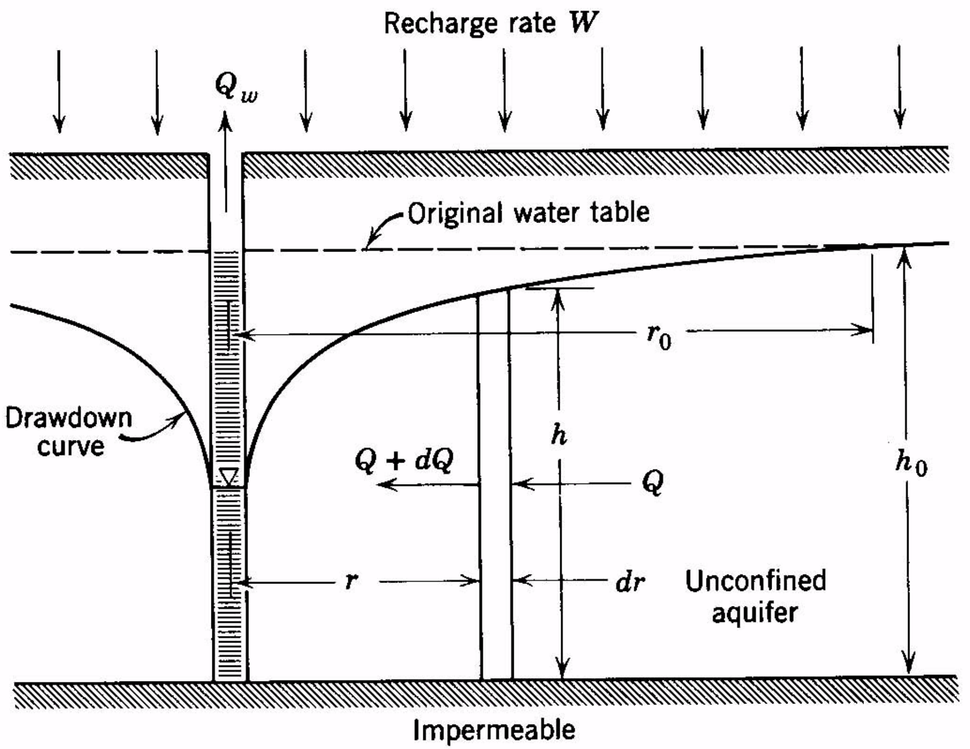



**Fig. 4.7** Steady flow to a well penetrating a uniformly recharged unconfined aquifer.

- **Uniform Recharge Rate**

  **= w  cfs/ft$^2$**

- **Well Flow**

  $Q = \pi r_0^2 w$

  $r_0$ = ***radius of influence***

- **Horizontal flow  thru vertical cylinder (*r* < *ro*)**

$$q = Q - \pi r^2 w$$

- **Also, flux *q***

$$q = (2\pi r h) K \frac{dh}{dr}$$

$$\pi r_0^2 w - \pi r^2 w = 2\pi Khr \frac{dh}{dr}$$

$$\pi w (r_0^2 - r^2) = 2\pi Khr \frac{dh}{dr}$$

$$\int_{r_w}^{r_0} \left(\frac{r_0^2}{r} - r\right) dr = 2 \frac{K}{w} \int_{h_w}^{h_0} dh$$

**Integrating and Substituting** $\frac{Q}{\pi w} = r_o^2$ **in ln term,**

**and multiplying by** $\frac{w}{K}$

$$\frac{Q}{\pi K} \ln \frac{r_o}{r_w} - \frac{w}{2K}(r_0^2 - r_w^2)$$

$$\frac{Q}{\pi K}\ln\frac{r_o}{r_w}-\frac{w}{2K}(r_0{}^2-r_w{}^2)=(h_0^2-h_w^2)$$

**If *w* known, compute $r_0$ for given *Q* and , or estimate *w* if other parameters known, or estimate  if *w* and other parameters known.**

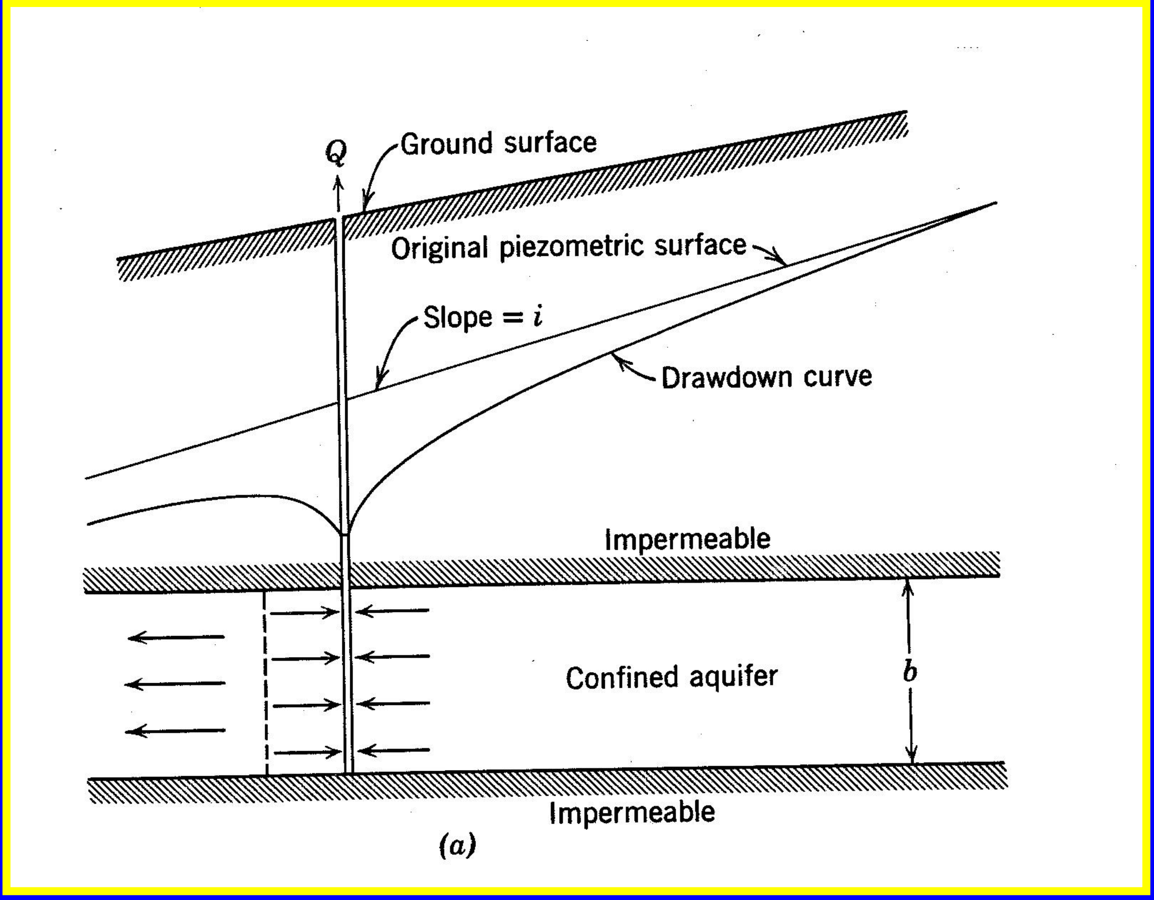
**Note:**

$$r_0 = f(Q, w)$$

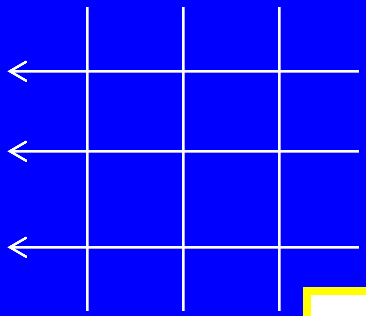
$r_0$ **independent of h and K**

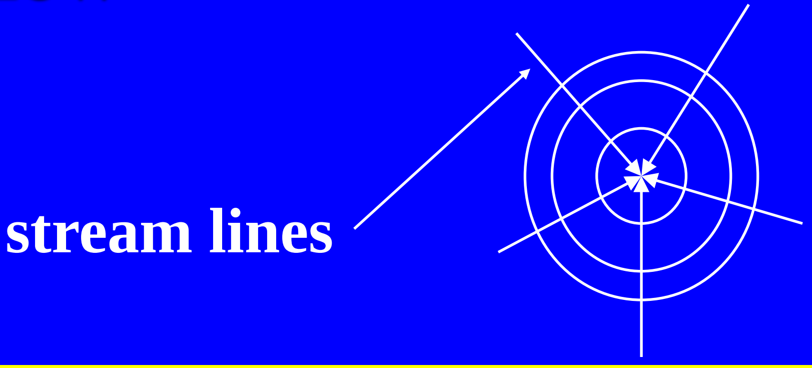
# D. Well in a Uniform Flow

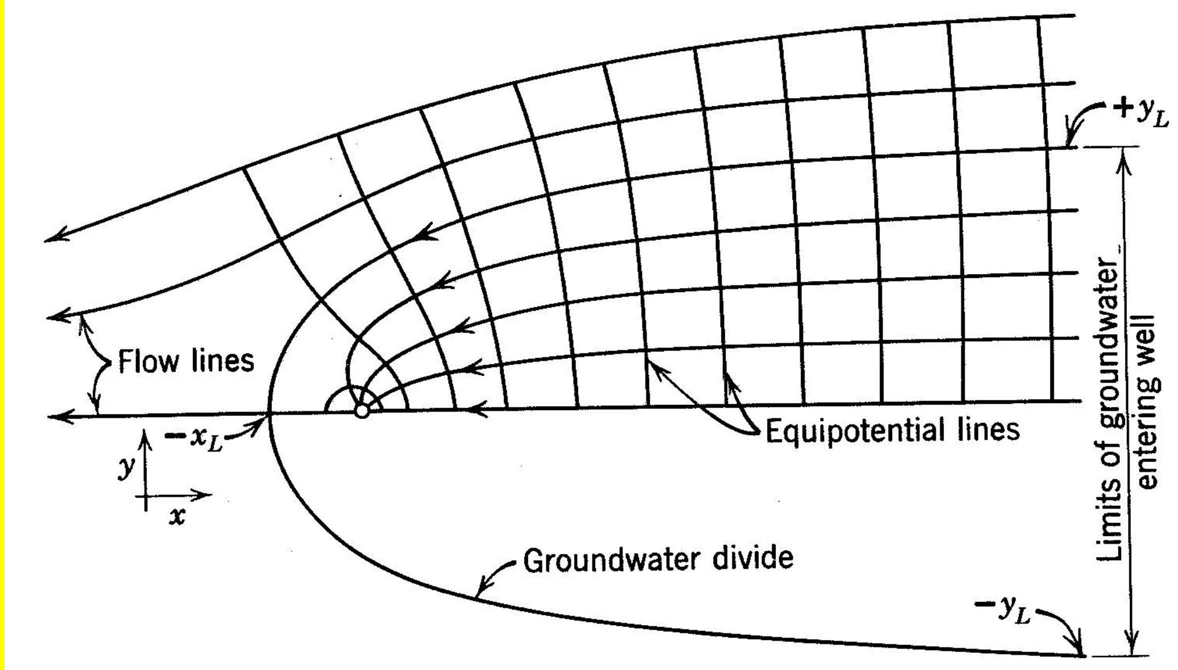
- P - Stagnation Point



# Uniform Flow






*(b)*

# Radial Flow

- **Used in Well Head Protection Plan (WHPA)**

**•Circular area of influence for radial flow becomes distorted. Wenzel -**

$$Q = K\bar{i}(2\pi r\bar{h}) = z\pi Kr + \frac{(i_u + i_d)}{z} * \frac{(h_u + h_d)}{z}$$

$$2Q = \pi Kr(i_u + i_d)(h_u + h_d)$$

$$K = \frac{2Q}{\pi r(h_u + h_d)(i_u + i_d)}$$

$Q =$ discharge ;

$i_u$ & $i_d$ – hyd. grads. U/S & D/S at a distance r from well;

$h_u$ & $h_d$ – hyd. heads U/S & D/S

- For unconfined aquifer, $h_0$ – sat. thickness
- For confined aquifer,

$$(h_u + h_d) = 2b; b - \text{aquifer thickness}$$

- Boundary of the flow area -

$$-\frac{y}{x} = \tan\frac{(2\pi\, K\, b\, i\, y)}{Q}$$

- Origin at well

*b* - aquifer thickness

*Q* – discharge

*i* - natural piez. slope

K - Perm

- **Boundary asymptotically approaches as**

$$x \to \infty, \quad -\frac{y}{x} \to -0$$

$$\tan(\alpha) \to -0$$

$$\alpha \to \pi$$

$$\frac{2\pi Kbiy}{Q} = \pi$$

$$y_L = \pm \frac{Q}{2Kbi}$$

- **Boundary of contributing area extends to stagnation point P, where**

$$x = -\frac{Q}{2\pi Kbi}$$

- **Boundary equation, *Y* and *X* applicable to unconfined aquifer, replace *b* by $h_0$ - sat. aquifer thickness, if drawdown is small compared to aquifer thickness.**

# E. Flow to Parallel Streams (Drainage Flow or Base Flow)

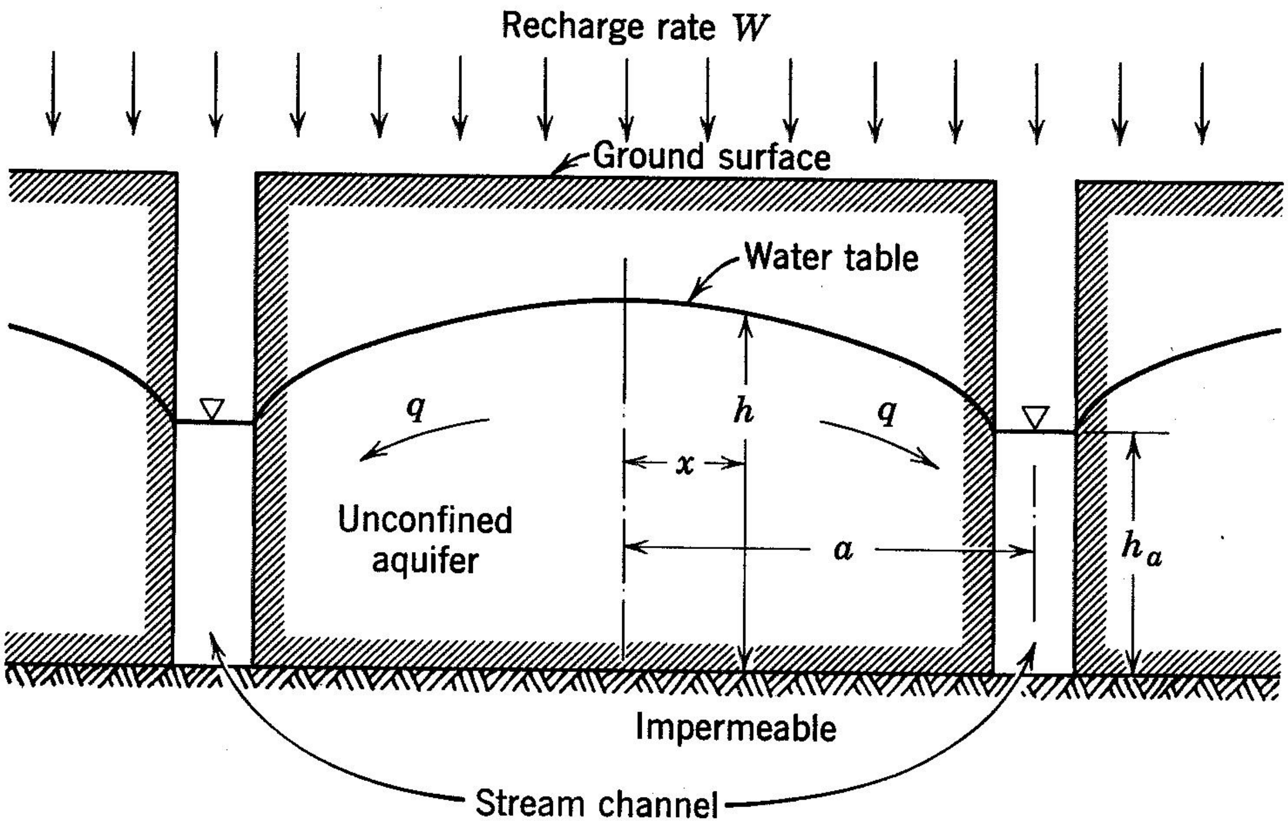



**Fig. 4.3** Steady flow to two parallel streams from a uniformly recharged unconfined aquifer.

- **Recharge rate continuously occurring over the area**

$$wx = [-Kh\frac{dh}{dx}]$$

$$W\int_{x}^{a} x\,dx = -K\int_{h}^{h_a} h\,dh$$

$$w\frac{(a^2 - x^2)}{2} = -K\frac{(h_a^2 - h^2)}{2}$$

$$\frac{w}{K}(a^2 - x^2) = h^2 - h_a{}^2$$

$$h^2 = h_a{}^2 + \frac{w}{K}(a^2 - x^2)$$

$$\left.\frac{\partial h}{\partial x}\right|_{x=a}$$

***Flux to stream***

$$q = Kha\left.\frac{\partial h}{\partial x}\right|_{x=a}$$

**EXAMPLE 4.3.1**

A fully penetrating production well with a radius of 0.5 m pumps at the rate of 15 L/s from a 35-m thick confined aquifer with a hydraulic conductivity of 20 m/day. If the distance and the observed piezometric head drop between two observation wells were 1000 m and 3 m, respectively, before the production well was installed, determine the longitudinal and transverse limits of groundwater entering the well.

***SOLUTION***

First determine the slope of the piezometric surface under natural conditions (i.e., before the production well was installed):

$$i = \frac{\Delta h}{\Delta x} = \frac{3\text{ m}}{1000\text{ m}} = 0.003$$

It is assumed that the observation wells were aligned with the groundwater flow direction. Then, using Equations 4.3.3 and 4.3.4, compute the limits of groundwater entering the well on a horizontal plane (i.e., plan view) for $Q = 15$ L/s $= 1296$ m$^3$/day:

$$y_L = \pm\frac{Q}{2Kbi} = \pm\frac{1296\text{ m}^3/\text{day}}{2(20\text{ m/day})(35\text{ m})\times 0.003} = \pm 308\text{ m}$$

$$x_L = -\frac{Q}{2\pi Kbi} = -\frac{1296\text{ m}^3/\text{day}}{2\pi(20\text{ m/day})(35\text{ m})\times 0.003} = -98.2\text{ m}$$

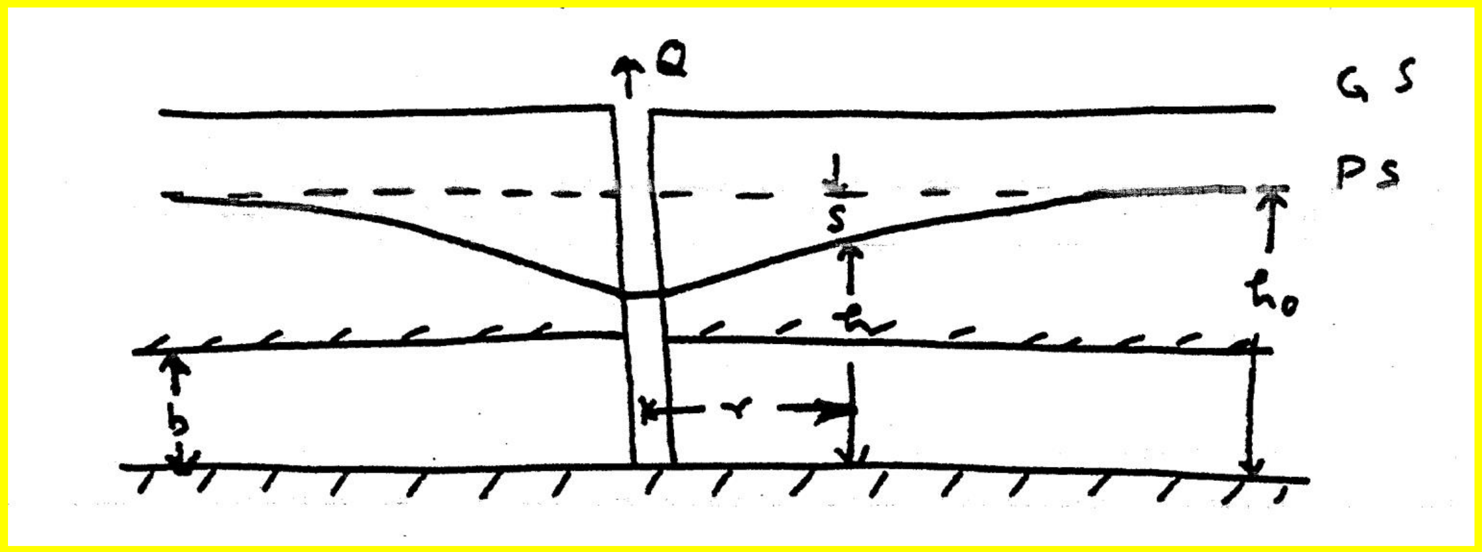
A practical result is that contaminant sources farther than 98.2 m downstream of the well or ± 308 m in the transverse direction do not impact the well. ■

# Unsteady Radial Flow to a Well

- **Extensive Confined Aquifer**

$$T\{\frac{\partial^2 h}{\partial x^2}+\frac{\partial^2 h}{\partial y^2}\}=S\,\frac{\partial h}{\partial t}$$

$$Q = S \cdot \Delta h \cdot \text{area of influence}$$




- **Polar coordinate system**

$$\frac{\partial^2 h}{\partial r^2}+\frac{1}{r}\left(\frac{\partial h}{\partial r}\right)=\frac{S}{T}\frac{\partial h}{\partial t}$$

$$T = Kb$$

- **Boundary Conditions**

$$h = h_o \qquad \text{at} \quad t = o$$

$$h = h_o \qquad \text{as} \quad r \to \infty \quad t > o$$

$$\lim_{r \to o} \left( r \frac{\partial h}{\partial r} \right) = \frac{Q}{2\pi T}$$

$$s = (h_o - h) \qquad \textit{where}: \; W(u) \quad \text{well function (Tables available)}$$

$$= \frac{Q}{4\pi T} \int_u^\infty \frac{e^{-u}}{u} du \qquad u = \frac{r^2 S}{4Tt}$$

$$= \frac{Q}{4\pi T} W(u)$$

$$= \frac{Q}{4\pi T} [-0.5772 - \ln u + u - \frac{u^2}{2.21} + \frac{u^3}{3.31} - \frac{u^4}{4.41} + \ldots]$$

## Application

1. To find the aquifer parameters or formation constants S & T
2. To determine drawdown for specified Q, S, T, & t

## Assumptions

1. Extensive confined aquifer
2. Homogeneous and isotropic aquifer
3. Well penetrates the entire aquifer
4. Well diameter is small
5. Water is removed instantaneously from storage with decline in head

# A. Theis Method

$$s = \left(h_o - h\right) = \frac{QW(u)}{4\pi T}$$

where :

$s$ = drawdown, ft or m

$$u = \frac{r^2 s}{4Tt}$$

$T$ = Trans., ft $^2/d$ or $m^2/d$, $gpd/ft$

$Q$ = disch; ft $^3/d$ or $m^3/d$, $gpm$

# Converting to field units

$$s = (h_o - h) = \frac{114.6}{T} Q W(u) \qquad \left( 114.6 = \frac{1440}{4\pi} \right)$$

$(h_0 - h) = s$ drawdown, ft.

$Q$ – discharge, gpm

$W(u)$ – dimensionl ess, Well Function

$T$ – Trans., gpd/ft.

$$u = \frac{1.87 r^2 S}{Tt}$$

$r =$ distance from well, ft.

$S =$ storage coeff., dimensionless

$t =$ time, days

$$1.87 = \left(\frac{7.48}{4}\right)$$

$$s = \left(\frac{114.6Q}{T}\right) W(u) \quad (1) \quad \log s = \log\left(\frac{114.6Q}{T}\right) + \log W(u)$$

$$\frac{r^2}{t} = \left(\frac{T}{1.87S}\right) u \quad (2) \quad \log\frac{r^2}{t} = \log\left(\frac{T}{1.87S}\right) + \log u$$

$\frac{114.6Q}{T}$ and $\frac{T}{1.87S}$ are constant or a test.

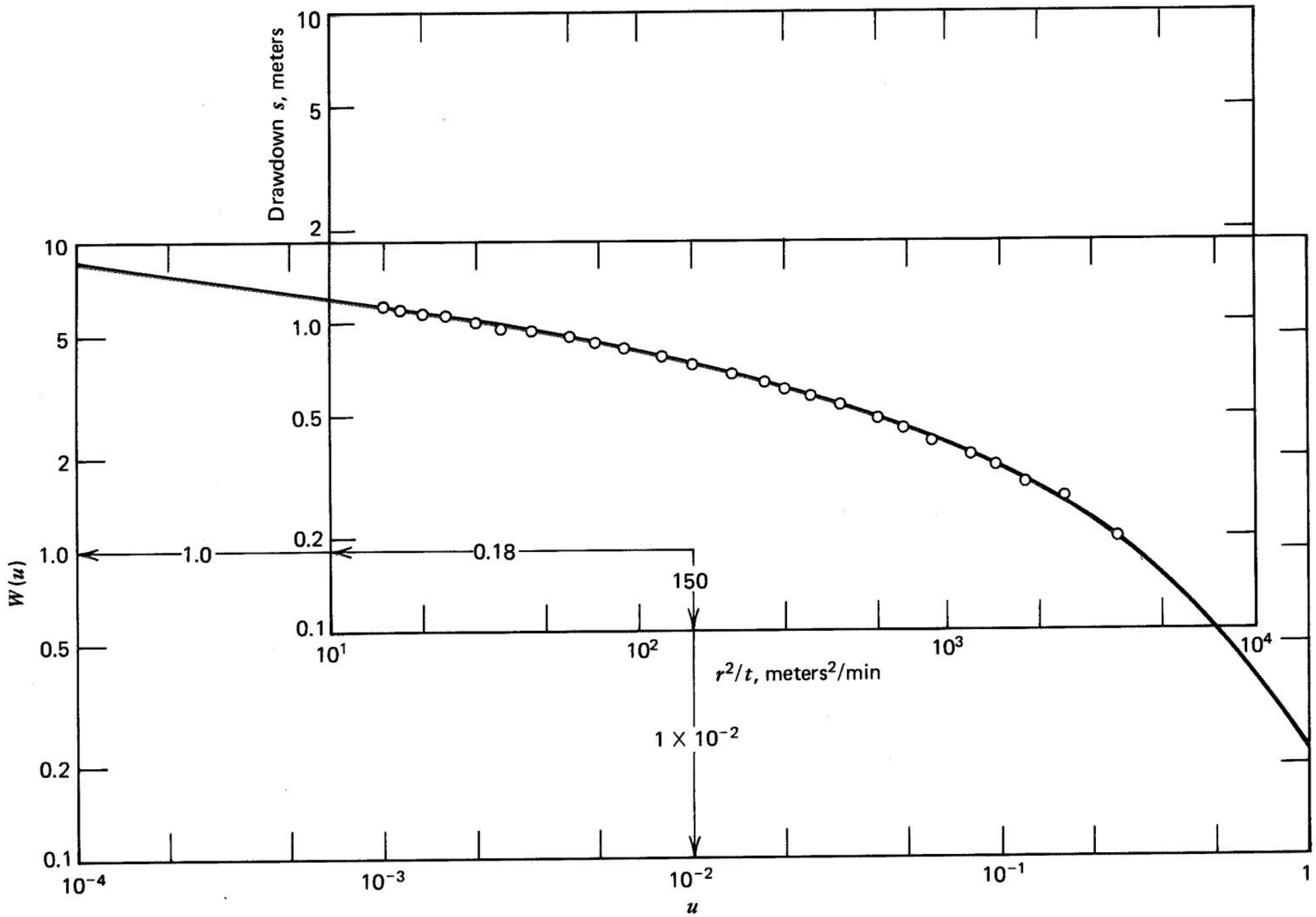



**Fig. 4.9** Theis method of superposition for solution of the nonequilibrium equation.

**Match the two curves. Locate a match point and obtain all coordinates. Solve for S & T.**

**S, $r^2/t$**

**W(u), u**

**(1) Insert s, W(u), and Q in Eq. (1) ---- T**

**(2) Substitute $r^2/t$, u, T in Eq. (2) --- S**

# For metric system:

$$s = \left(\frac{Q}{4\pi T}\right) W(u) \qquad T = \frac{Q}{4\pi s} W(u)$$

$$\frac{r^2}{t} = \left(\frac{4T}{S}\right) u \qquad \text{as} \ \ u = \frac{r^2 S}{4Tt}, \qquad S = \frac{4Ttu}{r^2}$$

**EXAMPLE 4.4.1** Drawdown was measured during a pumping test at frequent intervals in an observation well 200 ft from a well that was pumped at a constant rate of 500 gpm. The data for this pump test is listed in the table. These measurements show that the water level is still dropping after 4,000 minutes of pumping; therefore, analysis of the test data requires use of the Theis nonequilibrium procedure. Determine $T$ and $S$ for this aquifer.

Pump test data

| Time (min) | Drawdown (ft) |
|---|---|
| 1 | 0.05 |
| 2 | 0.22 |
| 3 | 0.40 |
| 4 | 0.56 |
| 5 | 0.70 |
| 7 | 0.94 |
| 10 | 1.2 |
| 20 | 1.8 |
| 40 | 2.5 |
| 100 | 3.4 |
| 300 | 4.5 |
| 1,000 | 5.6 |
| 4,000 | 7.0 |

***SOLUTION***

**Step 1.** Plot the time–drawdown data on log–log graph paper. The drawdown is plotted on the vertical axis and the time since pumping started on the horizontal axis (not shown).

**Step 2.** Superimpose this plot on the type curve sheet of the same size and scale as the time-drawdown plot, so that the plotted points match the type curve. The axes of both graphs must be kept parallel.

**Step 3.** Select a match point, which can be any point in the overlap area of the curve sheets. It is usually most convenient to select a match point where the coordinates on the type curve are known in advance (e.g., $W(u) = 1$ and $1/u = 1$ or $W(u) = 1$ and $1/u = 10$, etc.). Then determine the value of $s$ and $t$ for this match point:

$$W(u) = 1 \qquad s = 1 \text{ ft} \qquad 1/u = 1 \qquad t = 2 \text{ min}$$

**Step 4.** Determine $T$

$$T = \frac{114.6\,Q}{s} W(u)$$

$$= \frac{114.6 \times 500}{1} \times 1 = 57300 \text{ gpd / ft}$$

**Step 5.** Determine $S$

$$S = \frac{Tt}{\frac{1}{u} \times 2693 r^2}$$

$$= \frac{57300 \times 2}{1 \times 2693 \times 200^2}$$

$$= 1.06 \times 10^{-3}$$

■

**EXAMPLE 4.4.2** A well penetrating a confined aquifer is pumped at a uniform rate of 2,500 $m^3$/day. Drawdowns during the pumping period are measured in an observation well 60 m away; observations of $t$ and $s$ are listed in Table 4.4.2. Using the Theis method determine $T$ and $S$ for this confined aquifer.

**SOLUTION** Values of $r^2/t$ in $m^2$/min are computed and appear in the right column of Table 4.4.2. Values of $s$ and $r^2/t$ are plotted on logarithmic paper. Values of $W(u)$ and $u$ from Table 4.4.1 are plotted on another sheet of logarithmic paper of the same size and scale, and a curve is drawn through the points. The two sheets are superposed and shifted with coordinate axes parallel until the observational points coincide with the curve, as shown in Figure 4.4.1. A convenient match point is selected with $W(u) = 1.00$ and $u = 1 \times 10^{-2}$, so that $s = 0.18$ m and $r^2/t = 150$ $m^2$/min $= 216{,}000$ $m^2$/day. Thus, from Equation 4.4.5,

$$T = \frac{Q}{4\pi s} W(u) = \frac{2500(1.00)}{4\pi(0.18)} = 1110 \text{ m}^2/\text{day}$$

and from Equation 4.4.6,

$$S = \frac{4Tu}{r^2/t} = \frac{4(1110)(1\times10^{-2})}{216{,}000} = 0.000206$$

■

**Table 4.4.2** Pumping Test Data

($r$ = 60 m)

| $t$, min | s, m | $r^2/t$, $m^2$/min | $t$, min | s, m | $r^2/t$, $m^2$/min |
|---|---|---|---|---|---|
| 0 | 0 | ∞ | 18 | 0.67 | 200 |
| 1 | 0.20 | 3,600 | 24 | 0.72 | 150 |
| 1.5 | 0.27 | 2,400 | 30 | 0.76 | 120 |
| 2 | 0.30 | 1,800 | 40 | 0.81 | 90 |
| 2.5 | 0.34 | 1,440 | 50 | 0.85 | 72 |
| 3 | 0.37 | 1,200 | 60 | 0.90 | 60 |
| 4 | 0.41 | 900 | 80 | 0.93 | 45 |
| 5 | 0.45 | 720 | 100 | 0.96 | 36 |
| 6 | 0.48 | 600 | 120 | 1.00 | 30 |
| 8 | 0.53 | 450 | 150 | 1.04 | 24 |
| 10 | 0.57 | 360 | 180 | 1.07 | 20 |
| 12 | 0.60 | 300 | 210 | 1.10 | 17 |
| 14 | 0.63 | 257 | 240 | 1.12 | 15 |

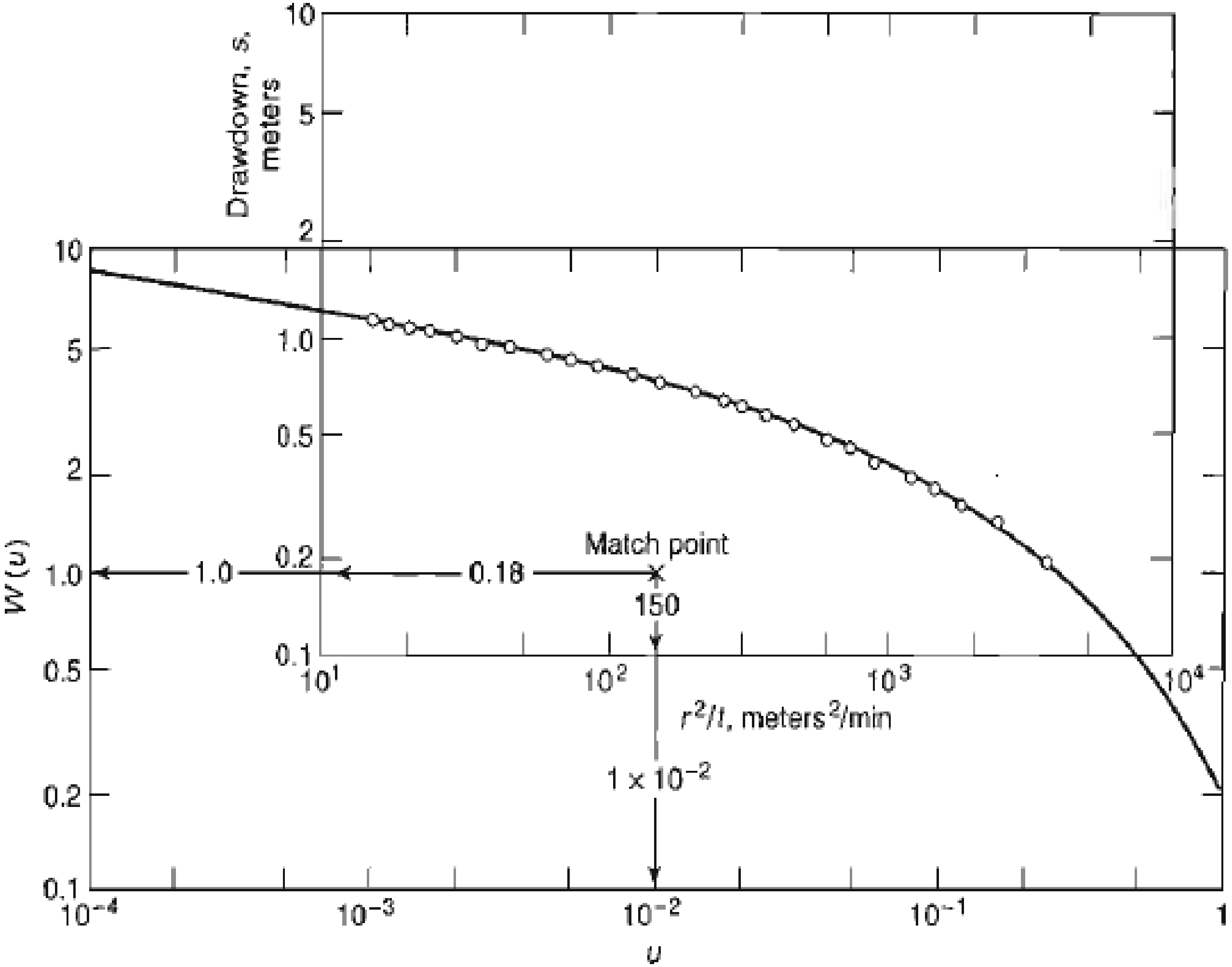



**Figure 4.4.1.** Theis method of superposition for solution of the nonequilibrium equation.

# B. Jacob-Cooper Method

$$u = \frac{r^2 S}{4Tt}$$

For small r and large t, u is small so that series terms become negligible after the first two terms.

$$s = \frac{Q}{4\pi T}\left(-0.5772 - \ln\ u\right) = \frac{Q}{4\pi T}\left[-\ln 1.781 - \ln \frac{r^2 s}{4rT}\right]$$

$$= \frac{Q}{4\pi T}\left[-\ln 1.781 \frac{r^2 s}{4Tt}\right] = \frac{Q}{4\pi T}\left[\ln \frac{4\pi T}{1.781 r^2 s}\right]$$

$$\ln = 2.3 \log_{10} \Rightarrow \ln = \log_e u$$

$$\frac{2.25Tt_o}{r^2 S} = 1$$

**or**

$$S = \frac{2.25Tt_0}{r^2}$$

**Thus, a plot of s vs. *t* forms a st. line.**
**Plot drawdown, s, from an OBS. well against time, $t_0$**
**Slope of the line gives *S & T values*.**

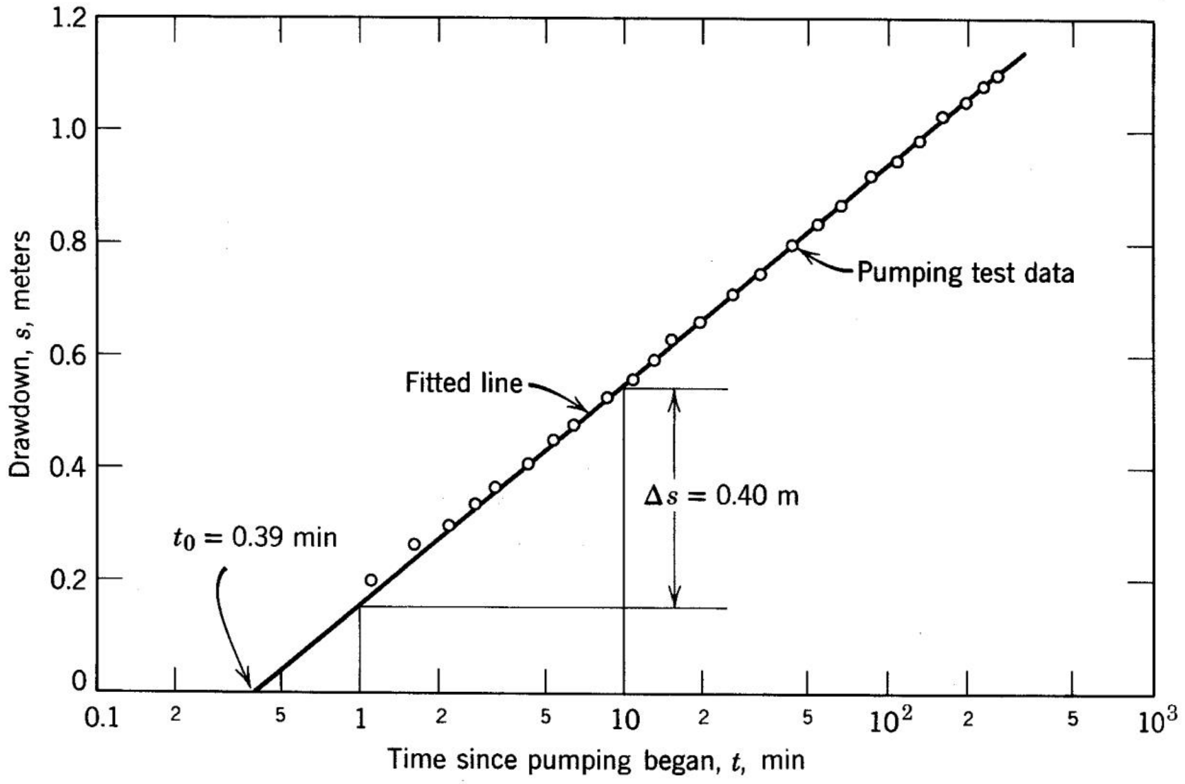



**Fig. 4.10** Cooper-Jacob method for solution of the nonequilibrium equation.

$$s = \frac{2.3Q}{4\pi T} \log \frac{2.25Tt}{r^2 S}$$

$$s = a \log bt$$

$$0 = \frac{2.3Q}{4\pi T} \log \frac{2.25Tt_0}{r^2 S}$$

**All parameters constant except t**

**Metric system** : From $\Delta S$ for one log cycle in Eq 1

$$\Delta s = s_2 - s_1 = \frac{2.3Q}{4\pi T} \log \frac{t_2}{t_1}$$

$$t_2 / t = 10, \quad \log t_2 / t_1 = 1$$

$$\Delta s = \frac{2 \cdot 3Q}{4\pi T}$$

$$T = \frac{2 \cdot 3Q}{4\pi \Delta s}$$

# Field Units

$$T = \frac{264Q}{\Delta s}$$

$$S = \frac{0.3Tt_0}{r^2}$$

$\Delta s =$ drawdown diff per log cycle of time

$t_0 =$ time at zero drawdown

$S =$ storage coefficien t

T = gpd/ft

Q = gpm

$$\frac{1440\text{x}2.3}{4\text{A}} = 264$$

$$\frac{2.25}{7.48} = 0.3$$

**To avoid large errors, *u* < *0.01* in this method.**

# Distance - Drawdown Method

**Theis:** ***s*** **vs** ***r*²/*t*  ; *t - constant; r- variable***

**Jacob Method:**

**Need 3 or more observation wells**

$$s = \frac{264Q}{T}\log\frac{0.3Tt}{r^2S}$$

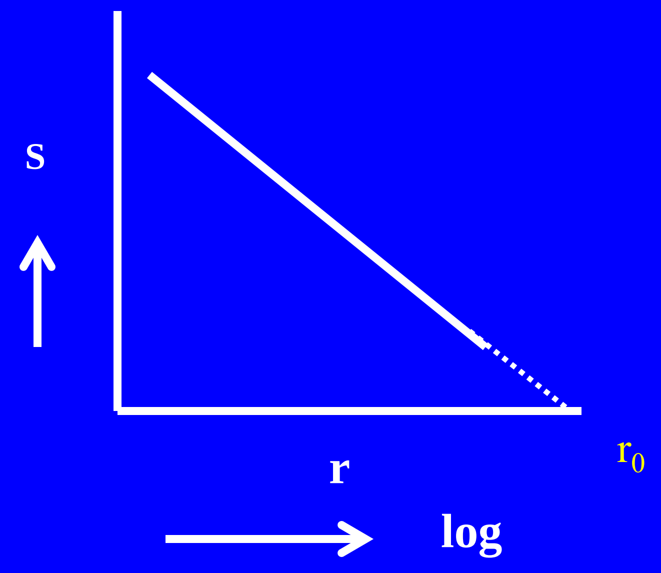
$$= \frac{528Q}{T}[\log\left(\frac{0.3Tt}{S}\right)\frac{1}{r}]$$

**or**

$$s = a \log \frac{b}{r}$$

$$\Delta s = s_2 - s_1 = \frac{528Q}{T}\log\frac{r_1}{r_2}$$

$$T = \frac{528 \quad Q}{\Delta s}$$




$$S = \frac{0.3Tt}{r^2}$$

$r_o$ = distance at zero breakdown, ft.

**Time - drawdown and distance - drawdown methods provide *S* & *T* values, which should be closely agreeable.**

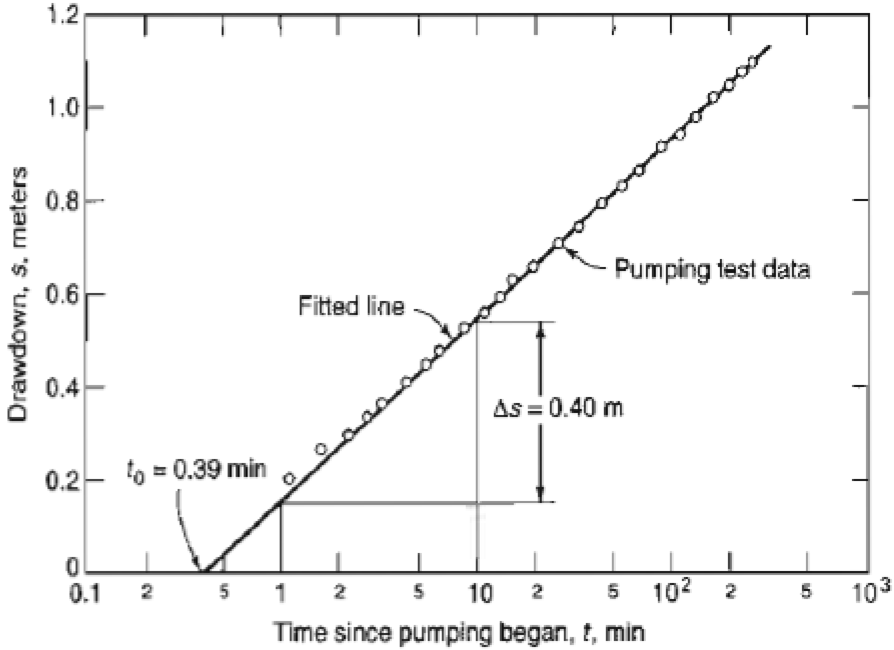



**Figure 4.4.2.** Cooper–Jacob method for solution of the nonequilibrium equation.

Thus, the procedure is first to solve for $T$ with Equation 4.4.12 and then to solve for $S$ with Equation 4.4.11. The straight-line approximation for this method should be restricted to small values of $u$ ($u < 0.01$) to avoid large errors.

***EXAMPLE 4.4.3***

Rework Example 4.4.2 using the Cooper–Jacob method.

***SOLUTION***

From the pumping test data in Table 4.4.2, $s$ and $t$ are plotted on semilogarithmic paper, as shown in Figure 4.4.2. A straight line is fitted through the points, and $\Delta s = 0.40$ m and $t_0 = 0.39$ min $= 2.70 \times 10^{-4}$ day are read. Thus,

$$T = \frac{2.30(2500)}{4\pi(0.40)} = 1144 \text{ m}^2/\text{day}$$

and

$$S = \frac{2.25Tt_0}{r^2} = \frac{2.25(1144)\left(2.70\times10^{-4}\right)}{(60)^2} = 0.000193$$

■

**EXAMPLE 4.4.4**

Using the Cooper–Jacob approximation, compute the rate of piezometric drawdown around a pumping well with respect to time. If the well is pumping at a constant rate of 55 gpm from a sandy confined aquifer with $T = 3{,}600\ \text{ft}^2/\text{day}$ and $S = 10^{-4}$, what is the time to reach near–steady-state conditions 200 ft from the pumping well? Assume that near–steady-state conditions are achieved when the drawdown rate falls below 0.5 in/hr (based on accuracy of groundwater level measurements with the available equipment). How does the answer change if the transmissivity of the aquifer is $1{,}200\ \text{ft}^2/\text{day}$?

**SOLUTION**

First, we must compute the critical time after which the Cooper–Jacob method becomes valid (i.e., $u < 0.01$) at 200 ft:

$$t \geq \frac{r^2 S}{4Tu} = \frac{(200\ \text{ft})^2 (1 \times 10^{-4})}{4(3600\ \text{ft}^2/\text{day})(0.01)} \rightarrow t \geq 40\ \text{min}$$

The drawdown is approximated by

$$s = \frac{Q}{4\pi T}\left(-0.5772 - \ln \frac{r^2 S}{4Tt}\right)$$

which can be rearranged to

$$s = \frac{Q}{4\pi T}\left(-0.5772 - \ln \frac{r^2 S}{4T} + \ln t\right)$$

Taking the derivative of drawdown with respect to time yields

$$\frac{ds}{dt} = \frac{Q}{4\pi T}\frac{1}{t}$$

This relationship implies that according to the Cooper–Jacob approximation, the rate of drawdown is independent of radial distance and is inversely proportional with time. The change in drawdown with respect to time and the time are, respectively,

$$\frac{ds}{dt} = \frac{Q}{4\pi T}\frac{1}{t} = 0.5\ \text{in/hr}$$

$$\frac{10587\ \text{ft}^3/\text{day}}{4\pi(3600\ \text{ft}^2/\text{day})}\frac{1}{t} = \frac{0.5}{12}\ \text{ft/hr}$$

$$t = 5.6\ \text{hr}$$

Note that the Cooper–Jacob approximation is satisfied so that the near–steady-state conditions at 200 ft will be reached after 5.6 hrs of pumping at this location. If the transmissivity were $1200\ \text{ft}^2/\text{day}$, the approximation would be valid when $t \geq 120$ min and the drawdown rate at 200 ft would be negligible after 16.8 hours of pumping. Thus, it would take longer to reach steady conditions with a lower transmissivity. ■

# Recovery Method

**Time - drawdown measurements during pumping and time-recovery measurements during recovery provide two sets of data from an aquifer test.**

- **Values of *T & S* serve to check calculations during pumping.**
- **If an obs. well available, take water level recovery data to obtain *T & S*.**
- **Where no obs. available, water level recovery data from pumped well used to calculate *T* only.**

# Recovery Method

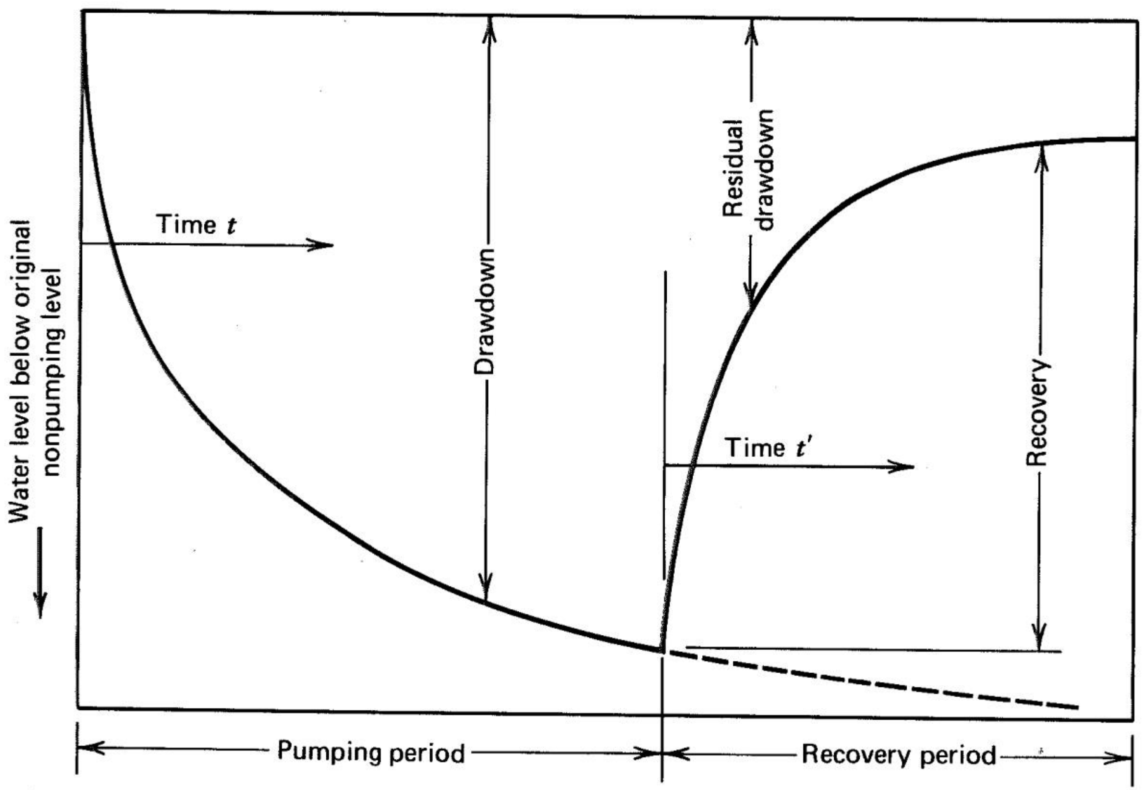



**Fig. 4.13** Drawdown and recovery curves in an observation well near a pumping well.

# A. Residual Drawdown Method

## Find T in pumped well

$t' =$ time since pumping stopped

$t =$ time since pumping began

- **Recovery measured in <u>pumped well</u>**

$$s' = \left(h_o - h'\right) = \frac{Q}{4\pi T}\left[\underbrace{\int_{\frac{r^2 s}{4Tt}}^{\infty} \frac{e^{-u}}{u}\,du}_{\text{Drawdown}} - \underbrace{\int_{\frac{r^2 s}{4Tt'}}^{\infty} \frac{e^{-u}}{u}\,du}_{\text{Recovery}}\right] = \frac{Q}{4\pi T}\left(W(u) - W(u')\right)$$

- **For small *r* and large *t'* , integrals approximated by first two terms in series.**

$$u = \frac{r^2 s}{4\pi t}$$

$$u' = \frac{r^2 s}{4\pi t'}$$

$$T = \frac{2 \cdot 3Q}{4\pi(h_o - h')} \log \frac{t}{t'}$$

$$= \frac{2 \cdot 3Q}{4\pi s'} \log {}^{t}\!/_{t'}$$

**over 1 log cycle, log** $\frac{t}{t'}$ **, log** ***10 = 1***

## Metric System :

$$T = \frac{2 \cdot 3Q}{4\pi \Delta s'}$$

$\Delta s'$ **= water level recover per log cycle of** $^{t}\!/_{t'}$

**Field Units:**

or $$T = \frac{264Q}{\Delta s'}$$

**S can't be determined from this method**

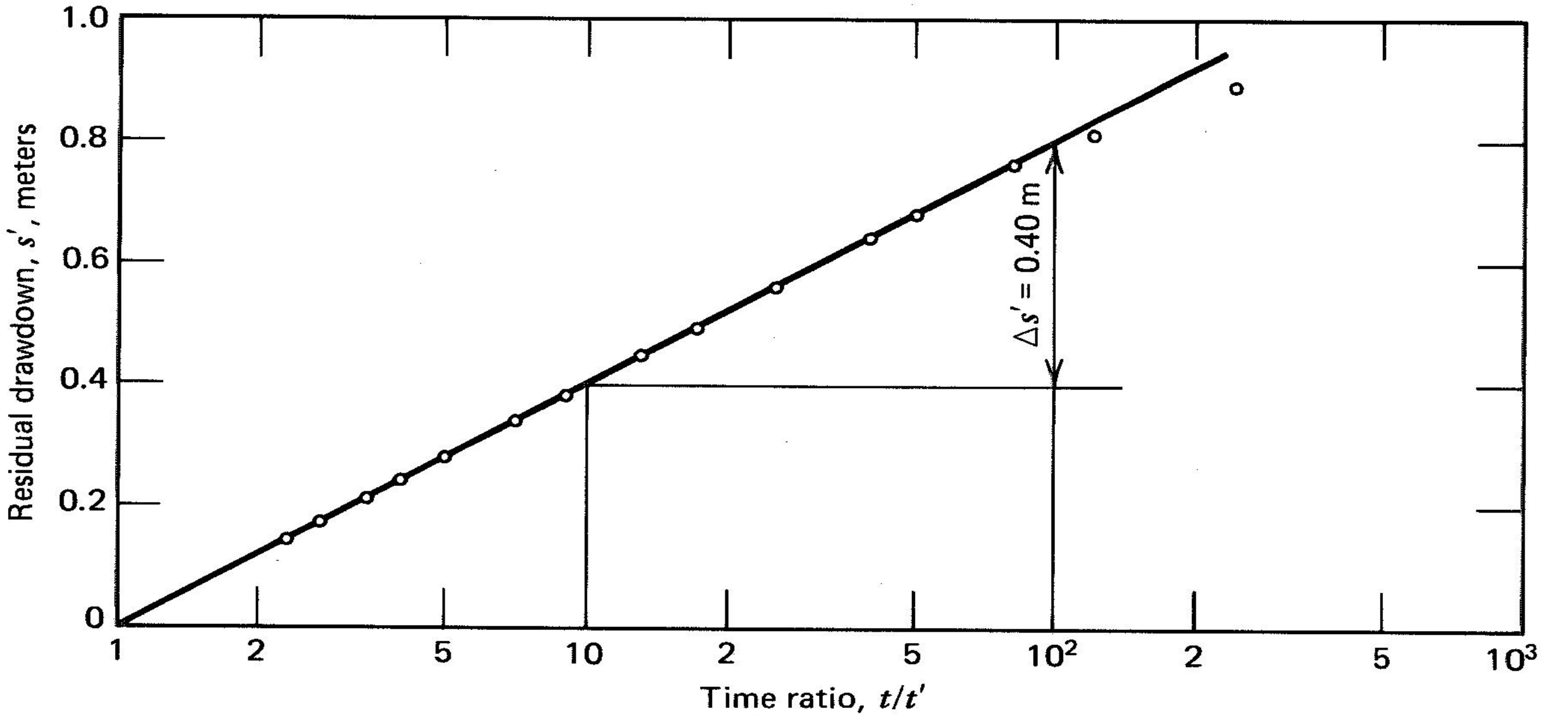



**Fig. 4.14** Recovery test method for solution of the nonequilibrium equation.

# B. Time - Recovery Method

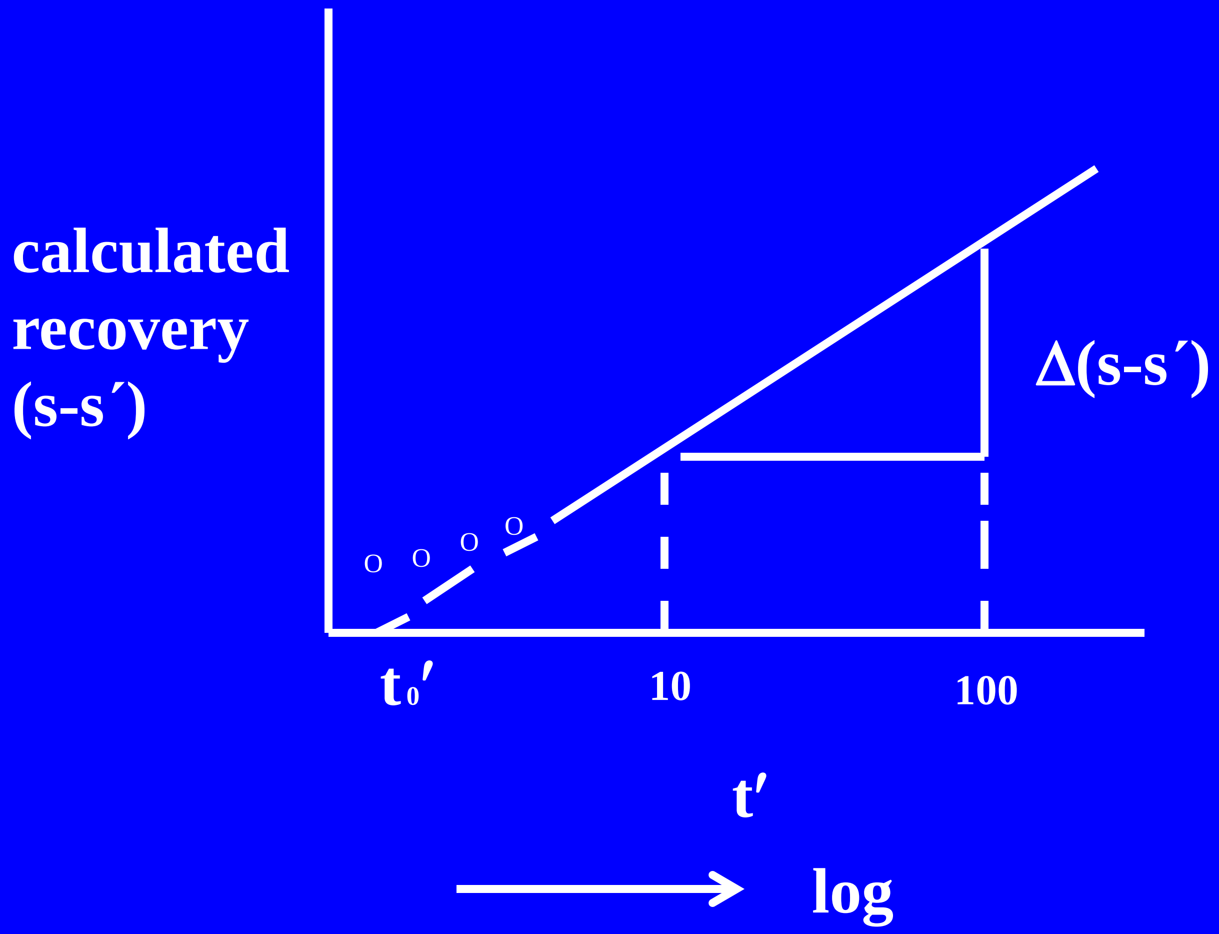
## Find S & T in observation well

- Recovery measured in observation well
- Plot $(s - s')$ , recovery, with $t'$
- Use Jacob method

**Field Units -** : **Metric System –**

$$T = \frac{264Q}{\Delta(s - s')} \quad : \quad T = \frac{2 \cdot 3Q}{4\pi\Delta(s - s')}$$

$$S = \frac{0 \cdot 3Tt'_o}{r^2} \quad : \quad S = \frac{2 \cdot 25Tt'_o}{r^2}$$




**Time - Recovery Method and Time-drawdown Method give close values of S and *T*.**

**EXAMPLE 4.4.6** A well pumping at a uniform rate of 2,500 m$^3$/day was shut down after 240 min; thereafter, measurements of $s'$ and $t'$ tabulated in Table 4.4.3 were made in an observation well. Determine the transmissivity.

**SOLUTION** Values of $t/t'$ are computed, as shown in Table 4.4.3, and then plotted versus $s'$ on semilogarithmic paper (see Figure 4.4.6). A straight line is fitted through the points and $\Delta s' = 0.40$ m is determined; then,

$$T = \frac{2.30Q}{4\pi\Delta s'} = \frac{2.30(2500)}{4\pi(0.40)} = 1140 \text{ m}^2/\text{day}$$

**Table 4.4.3** Recovery Test Data (pump shut down at $t$ = 240 min)

| $t'$, min | $t$, min | $t/t'$ | $s'$, m |
|---|---|---|---|
| 1 | 241 | 241 | 0.89 |
| 2 | 242 | 121 | 0.81 |
| 3 | 243 | 81 | 0.76 |
| 5 | 245 | 49 | 0.68 |
| 7 | 247 | 35 | 0.64 |
| 10 | 250 | 25 | 0.56 |
| 15 | 255 | 17 | 0.49 |
| 20 | 260 | 13 | 0.55 |
| 30 | 270 | 9 | 0.38 |
| 40 | 280 | 7 | 0.34 |
| 60 | 300 | 5 | 0.28 |
| 80 | 320 | 4 | 0.24 |
| 100 | 340 | 3.4 | 0.21 |
| 140 | 380 | 2.7 | 0.17 |
| 180 | 420 | 2.3 | 0.14 |

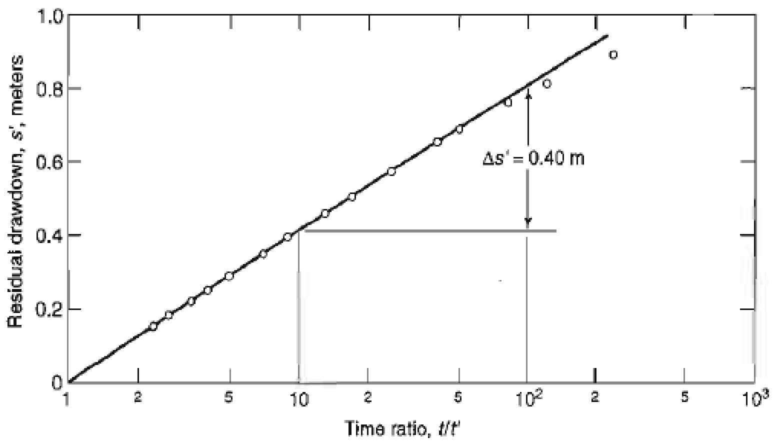
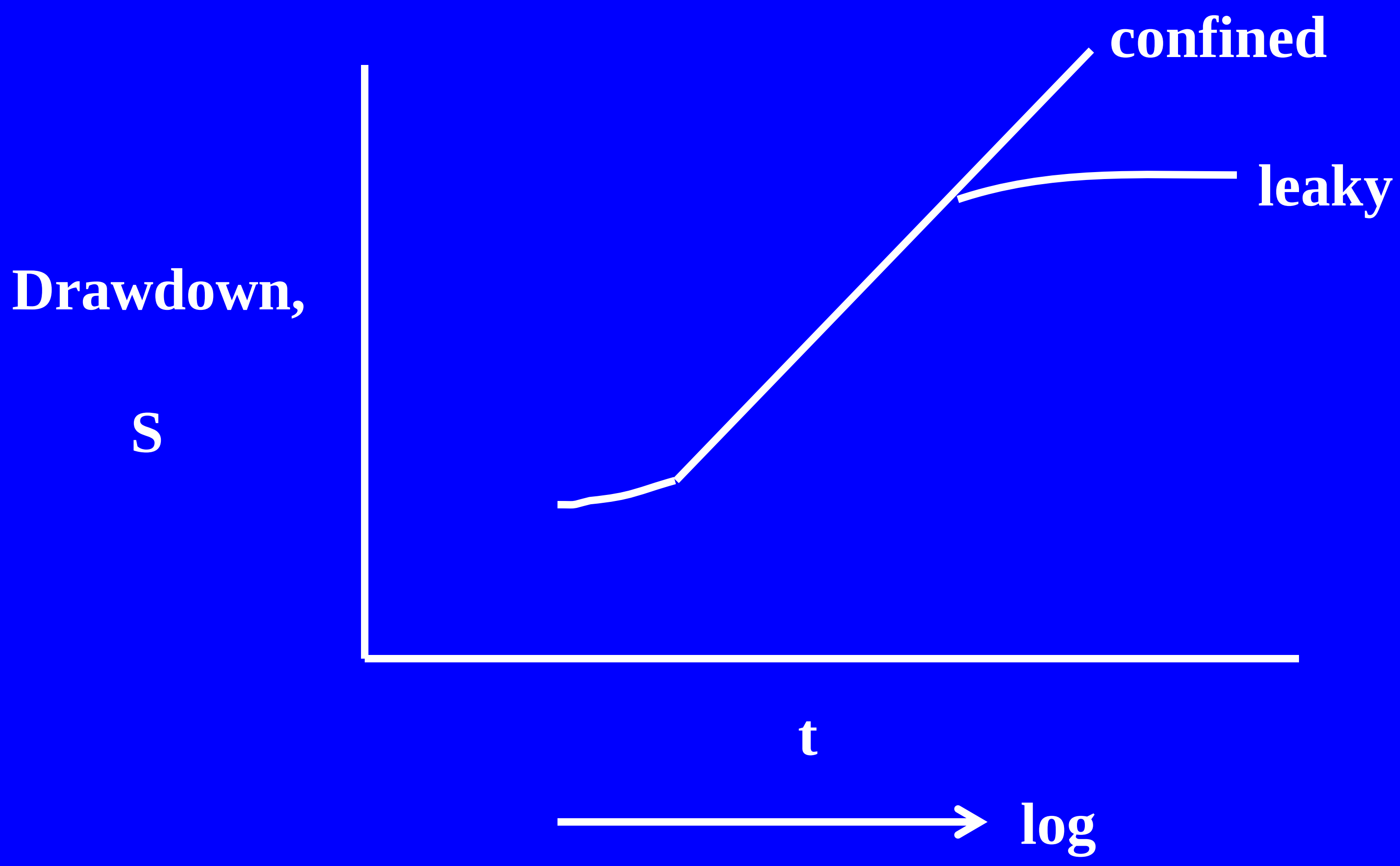



**Figure 4.4.6.** Recovery test method for solution of the nonequilibrium equation.

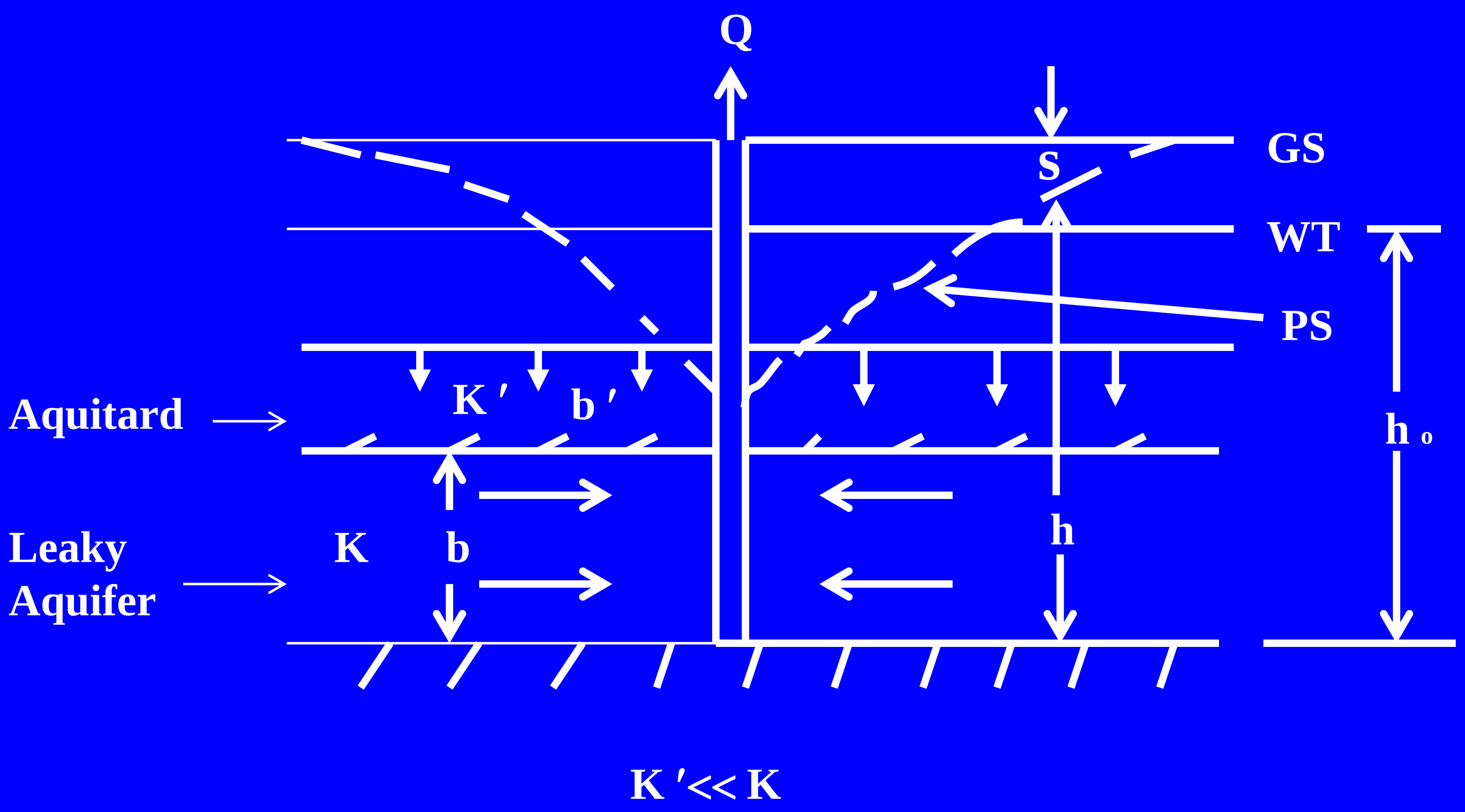
# Leaky Aquifers

- Due to recharge the top of the curve is flat.



# Hantush & Jacob Method for Leaky Aquifers

**Determine S, T, K´**



## Assumptions:

1. Leakage is vertical
2. Leakage $\alpha$ Drawdown
3. Water level in upper supply aquifer is constant
4. W.T. & P.S. are initially same

## Field Units

$$(h_o - h) = s = \frac{114.6Q}{T} \quad W(u, r/B)$$

$W(u,\ r/B) =$ well function for a leaky aquifer

$$u = \frac{1.87 r^2 S}{Tt} \quad , \qquad \frac{r}{B} = \frac{r}{\sqrt{T/K'/b'}}$$

**K´ = vertical hydraulic gradient of aquitard**

**b´ = thickness of aquitard**

**Field Units** | **Metric System**

$$s = \frac{114.6Q}{T} W(u, r/B) \quad : \quad s = \frac{Q}{4\pi T} W(u, r/B)$$

$$t = \frac{1.87 r^2 S}{T} \left( \frac{1}{u} \right) \quad : \quad t = \frac{r^2 S}{T} \left( \frac{1}{u} \right)$$

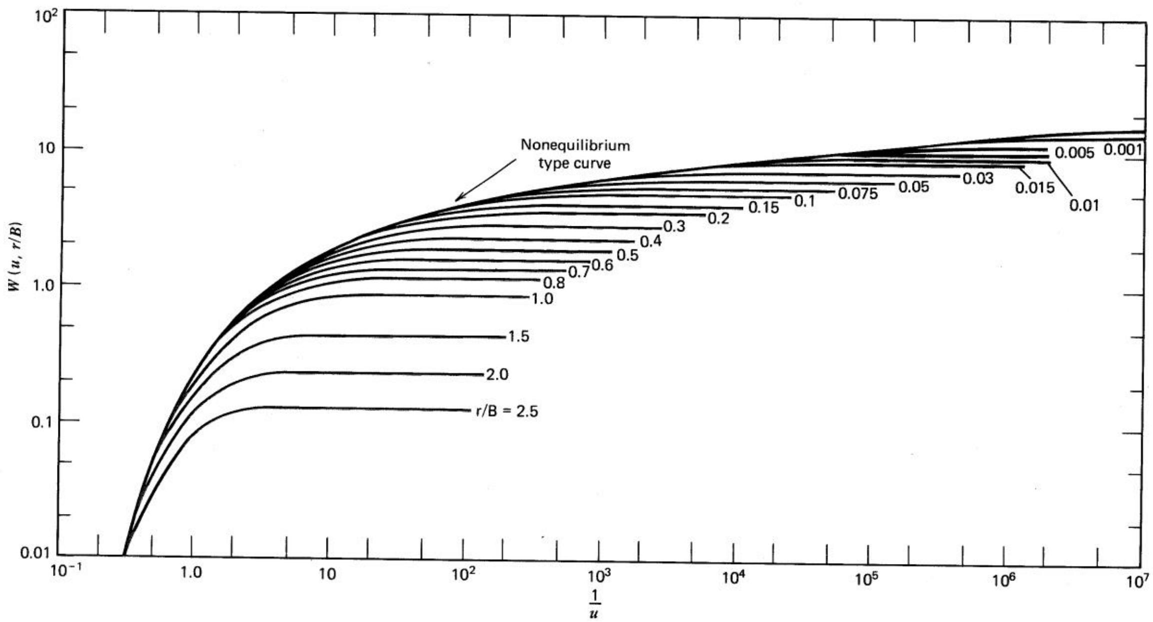
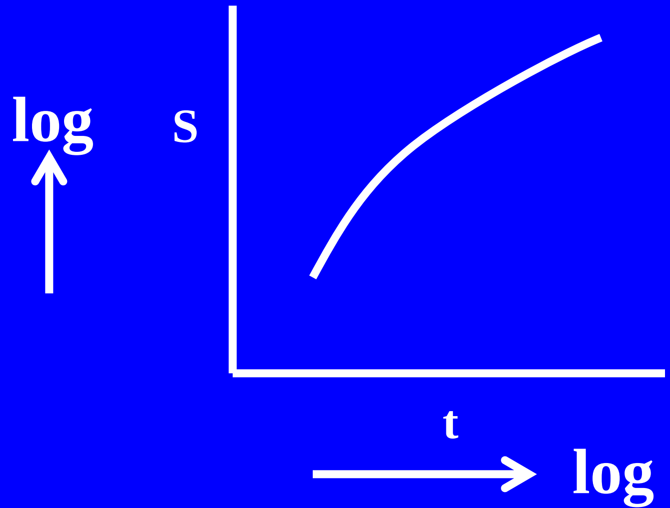



**Fig. 4.18** Type curves for analysis of pumping test data to evaluate storage coefficient and transmissivity of leaky aquifers (after Walton[69]).



- **Superimpose the s - t curve on well function curve.**

- **Select a match point and get** $s, t, \dfrac{1}{\mathrm{u}}, W\left(u, r/B\right) \& r/B$

$$T = \frac{114.6Q}{s} \; W(u, r/B)$$

$$S = \frac{Tut}{1.87r^2}$$

$$K' = \frac{Tb'\left(r/B\right)^2}{r^2}$$

***EXAMPLE 4.6.1***

(adapted from U.S. Department of the Interior)[62].

A well pumping at 600 $ft^3$/min fully penetrates a confined aquifer overlain by a leaky confining layer of 14-ft thickness. Using the tabulated time–drawdown data for an observation well 40 ft away from the pumping well, estimate the transmissivity and storage coefficient of the confined aquifer, and the permeability of the aquitard. Assume that the confining layer does not release water from storage.

| Time (min) | Drawdown (ft) | Time (min) | Drawdown (ft) |
|---|---|---|---|
| 0 | 0.00 | 80 | 12.02 |
| 2 | 5.65 | 90 | 12.26 |
| 4 | 6.96 | 100 | 12.33 |
| 6 | 7.72 | 110 | 12.37 |
| 8 | 8.00 | 120 | 12.41 |
| 10 | 8.71 | 150 | 12.69 |
| 15 | 9.47 | 180 | 12.85 |
| 20 | 9.99 | 210 | 13.09 |
| 25 | 10.35 | 240 | 13.13 |
| 30 | 10.70 | 270 | 13.25 |
| 40 | 11.14 | 300 | 13.33 |
| 50 | 11.46 | 360 | 13.37 |
| 60 | 11.62 | 420 | 13.41 |
| 70 | 11.86 | | |

***SOLUTION***

The time–drawdown field data were superimposed on the family type curves for leaky aquifers (Figure 4.6.3). Comparison shows that the best fit occurs for $r/B = 0.03$. The coordinates of the match point selected are

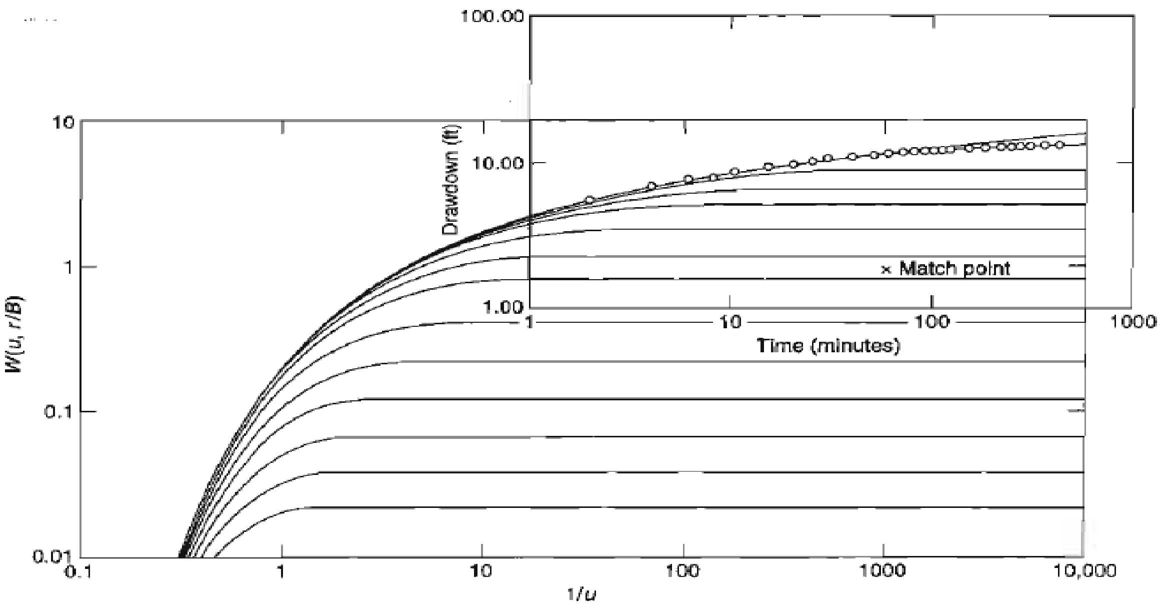



**Figure 4.6.3.** Leaky type curve matching for Example 4.6.1.

$$\frac{1}{u} = 1000, \qquad W\left(u, \frac{r}{B}\right) = 1.0$$

$$t = 59 \text{ min}, \qquad s = 1.93 \text{ ft}$$

Next we must perform the following unit conversions in order to obtain the transmissivity in units of $\text{ft}^2$/day and hydraulic conductivity of the aquitard in units of ft/day for $Q = 600 \text{ ft}^3/\text{min} = 864{,}000 \text{ ft}^3/\text{day}$ and $t = 59 \text{ min} = 0.041$ days. The transmissivity and storage coefficient of the confined aquifer are computed using Equations 4.6.1 and 4.6.2 rearranged respectively as

$$T = \frac{Q}{4\pi s} W(u, r/B) = \frac{864{,}000 \text{ ft}^3/\text{day}}{4\pi(1.93 \text{ ft})}(1.0) = 35{,}624 \text{ ft}^2/\text{day}$$

$$S = \frac{4Ttu}{r^2} = \frac{4(35{,}624 \text{ ft}^2/\text{day})(0.041 \text{ days})(0.001)}{(40 \text{ ft})^2} = 0.00365$$

The hydraulic conductivity of the aquitard is computed by rearranging Equation 4.6.3

$$K' = \frac{Tb'(r/B)^2}{r^2} = \frac{(35{,}624 \text{ ft}^2/\text{day})(14 \text{ ft})(0.03)^2}{(40 \text{ ft})^2} = 0.28 \text{ ft/day}$$

■

# Unconfined Aquifers

- Exact solution difficult because:
  - T varies w/ *r* and *t* with decline of W.T.
  - vertical flow component significant, especially near well casing.

- If *s* is small compared to , Theis or Jacob solutions can be used for unconfined aquifers.

- Jacob suggested that more accurate values of *S & T* obtained by subtracting $\frac{(h_0 - h)^2}{2h_o}$ from each drawdown, *s*.

# Bolton Equation

- For larger drawdowns, $s < 0.5h_o$

$$s = \frac{Q}{2\pi k h_o}\left(1 + C_k\right)V(t', r')$$

where $C_k$ - correction factor

- $V(t', r')$ well function (table available for $t', r'$ )

$$t' = \frac{Kt}{Sh_o}$$

$C_k$ - varies - 0.30 to 0.16

$$r' = \frac{r}{h_o}$$

$h_o$ – max. sat. thickness at $r_o$

$$0.05 < t' < 5, \ C_k = o$$

$$t' < 0.05, \quad C_k \quad \text{varies}$$

**$t' < 5$ refers to early pumping & not of much interest in unconfined aquifers**

$t'75, \ C_k = f(r')$ **table or graph available**

**At** $r > 1.5h_o$, **effect of vertical seepage are negligible**

$t' 75,$

$$h^2_{iw} = h_o^2 - \frac{Q}{\pi K} \ln\left( \sqrt[1.5]{\frac{Kt}{sr_w}} \right)$$

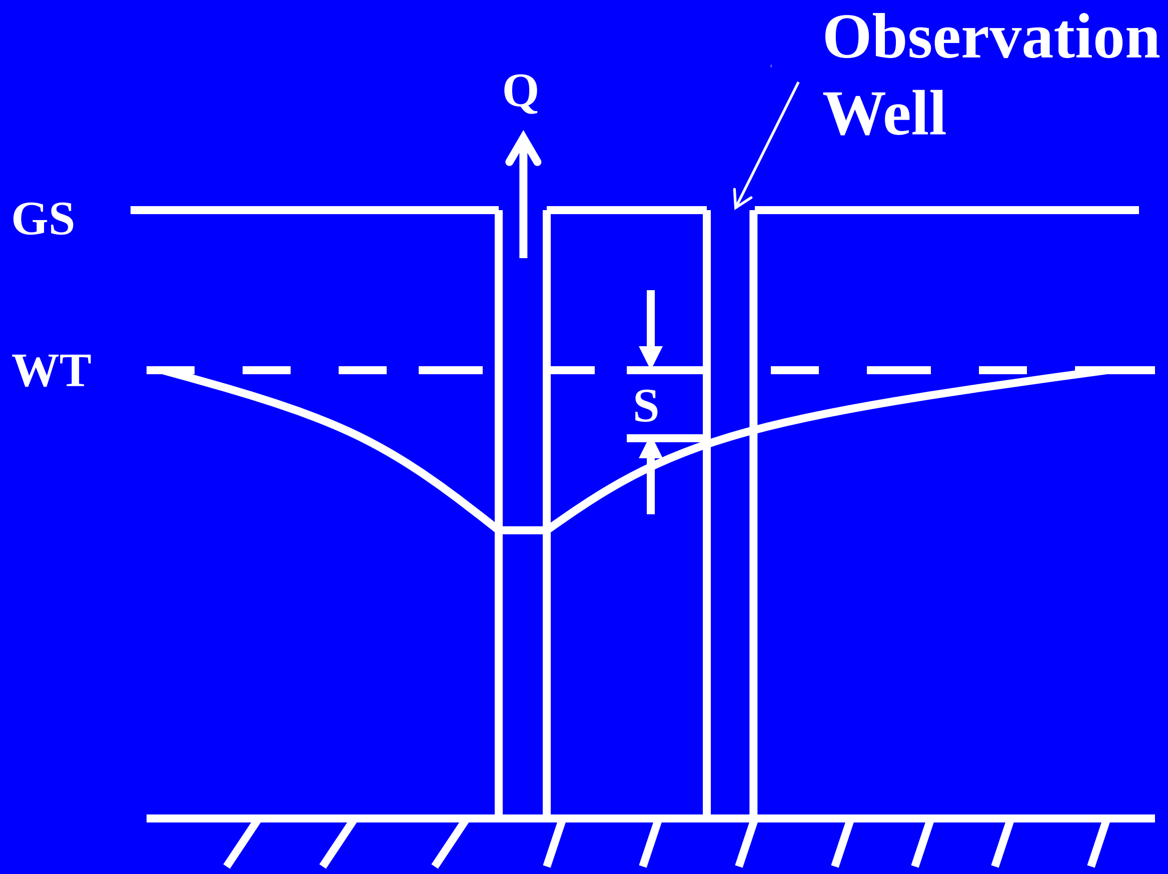
$0.05 < t' < 5,$

$$h_{iw} = h_o - \frac{Q}{2\pi k h_o}\left( m + \ln \frac{h_o}{r_w} \right)$$

**where** $m = f(t')$ **(Table or curve available)**

$t' < 0.05 -$ **minor significance, no eqn.**

# Unconfined Aquifer



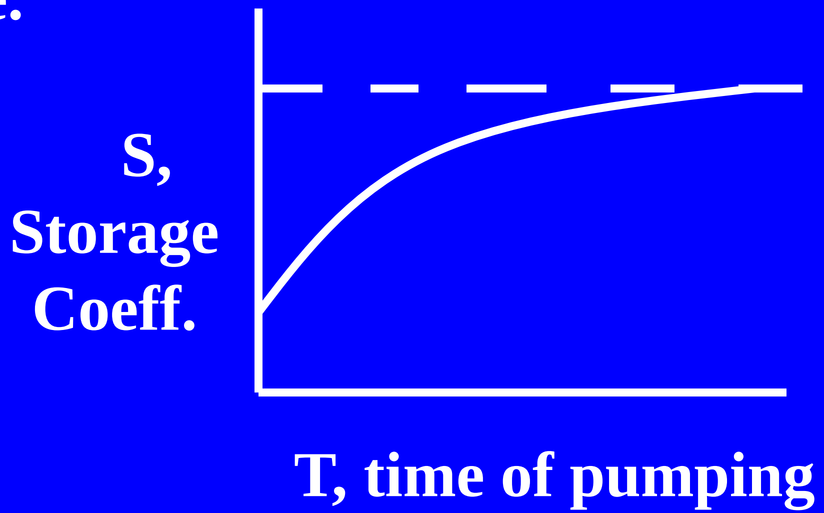
# Delayed drainage

When a well is pumped, water continuously withdraws from storage within the aquifer as cone of depression progresses radially outward from the well.

- Since no recharge source is there, no steady-state flow, and head will continue to decline as long as aquifer is infinite.
- However, rate of decline of head decreases as cone of depression spreads.

- Water is released from storage by gravity drainage of pores in the portion of the aquifer drained by pumping and by the compaction of aquifer and the expansion of water.

- Gravity drainage of water from sediments is not immediate; *S* varies and increases at a diminishing rate with time.

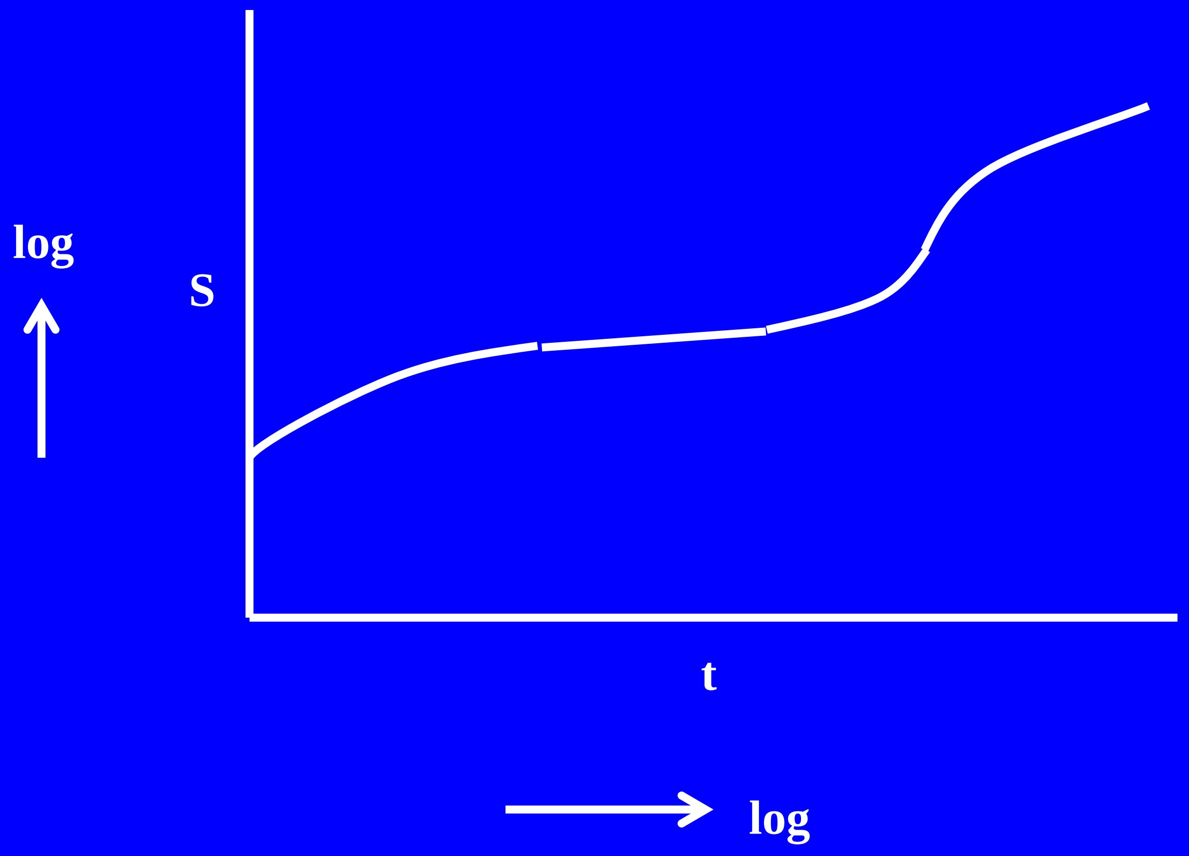


- First, water is released instantaneously from storage by compaction of aquifer and expansion of water.

- After a short time, cone of depression grows at a slow rate as water is released from storage by gravity drainage reaches the cone.

- Finally, rate of cone expansion increases and cone continues to expand as gravity drainage keeps pace with declining water levels.

# S – t Curve for Delayed Drainage



**EXAMPLE 4.5.1**

(adapted from U.S. Department of the Interior)[62].

A well pumping at 144.4 $ft^3$/min fully penetrates an unconfined aquifer with a saturated thickness of 25 ft. Determine the transmissivity, storativity, specific yield, and horizontal and vertical hydraulic conductivities using the tabulated time–drawdown data in Table 4.5.1 for an observation well located 73 ft away.

**SOLUTION**

Time–drawdown data (Table 4.5.1) are plotted in Figure 4.5.4, which shows the typical three phases of drawdown for unconfined aquifers. The early drawdown versus time data fit best on the type-a curves for $\eta = 0.06$. The selected match point in Figure 4.5.4 has the following coordinates: ($t = 0.17$ min, $s = 0.57$ ft) and ($1/u_a = 1.0$, $W(u_a,u_y,\eta) = 1.0$). Using Equation 4.5.1 with a discharge of $Q = 144.4\ \text{ft}^3/\text{min}$, we find the transmissivity to be

$$T = \frac{Q}{4\pi s} W(u_a,u_y,\eta) = \frac{(144.4\ \text{ft}^3/\text{min})}{4\pi(0.57\ \text{ft})}(1.0) = 20.16\ \text{ft}^2/\text{min} \cong 29{,}900\ \text{ft}^2/\text{day}$$

Next, the storativity value is computed using Equation 4.5.2:

**Table 4.5.1** Time–Drawdown Data for Example 4.5.1.

| $t$ (min) | $s$, feet | $t$ (min) | $s$, feet | $t$ (min) | $s$, feet | $t$ (min) | $s$, feet |
|---|---|---|---|---|---|---|---|
| 0.165 | 0.12 | 1.68 | 0.82 | 10 | 1.02 | 200 | 1.52 |
| 0.25 | 0.195 | 1.85 | 0.84 | 12 | 1.03 | 250 | 1.59 |
| 0.34 | 0.255 | 2 | 0.86 | 15 | 1.04 | 300 | 1.65 |
| 0.42 | 0.33 | 2.15 | 0.87 | 18 | 1.05 | 350 | 1.7 |
| 0.5 | 0.39 | 2.35 | 0.9 | 20 | 1.06 | 400 | 1.75 |
| 0.58 | 0.43 | 2.5 | 0.91 | 25 | 1.08 | 500 | 1.85 |
| 0.66 | 0.49 | 2.65 | 0.92 | 30 | 1.13 | 600 | 1.95 |
| 0.75 | 0.53 | 2.8 | 0.93 | 35 | 1.15 | 700 | 2.01 |
| 0.83 | 0.57 | 3 | 0.94 | 40 | 1.17 | 800 | 2.09 |
| 0.92 | 0.61 | 3.5 | 0.95 | 50 | 1.19 | 900 | 2.15 |
| 1 | 0.64 | 4 | 0.97 | 60 | 1.22 | 1,000 | 2.2 |
| 1.08 | 0.67 | 4.5 | 0.975 | 70 | 1.25 | 1,200 | 2.27 |
| 1.16 | 0.7 | 5 | 0.98 | 80 | 1.28 | 1,500 | 2.35 |
| 1.24 | 0.72 | 6 | 0.99 | 90 | 1.29 | 2,000 | 2.49 |
| 1.33 | 0.74 | 7 | 1 | 100 | 1.31 | 2,500 | 2.59 |
| 1.42 | 0.76 | 8 | 1.01 | 120 | 1.36 | 3,000 | 2.66 |
| 1.5 | 0.78 | 9 | 1.015 | 150 | 1.45 | | |

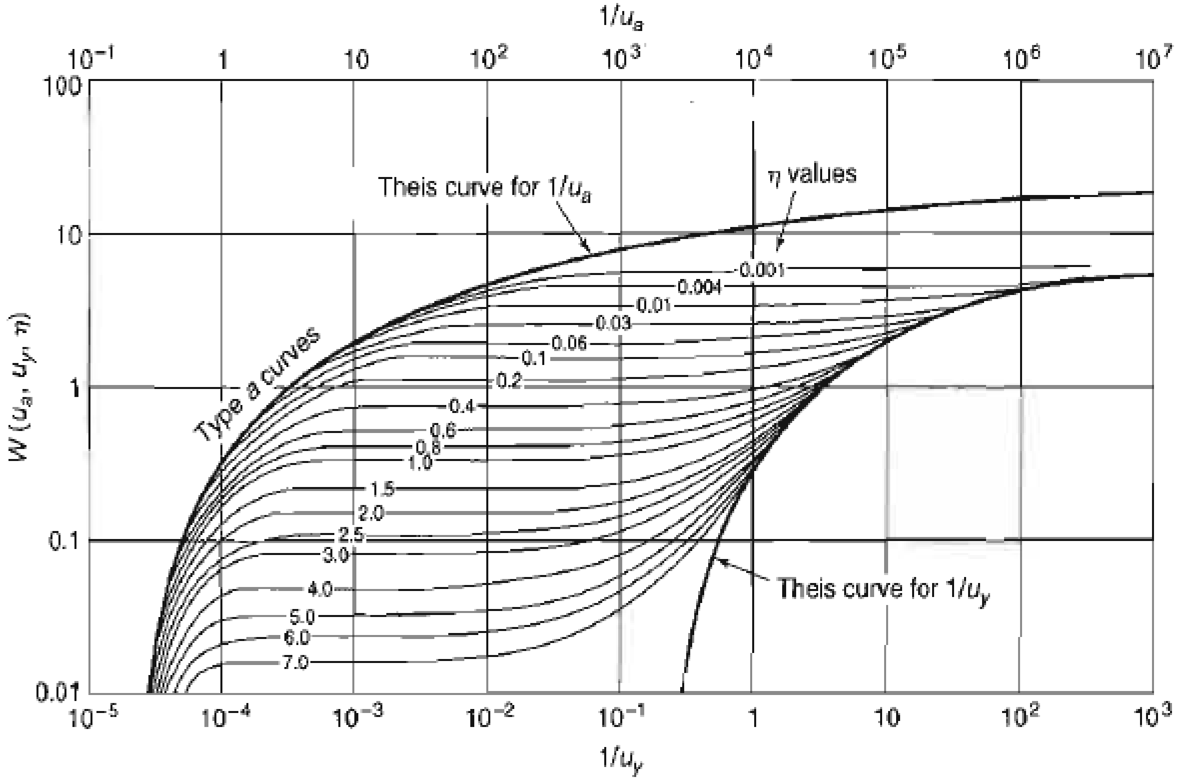



**Figure 4.5.3.** Theoretical curves of $W(u_a,u_y,\eta)$ versus $1/u_a$ and $1/u_y$ for an unconfined aquifer (after Neuman[43]).

$$S = \frac{4Tu_at}{r^2} = \frac{4\left(20.16\ \text{ft}^2/\text{min}\right)(1.0)(0.17\ \text{min})}{(73\ \text{ft})^2} = 0.00257$$

Moving the data curve to the right on the type curve to the best late-time match (for $\eta = 0.06$) where $s = 0.57$ ft (see the match point on Figure 4.5.5) yields ($t = 13$ min, $s = 0.57$ ft) and ($1/u_y = 0.1$, $W(u_y, \eta) = 1$). Inserting the appropriate values in Equation 4.5.1 does not change the transmissivity estimate, but using Equation 4.5.3 yields

$$S_y = \frac{4Tu_yt}{r^2} = \frac{4\left(20.16\ \text{ft}^2/\text{min}\right)(0.1)(13\ \text{min})}{(73\ \text{ft})^2} = 0.02$$

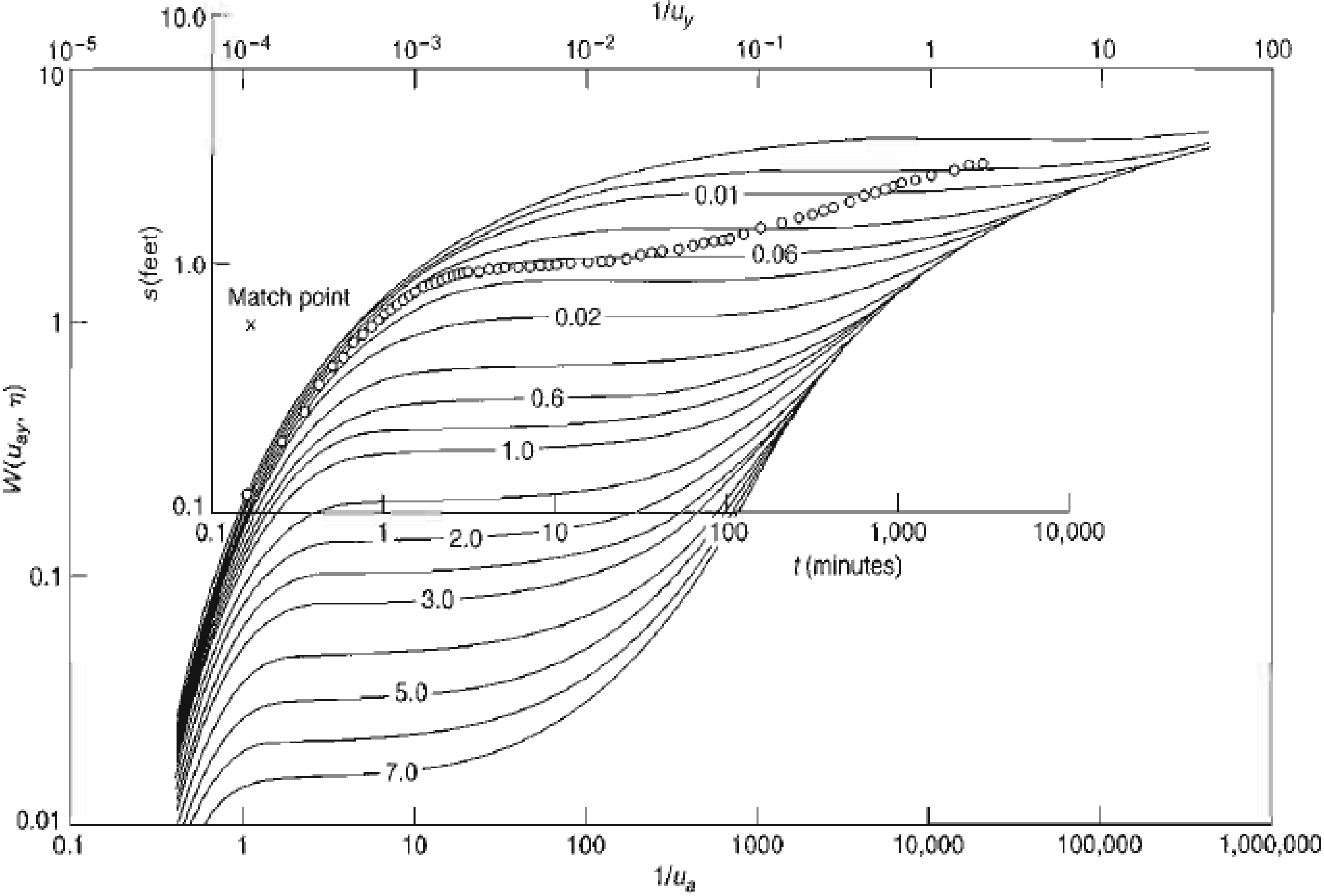



**Figure 4.5.4.** Type-a curve matching for Example 4.5.1.

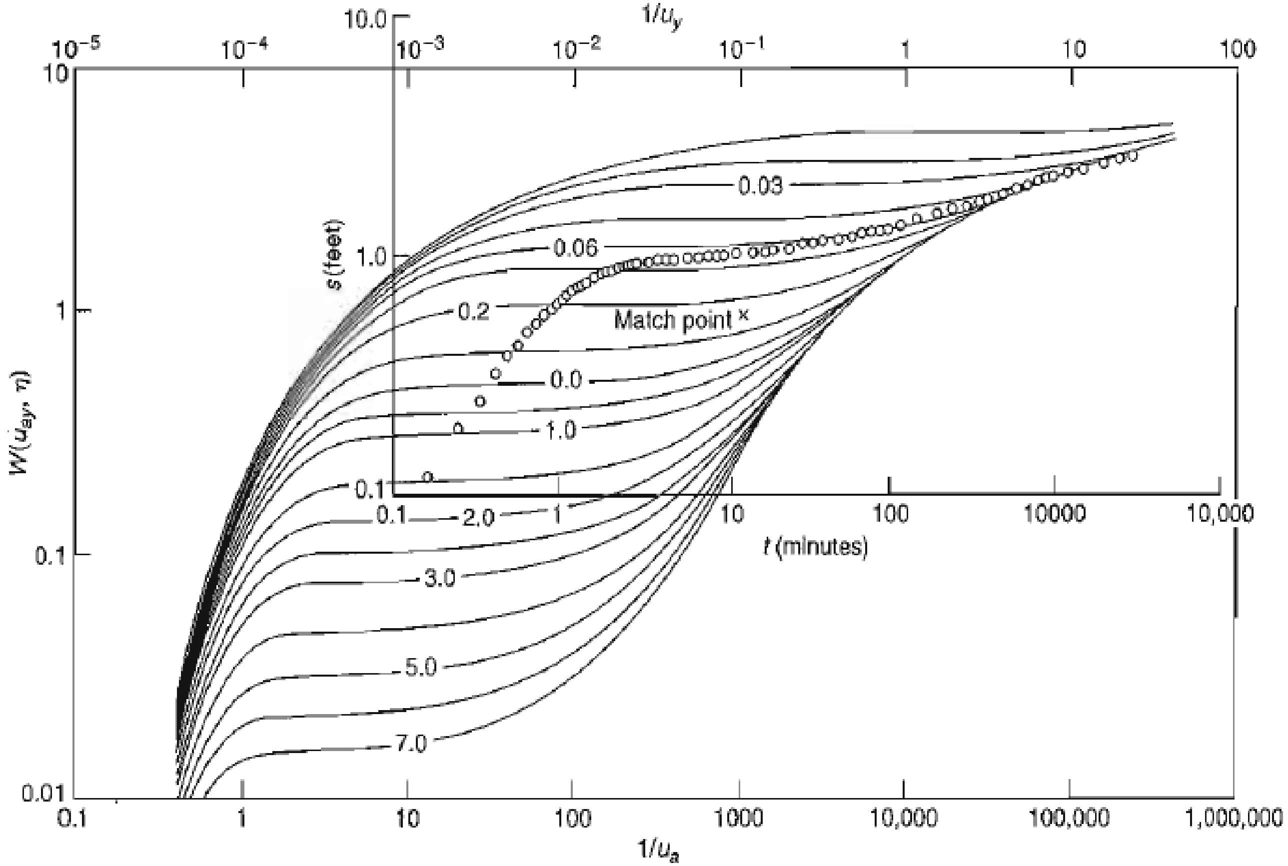



**Figure 4.5.5.** Type-y curve matching for Example 4.5.1.

The horizontal hydraulic conductivity, $K_r$ or $K_h$, is computed using

$$K_h = \frac{T}{b} = \frac{20.16\ \text{ft}^2/\text{min}}{25\ \text{ft}^2} = 0.806\ \text{ft/min or } 1160\ \text{ft/day}$$

and the vertical hydraulic conductivity, $K_z$ or $K_v$, is computed using Equation 4.5.4:

$$K_v = \frac{\eta b^2 K_h}{r^2} = \frac{(0.06)(25\ \text{ft})^2(1160\ \text{ft/day})}{(73\ \text{ft})^2} = 8.2\ \text{ft/day}$$

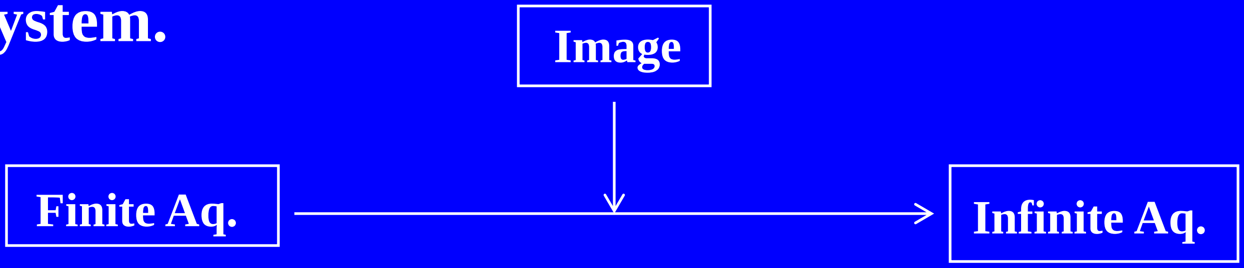
■

# Well Flow near Aquifer Boundaries

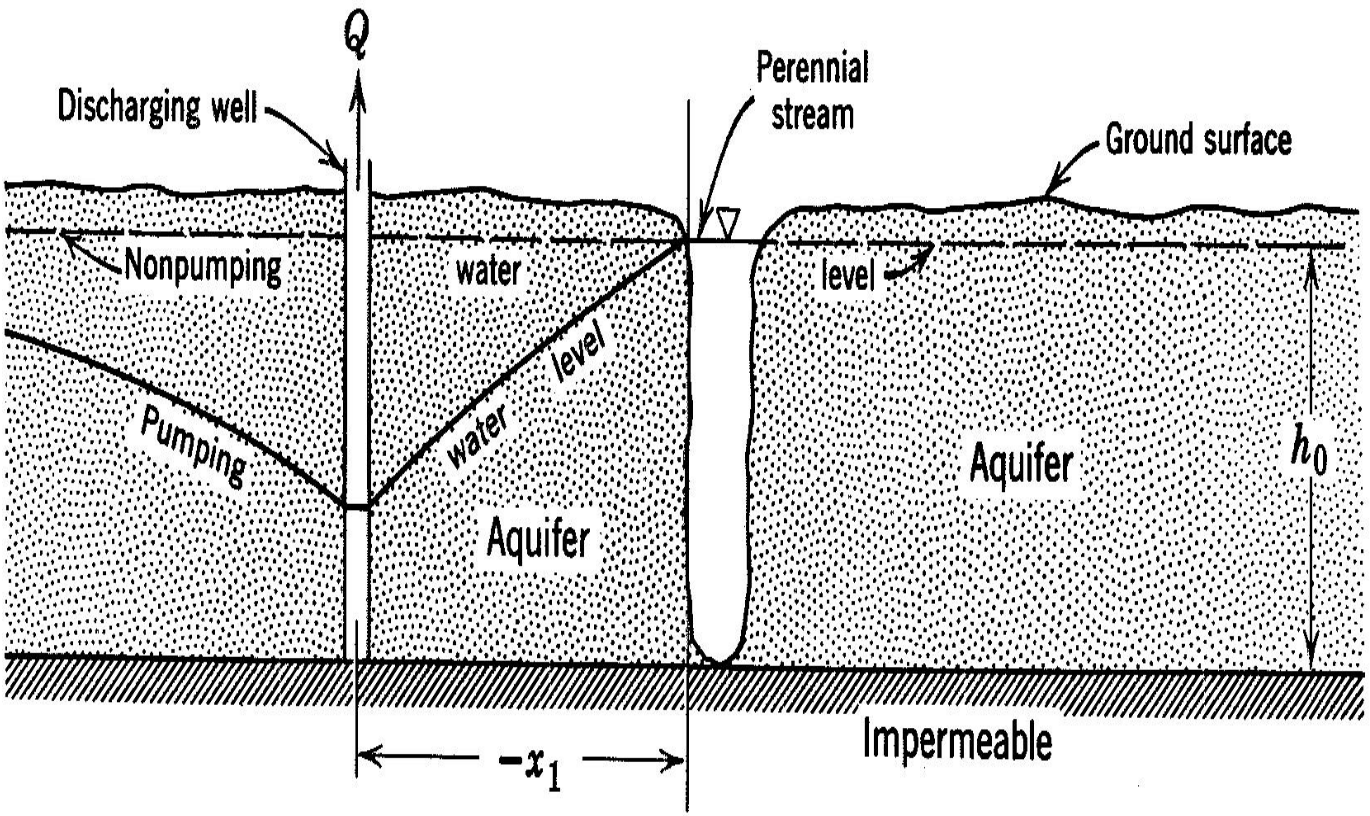
- Impermeable or negative boundary
- Permeable or positive boundary

Solution of boundary problem in well flow is simplified by applying the method of images.

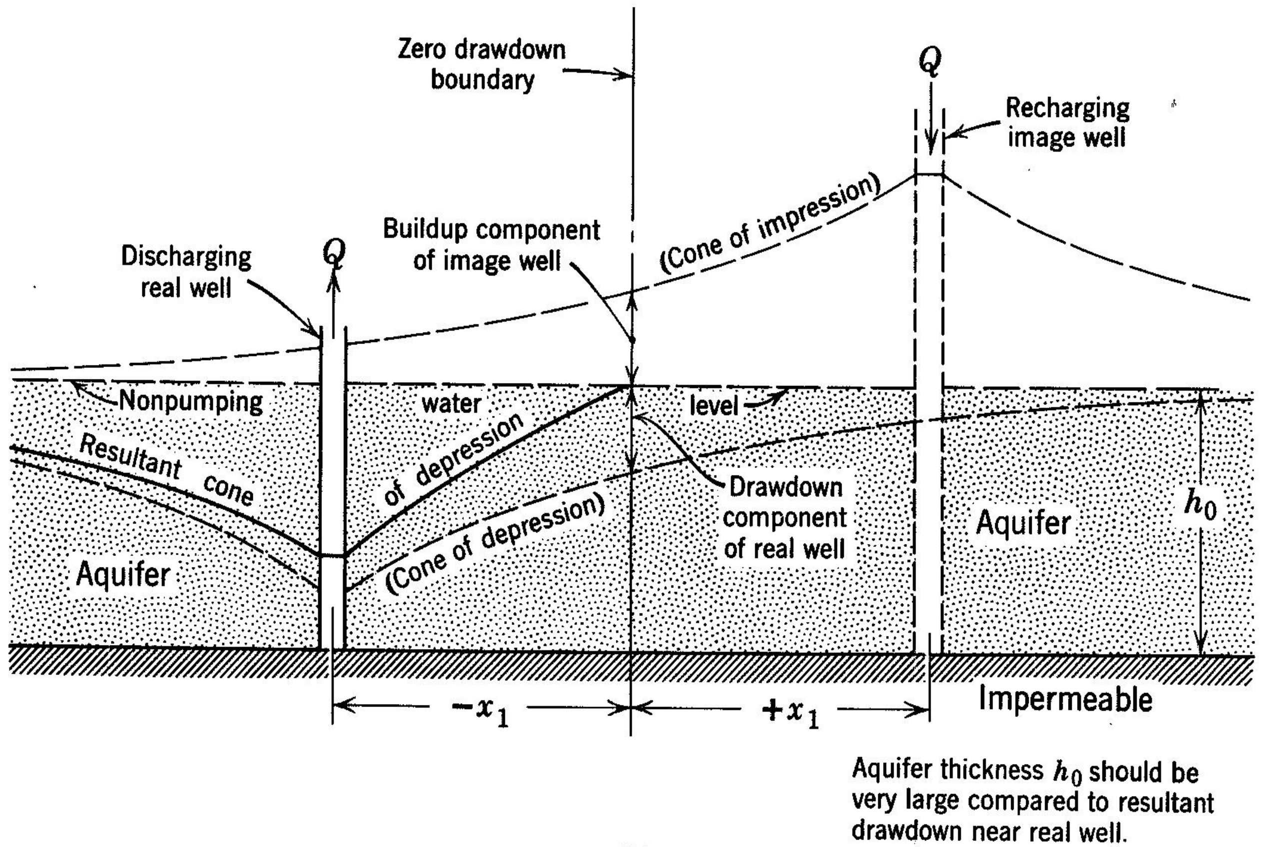
Image - an imaginary well introduced to create a hyd flow system which will be equivalent to the effects of a known flow system.



# a. Well near a stream - Permeable Boundary



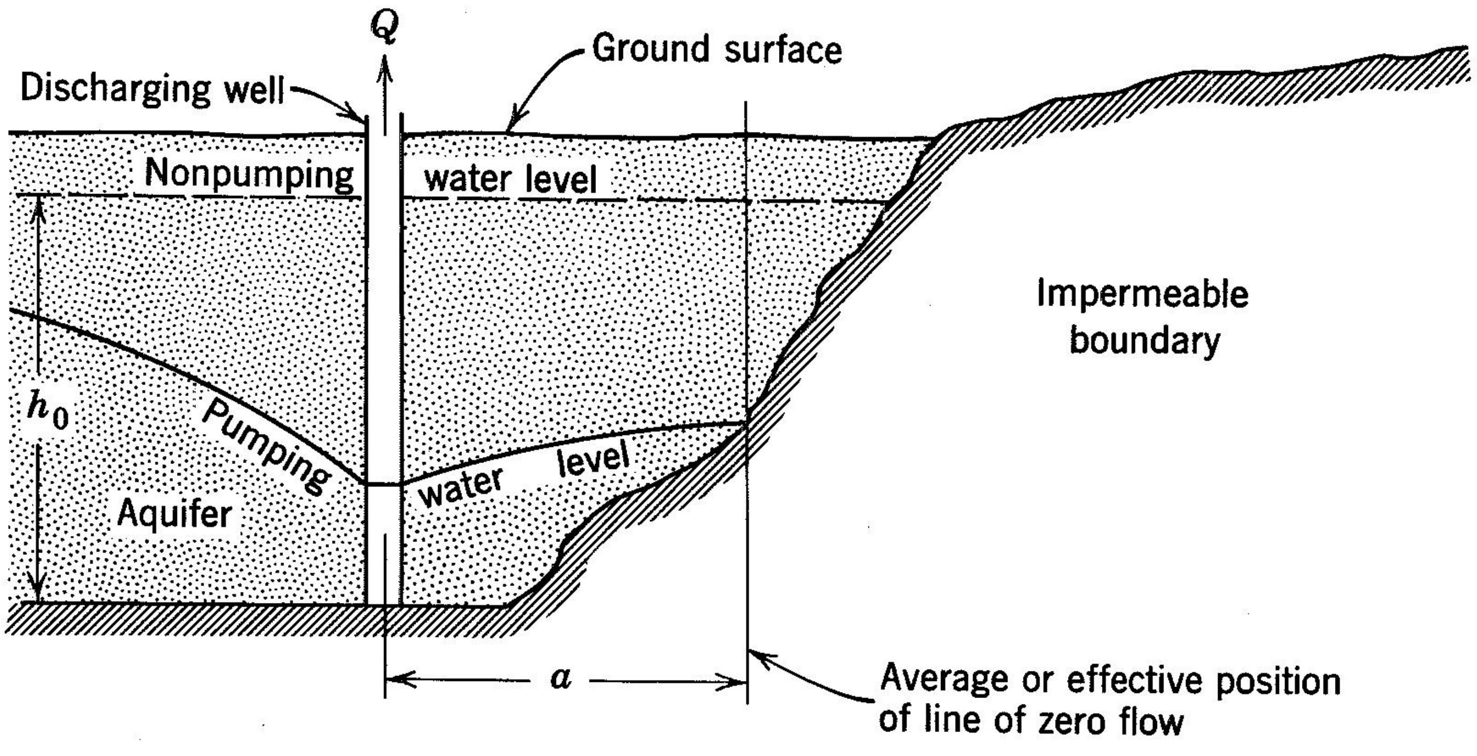

(a)



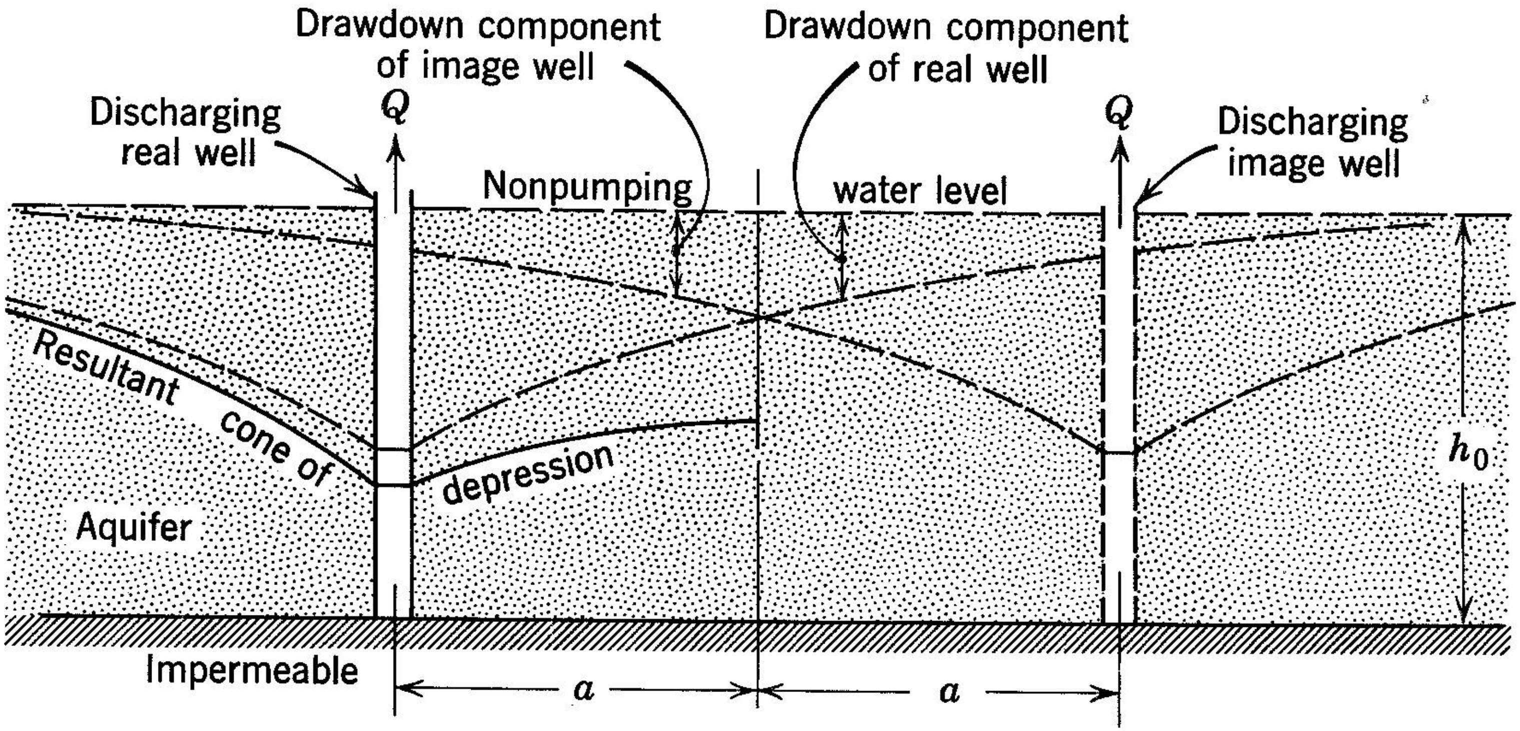

*(b)*

- **This system is converted to a discharging real well and a recharge imaginary well in an extensive aquifer.**

# b. Well near an Impermeable Boundary



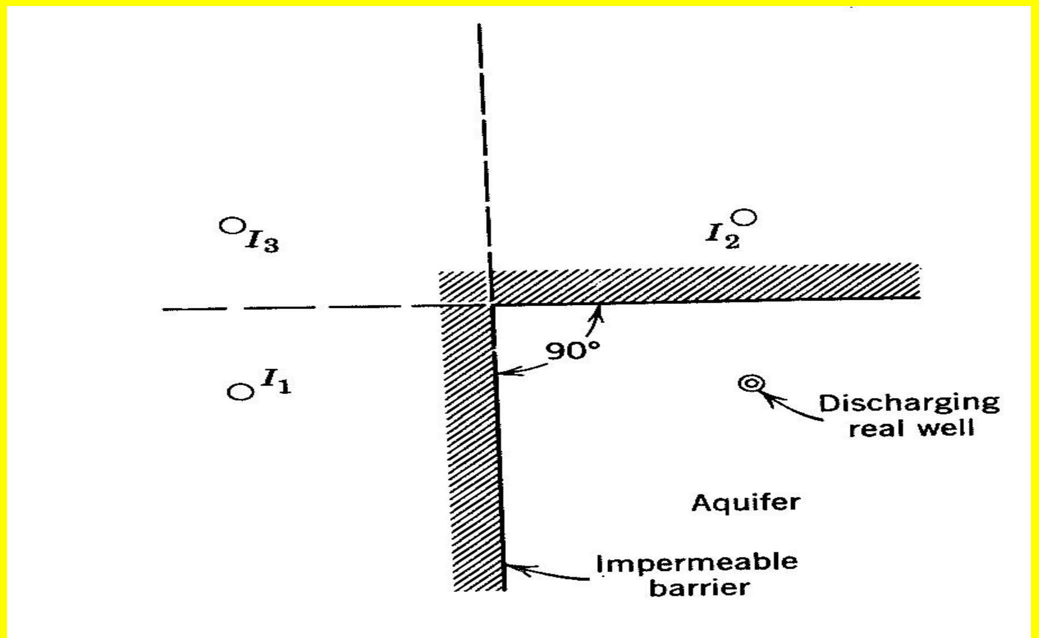

*(a)*



Aquifer thickness $h_0$ should be very large compared to resultant drawdown near real well

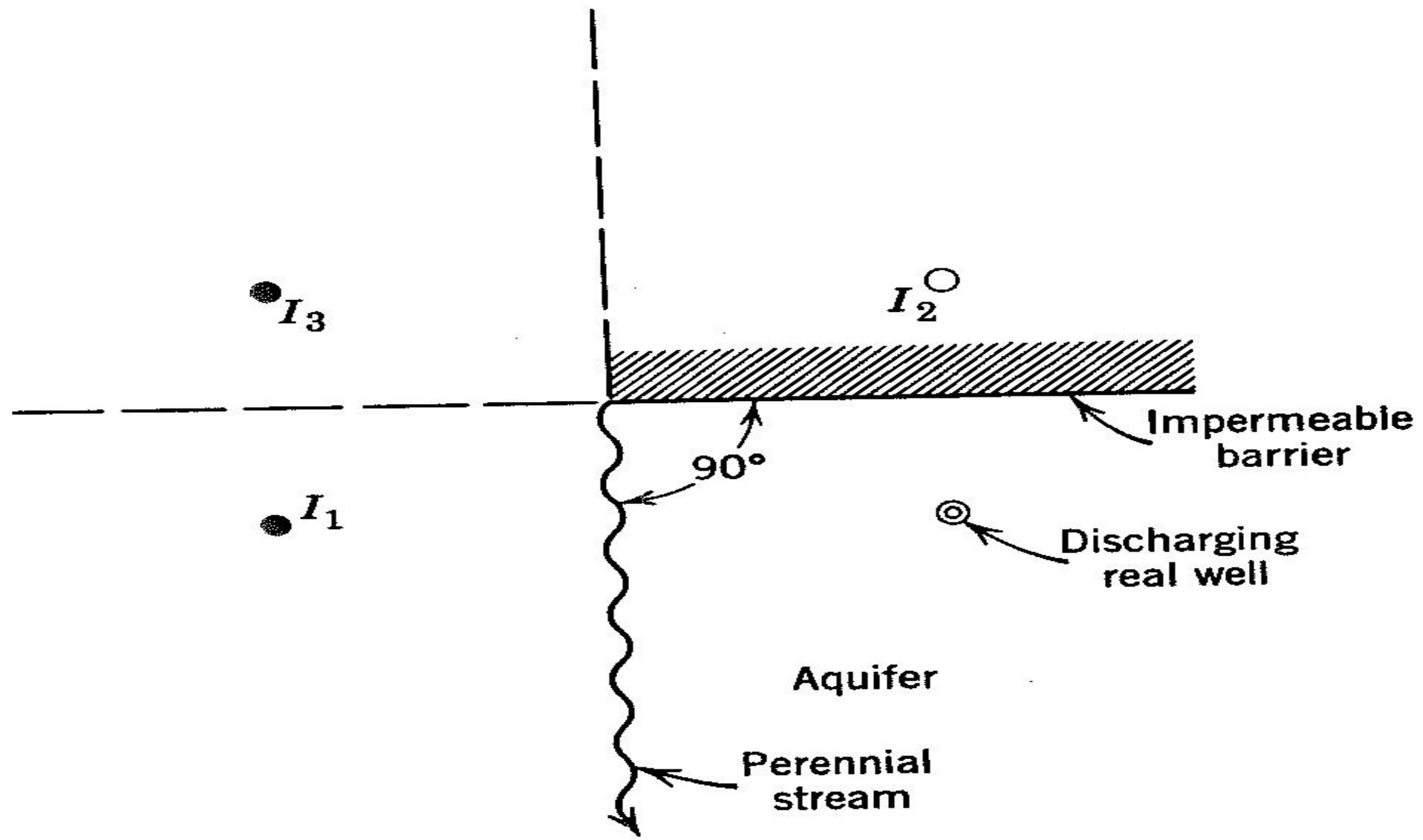
# c. Aquifer Bounded by Two Impermeable Boundaries

- $I_1$ and $I_2$ provide required flow, but $I_3$ required balance drawdowns along the extensions of the boundaries.



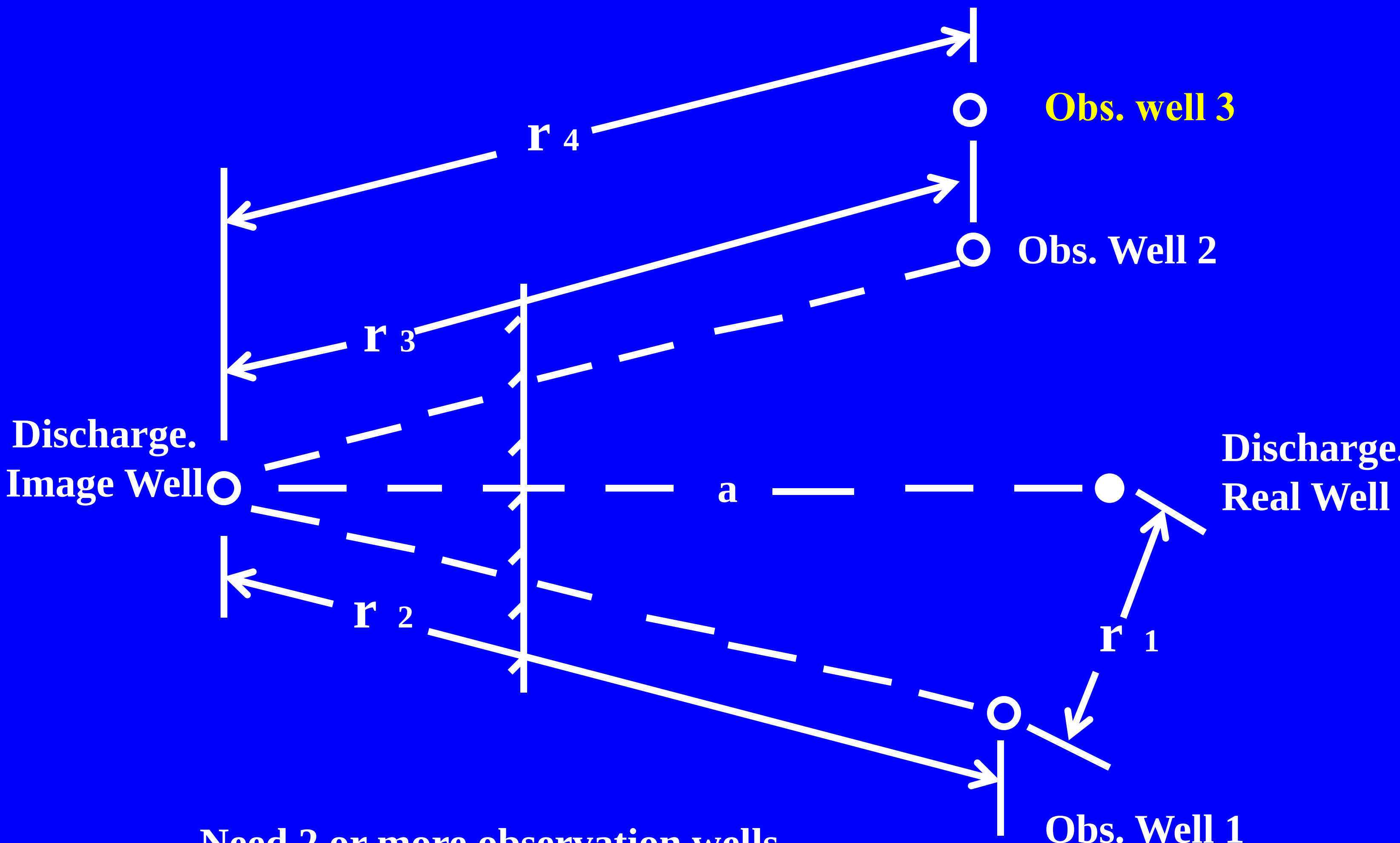

(a)

# d. Impermeable Boundary ⊥ to a stream



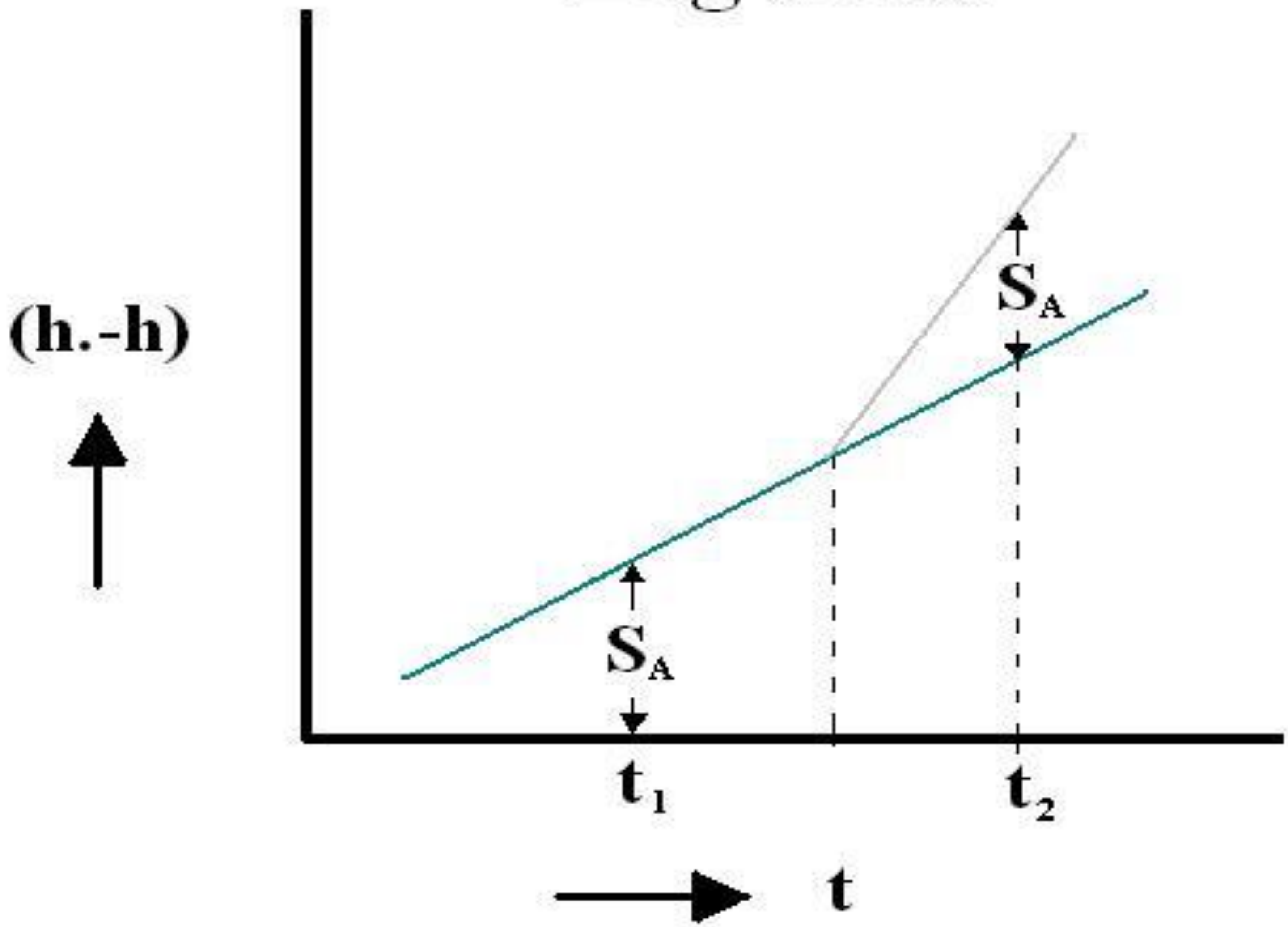

(b)

# Determination of a Boundary




-Need 2 or more observation wells

# Log Scale



$$(h_o - h)_T = (h_o - h)_1 + (h_0 - h)_2$$

$$= 114.6\frac{Q}{T}[W(u_1) + W(u_2)]$$

$$\mathrm{u} = \frac{1.87r^2S}{Tt}$$

**Assume the wells are pumped individually. At a given time interval**

$$(h_o - h)_1 = (h_o - h)_2$$

$$w(u_1) = w(u_2)$$

$$(-0.5772 - \ln u_1) = (-0.5772 - \ln u_2)$$

or $u_1 = u_2$ $u \propto r^2/t$

$$\frac{r_1^{\ 2}}{t_1} = \frac{r_2^{\ 2}}{t_2} = \frac{r_3^{\ 2}}{t_3} \qquad r_2 = r_1 \sqrt{\frac{t_2}{t_1}}$$

**Where $t_1$ - time since pumping began for a given value of $(h_o - h)$ to occur, before the boundary becomes effective.**

**$t_2$ -time since pumping began, after the boundary becomes effective, when the divergence of the drawdown curve caused by the influence of image well = to particular value of drawdown at $t_1$.**

# Rate-of-Rise Techniques

**Special Techniques:**

- **Determine local *K* around a well, without pumping the well.**

**Rate-of-Rise Techniques**

- **Slug Test**
- **Auger-Hole Method**
- **Piezometer Method**

- **Water is suddenly removed by a bucket, bailer, or cylinder, causing sudden lowering of water levels around the well.**

- **Rise of water level with time is measured and *K* is obtained.**

- **Remove enough water to lower water in the well 10 to 50 cm.**

## Advantages

1. Pumping not needed.
2. Observation wells not required.
3. Tests completed in short time.
4. Provides good preliminary estimate of *K*.
5. Test useful where continuous *Q* is difficult, where obs. wells not available, and where interference from other wells.

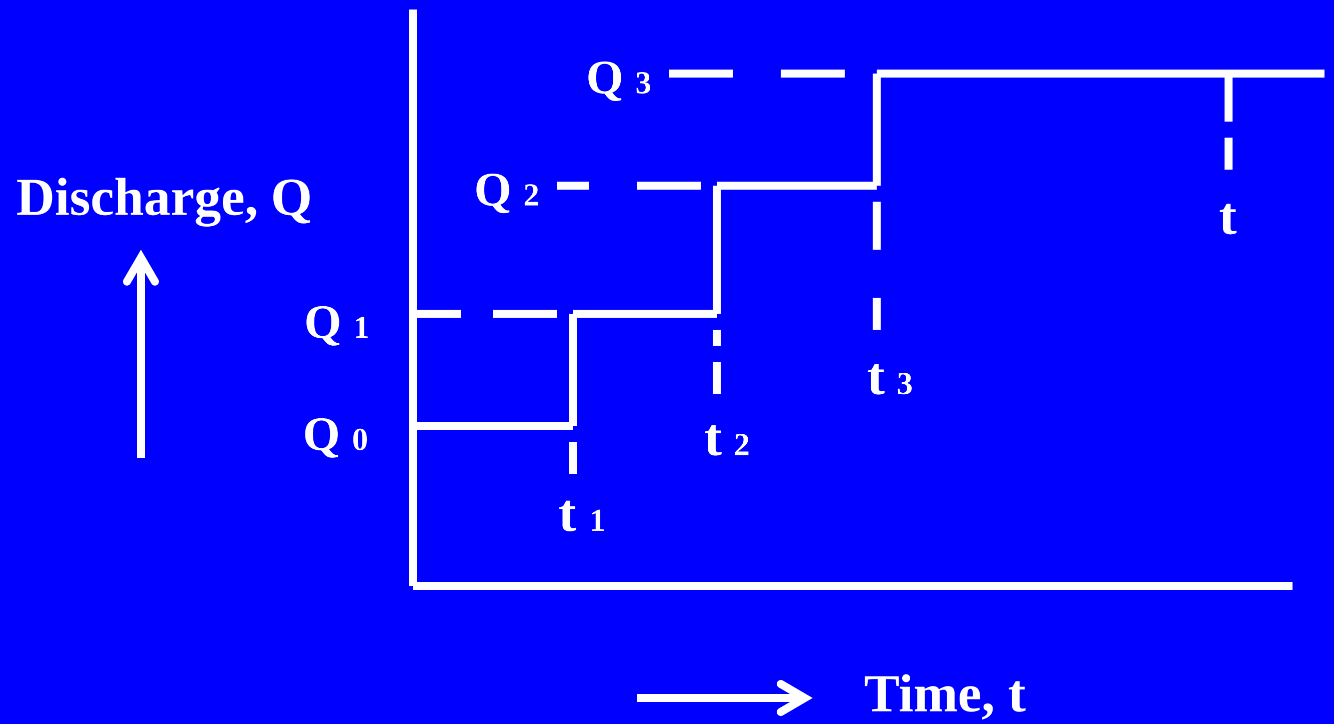
## Disadvantages

1. *K* measured on small area of aquifer.
2. *S* generally not evaluated.

## Step - Type Pumping Test

Rorabaugh (1953) AGU Tran.

Sternberg (1967) J. Groundwater




**Discharge increased in steps of time. Theis Eq.**

$$(h_o - h) = (h_o - h_1) + (h_o - h_2) + \ldots$$

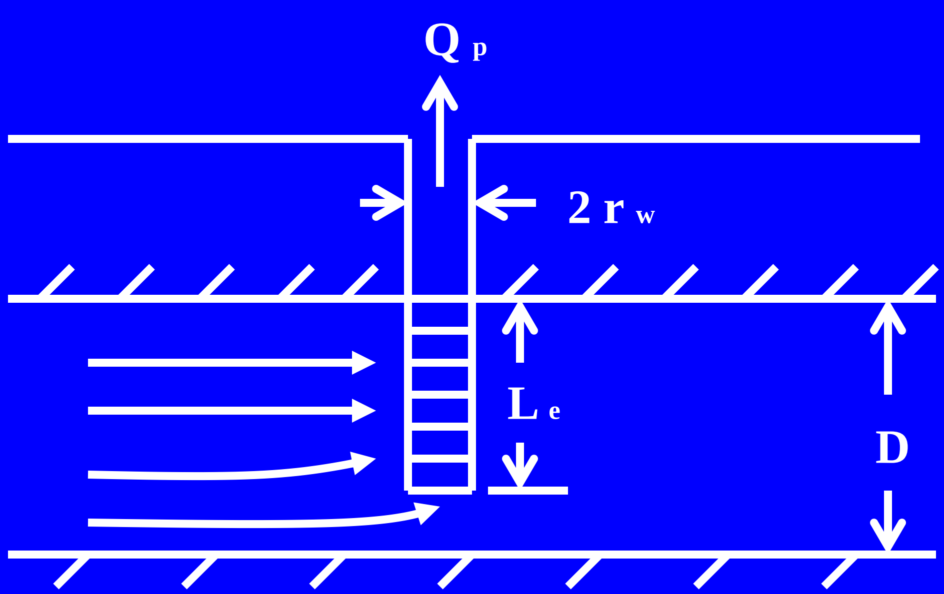
$$\mathrm{Q_o} \qquad\qquad \mathrm{Q_1}$$

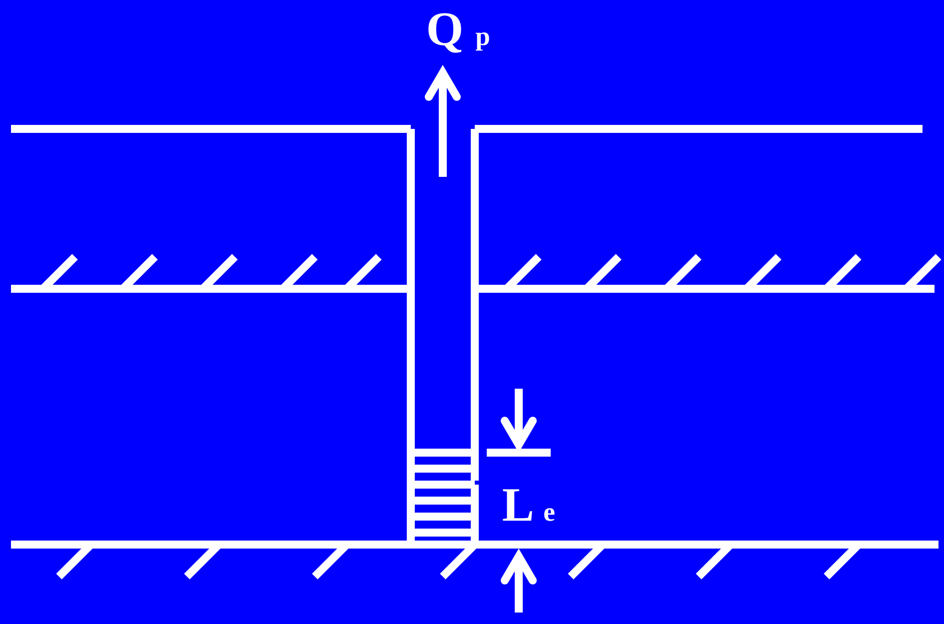
$$< t_1 \qquad\qquad t_1 < t < t_2$$

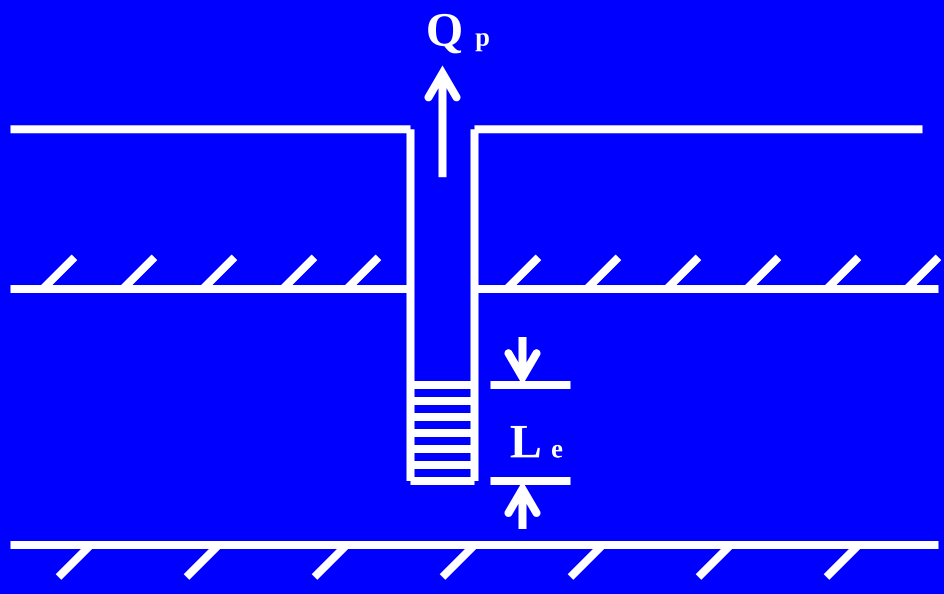
# Partially Penetrating Wells

- A well having length of water entry less than the aquifer is known as partially penetrating well.

- Flow pattern to such wells differs from radial flow around fully penetrating wells.




Q p
L e


Q p
L e

- Average length of flow line in a P.P.W. > that in F.P.W. so that a greater resistance to flow is encountered. Consider two wells – P.P.W. and F.P.W.
- If $Q_p = Q$, then $(\Delta h)_p > \Delta h$

  and if $(\Delta h)_p = \Delta h$, then $Q_p < Q$

  where $Q$ - well discharge

  $\Delta h$ - drawdown at the well

  $p$ - refers to P.P.W.

## Drawdown of P.S. at the well

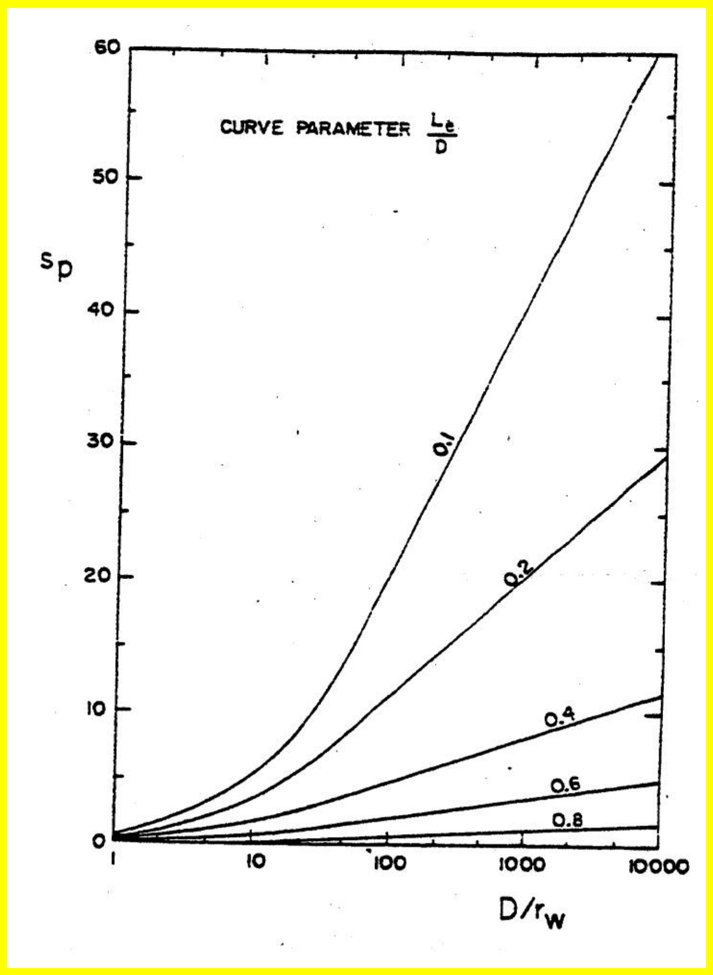
$$S_{wp} = \frac{Q}{4\pi T}\left(\ln\frac{2.25Tt}{r^2 S} + 2S_p\right)$$

$S_p$ - a dimensionless term,

$$= f\left(\frac{Le}{D}, \frac{D}{r_w}\right)$$

$$\frac{Q_p}{Q} = \frac{\ln(r_o / r_w)}{\ln(r_o / r_w) + S_p} \qquad (1)$$

$r_0$= radius of influence




ratio $\frac{Q_b}{Q}$ for P.P.W. > penetration ration $\frac{Le}{D}$

- **For screen at top or bottom, use equation 1 and figure to compute** $\frac{Q_b}{Q}$ **.**

- **For screen at center, use** $\frac{Le}{2}$ **for obtaining** $S_p$ **.**
- **Example:** $2r_w = 12''$ diameter well; $r_o = 2000\,ft$ (radius of influence)

$$D = 50'$$

$$Le = 20'$$

$$\frac{D}{r_w} = \frac{50}{1/2} = 100; \qquad \frac{Le}{D} = \frac{20}{50} = 0.40 \rightarrow S_p = 5$$

$$= \frac{10}{50} = 0.20 \qquad = 10 \ \text{(at center)}$$

$$\frac{Q_p}{Q} = \frac{\ln(4000)}{\ln(4000)+5} = \frac{8.29}{8.29+5} = 0.62$$

$$= \frac{8.29}{8.29+10} = 0.45$$

# Well Losses

**Drawdown at a well = Aquifer drawdown and drawdown caused by flow thru well screen and flow inside the well to pump intake.**

$$S_{iw} = S_w + S_e$$

**Total head loss = formation loss + well loss**

**Laminar flow** **Turbulent flow**

**Since flow in aquifer is laminar,** $s_w \propto Q$

**flow in well screen is turbulent,** $s_w \propto Q^n$

$n \approx 2,$ **but may be > 2 (2 - 4)**

$$s_{iw} = C_f Q + C_w Q^n$$

$$s_{iw} = BQ + CQ^n$$

**where** $C_{f,}\ C_W, (B,C) -$ **constants**

- **For steady flow in a confined aquifer**

$$s_{iw} = \frac{Q}{2\pi bK}\ln\frac{r_o}{r_w} + C_w Q^n$$

$$C_f = B = \frac{1}{2\pi T}\ln(r_o / r_w)$$

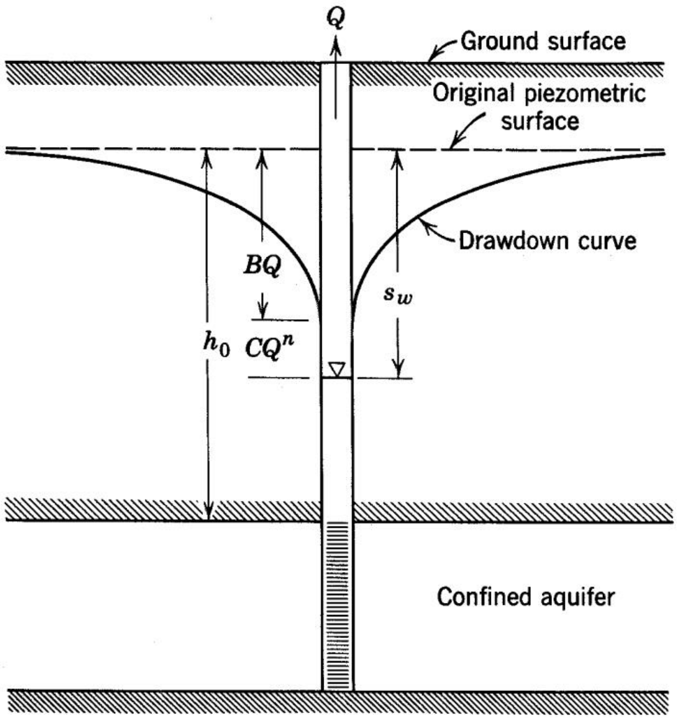



**Fig. 4.29** Relation of well loss $CQ^n$ to drawdown for a well penetrating a confined aquifer.

- **For low *Q*, well losses may be neglected,**

- **For high *Q*, well losses may represent a sizable fraction of total drawdown.**

- **For screen size compatible with surrounding porous media and which is not clogged, well loss caused by water entering is small than the portion resulting from axial movement within the well.**

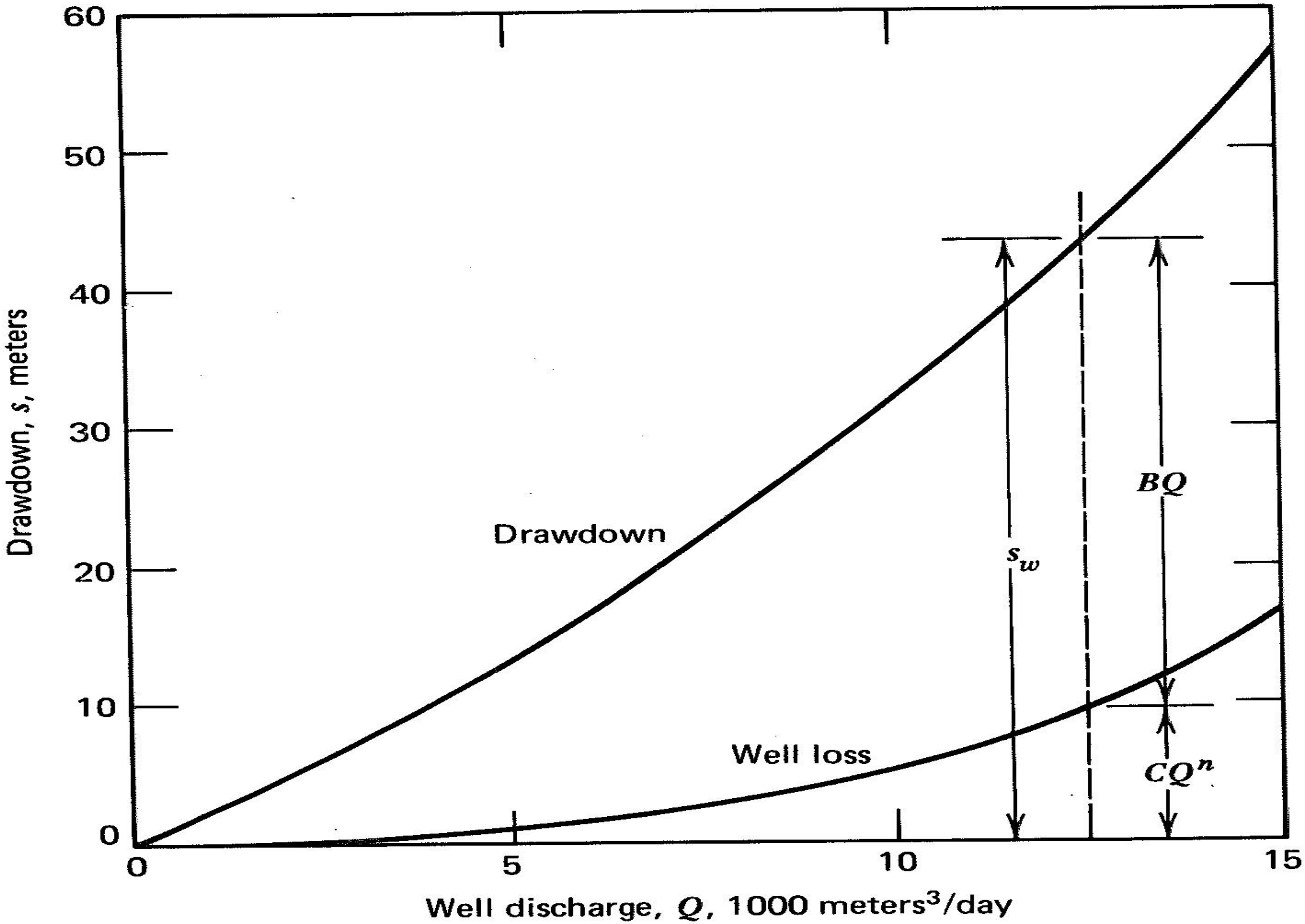



**Fig. 4.30** Variation of total drawdown $s_w$, aquifer loss $BQ$, and well loss $CQ^n$ with well discharge (after Rorabaugh[55]).

# Specific Capacity

$$\frac{Q}{s_{iw}} = \frac{1}{C_f + C_w Q^{n-1}} \quad \text{-} \qquad \text{specific capacity} = f(Q)$$

$$\frac{Q}{s_{iw}} = \frac{\text{Discharge}}{\text{Total Drawdown}}$$

$$s_{iw} = BQ + CQ^2 ; n = 2$$

## Empirical formulas developed in field

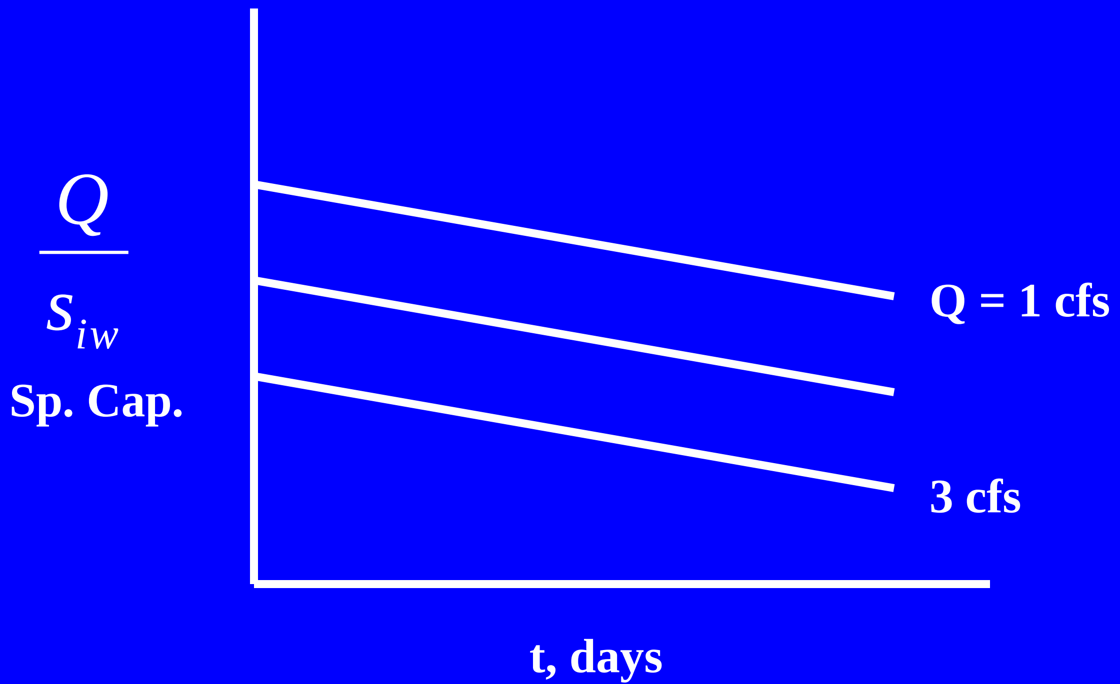
- $T = \text{Const}\left(\frac{Q}{s_{iw}}\right)$ **, const. varies depending upon geology - 0(2000).**

## Unsteady flow for a confined aquifer

$$s_{iw} = \frac{2.3Q}{4\pi T}\log\frac{2.25Tt}{r_w{}^2 S} + CQ^n$$

$$= f(Q,t)$$

$$\frac{Q}{s_{iw}} = \frac{1}{\frac{2.3}{4\pi T}\log\frac{2.25Tt}{r_w^2 S} + CQ^{n-1}}$$




- Hence, the concept that

  $Q \sim s_{iw} \rightarrow$ implying a constant S.C. $\rightarrow$

  Can introduce sizable errors.

# Multiple Well System

**To determine drawdowns (or interference) in a well field.**

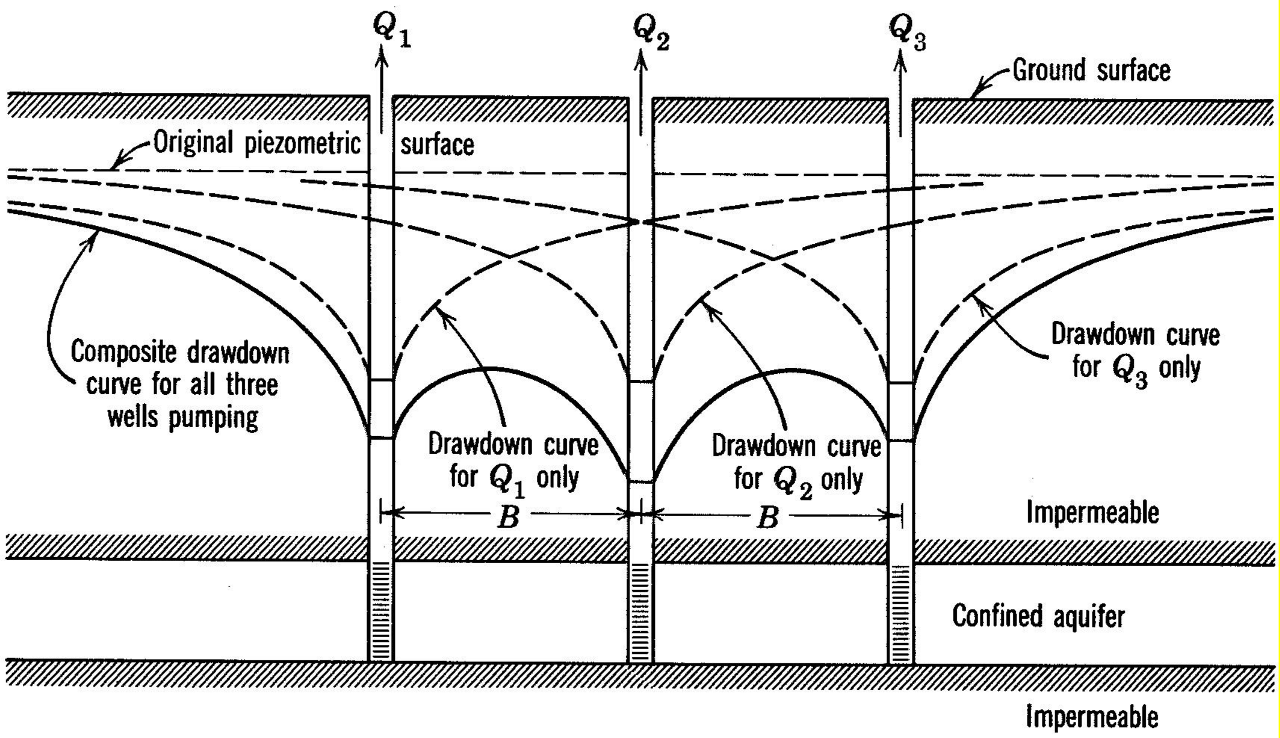



**Fig. 4.27** Individual and composite drawdown curves for three wells in a line.

Determine drawdowns at various points from known $Q$'s and add them together.

At a point, Total drawdown.

$$D_T = D_1 + D_2 + ... + D_n$$

Where

$D_T = D_1 + D_2 ,..D_n -$ drawdown at the point due to $Q_1, Q_2 ,..Q_n$

**At a distance of 2D from a well, the effect of partial penetration is negligible on the flow pattern and drawdown.**

**D = Average aquifer thickness**

**EXAMPLE 4.7.1**

A 0.5-m diameter well (200 m from a river) is pumping at an unknown rate from a confined aquifer (see Figure 4.7.6). The aquifer properties are $T = 432$ m$^2$/day and $S = 4.0 \times 10^{-4}$. After eight hours of pumping, the drawdown in the observation well (60 m from the river) is 0.8 m. Compute the rate of pumping and the drawdown in the pumped well. What is the effect of the river on drawdown in the observation well and in the pumped well?

**SOLUTION**

The following information is given in the above statement: $r_w = 0.25$ m, $T = 432$ m$^2$/day $= 5.0 \times 10^{-3}$ m$^2$/s, $S = 4 \times 10^{-5}$, $t = 8$ hr $= 28{,}800$ s, and $s = 0.8$ m. A recharging image well is placed at the same distance from the river as the pumped well as shown in Figure 4.7.6b.

Equation 4.7.5 is used to compute the discharge from the pumped well knowing the above information:

$$s = \frac{Q}{4\pi T} W(u_p) - \frac{Q}{4\pi T} W(u_i)$$

$$u_p = \frac{r_p^2 S}{4Tt} = \frac{(140)^2 (4 \times 10^{-4})}{4(5 \times 10^{-3})(28800)} = 1.36 \times 10^{-2}$$

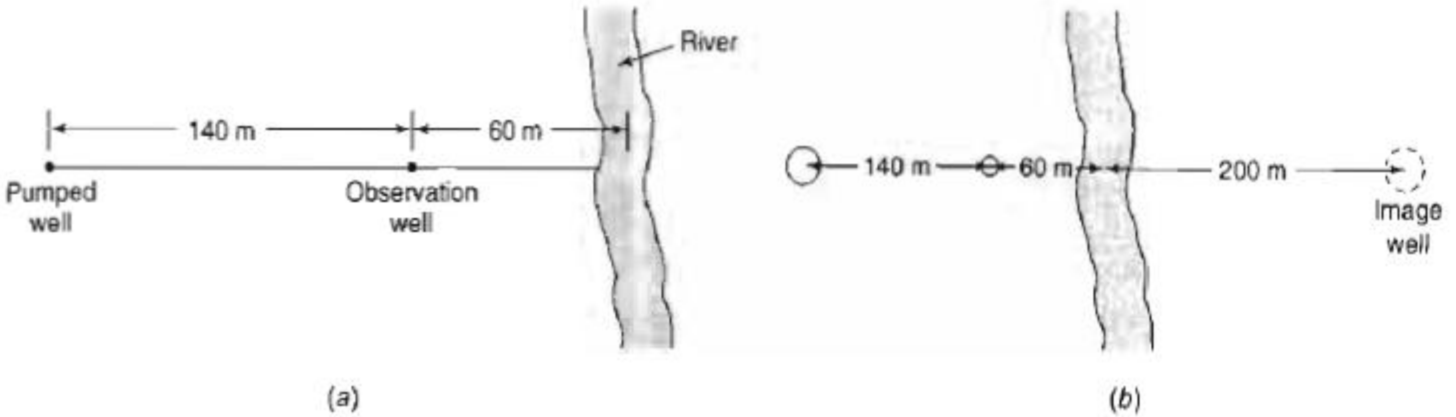



**Figure 4.7.6.** Example 4.7.1 system. (*a*) Well locations (*b*) Image well location

$$u_i = \frac{r_i^2 S}{4Tt} = \frac{(260)^2 \left(4\times10^{-4}\right)}{4\left(5\times10^{-3}\right)(28800)} = 4.69\times10^{-2}$$

$$W\left(u_p\right) = 3.79 \text{ for } u_p = 1.36\times10^{-2} \text{ and } W\left(u_i\right) = 2.54 \text{ for } u_i = 4.69\times10^{-2}$$

Thus the discharge is computed using

$$0.8 = \frac{Q}{4\pi\left(5\times10^{-3}\right)}(3.79) - \frac{Q}{4\pi\left(5\times10^{-3}\right)}(2.54)$$

so that $Q = 0.04\ \mathrm{m^3/s}$.

The drawdown in the pumped well is computed using equation 4.7.5:

$$u_w = \frac{r_w^2 S}{4Tt} = \frac{(0.25)^2\left(4\times10^{-4}\right)}{4\left(5\times10^{-3}\right)(28800)} = 4.34\times10^{-8}$$

$$u_i = \frac{(400)^2\left(4\times10^{-4}\right)}{4\left(5\times10^{-3}\right)(28800)} = 0.111$$

$$W(u_w) = 16.38 \text{ for } u_w = 4.39\times10^{-8} \text{ and } W(u_i) = 1.75 \text{ for } u_i = 0.111$$

Thus the drawdown is

$$s_w = \frac{0.04}{4\pi\left(5\times10^{-3}\right)}(16.38) - \frac{0.04}{4\pi\left(5\times10^{-3}\right)}(1.75) = 9.31 \text{ m}$$

The effect of the river on the wells is to decrease the drawdown, so the reduced drawdown in the observation well is

$$s_{river} = -\frac{Q}{4\pi T}W(u_i) = -\frac{0.04}{4\pi\left(5\times10^{-3}\right)}(2.54) = -1$$

Similarly, in the pumped well, the reduced drawdown is

$$s_{river} = -\frac{0.04}{4\pi\left(5\times10^{-3}\right)}(1.75) = -1.11 \text{ m}$$

■

$$u_w = \frac{r_w^2 S}{4Tt} = \frac{(0.25)^2\left(4\times10^{-4}\right)}{4\left(5\times10^{-3}\right)(28800)} = 4.34\times10^{-8}$$

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■

**EXAMPLE 4.7.2**

A well is pumping near a barrier boundary (see Figure 4.7.9) at a rate of 0.03 $m^3/s$ from a confined aquifer 20 m thick. The hydraulic conductivity of the aquifer is 27.65 m/day and its storativity is $3 \times 10^{-5}$. Determine the drawdown in the observation well after 10 hours of continuous pumping. What is the fraction of the drawdown attributable to the barrier boundary?

**SOLUTION**

The following information is given in the above problem statement: $Q = 0.03$ $m^3/s$, $b = 20$ m, $K = 27.65$ m/day $= 3.2 \times 10^{-4}$ m/s, $S = 3 \times 10^{-5}$, $t = 10$ hrs $= 36{,}000$ s. An image well is placed across the boundary at the same distance from the boundary as the pumped well (as shown in Figure 4.7.9b). The drawdown in the observation well is due to the real well and the imaginary well (which accounts for the barrier boundary). Hence, using Equation 4.7.17

$$s = \frac{Q}{4\pi T} W(u_p) + \frac{Q}{4\pi T} W(u_i)$$

$$u_p = \frac{r_p^2 S}{4Tt} = \frac{(240)^2 (3 \times 10^{-5})}{4(20)(3.2 \times 10^{-4})(36{,}000)} = 1.88 \times 10^{-3}$$

Next compute the distance from the observation well to the image well: $r_i^2 = 600^2 + 240^2 - 2(600)(300) \cos 30° = 16{,}8185$ $m^2$ so $r_i = 410$ m. Using $r_i$, compute

$$u_i = \frac{168185 (3 \times 10^{-5})}{4(20)(3.2 \times 10^{-4})(36{,}000)} = 5.47 \times 10^{-3}$$

The well functions are now computed or obtained from Table 4.4.1 as $W(u_p) = 5.72$ for $u_p = 1.88 \times 10^{-3}$ and $W(u_i) = 4.64$ for $u_i = 5.47 \times 10^{-3}$.

The drawdown at the observation well is computed as

$$s = \frac{0.03}{4\pi(20)\left(3.2\times10^{-4}\right)}(5.72+4.64) = 3.86 \text{ m}.$$

The drawdown attributable to the barrier boundary is computed as

$$s_i = \frac{Q}{4\pi T}W(u_i) = \frac{0.03}{4\pi(20)\left(3.2\times10^{-4}\right)}(4.64) = 1.73 \text{ m}$$

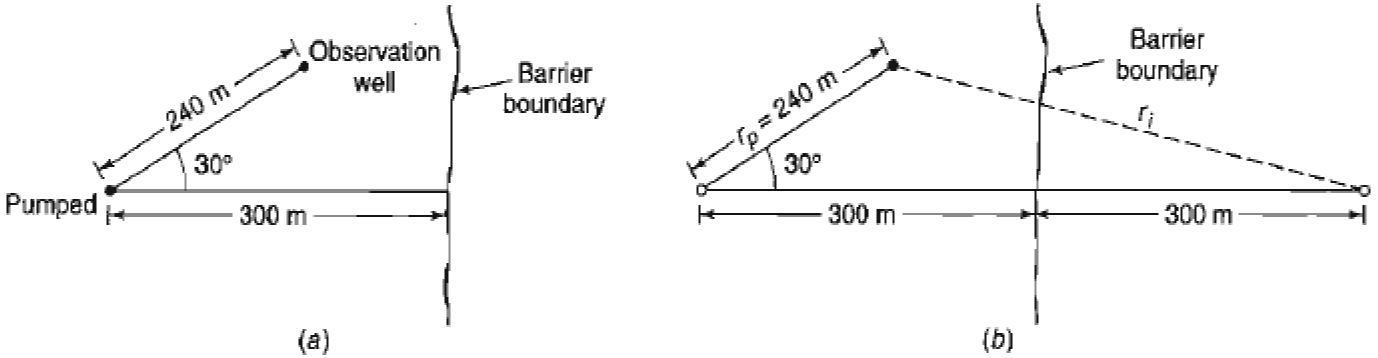



**Figure 4.7.9.** Example 4.7.2 system. (*a*) Well locations (*b*) Image well location

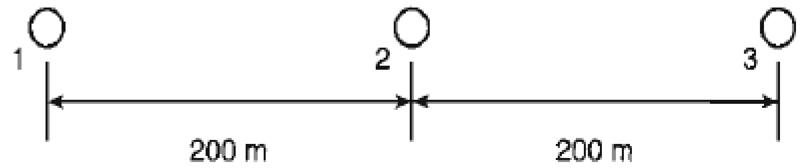
and the fraction of drawdown attributable to the impermeable boundary is

$$\frac{s_i}{s} = \frac{1.73}{3.86} = 0.45\,(45\%).$$

■

**EXAMPLE 4.8.1**

Three pumping wells located along a straight line are spaced at 200 m apart. What should be the steady-state pumping rate from each well so that the near steady-state drawdown in each well will not exceed 2 m? The transmissivity of the confined aquifer, which all the wells fully penetrate, is 2400 $m^2$/day and all the wells are 40 cm in diameter. The thickness of the aquifer is 40 m and the radius of influence of each well is 800 m.




**SOLUTION**

The following information is given in the above problem statement: $s_1 \le 2$ m, $s_2 \le 2$ m, and $s_3 \le 2$ m, $T =$ 2,400 $m^2$/day $= 27.8 \times 10^{-3}$ $m^2$/s, $r_w = 0.2$ m, $b = 40$ m, $r_0 = 800$ m, and $r = 200$ m. Let $Q$ be the pumping rate from each well and $h_0$ be the head before pumping started. For well **1**, $s_1 = s_{11} + s_{12} + s_{13}$ where $s_{ij}$ is the drawdown in well $i$ due to pumping in well $j$. Thus, for the other wells, $s_2 = s_{21} + s_{22} + s_{23}$, and $s_3 = s_{31} + s_{32} + s_{33}$. By symmetry, $s_1 = s_3$. The drawdowns in well 1 due to pumping in wells 1, 2, and 3 are respectively

$$s_{11} = \frac{Q\ln\left(\frac{r_0}{r_w}\right)}{2\pi T} = \frac{Q\ln\left(\frac{800}{0.2}\right)}{2\pi\left(27.8\times10^{-3}\right)} = 47.48Q$$

$$s_{12} = \frac{Q\ln\left(\frac{r_0}{r_{12}}\right)}{2\pi T} = \frac{Q\ln\left(\frac{800}{200}\right)}{2\pi\left(27.8\times10^{-3}\right)} = 7.94Q$$

$$s_{13} = \frac{Q\ln\left(\frac{r_0}{r_{13}}\right)}{2\pi T} = \frac{Q\ln\left(\frac{800}{400}\right)}{2\pi\left(27.8\times10^{-3}\right)} = 3.97Q$$

The drawdowns in wells 1 and 3 are identical so total drawdown in the wells is $s_1 = s_3 = 47.48Q + 7.94Q + 3.97Q = 59.39Q$. The drawdowns in well 2 due to pumping in wells 1, 2, and 3 are respectively

$$s_{21} = \frac{Q\ln\left(\frac{r_0}{r_{12}}\right)}{2\pi T} = \frac{Q\ln\left(\frac{800}{200}\right)}{2\pi\left(27.8\times10^{-3}\right)} = 7.94Q$$

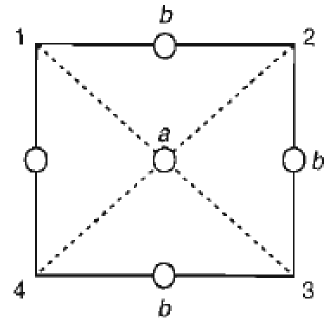
$$s_{22} = s_{11} = 47.48Q$$

$$s_{23} = s_{21} = 7.94Q$$

The total drawdown in well 2 is $s_2 = 7.94Q + 47.48Q + 7.94Q = 63.36Q$. The relationships for $s_1 = 59.39Q$ and $s_2 = 63.36Q$ show that for the same discharge from all the wells, more drawdown results at the middle well; therefore, the drawdown in this well governs. So using $s_2 \le 2$ or $63.36Q \le 2$, then the steady-state pumping rate from each well should be $Q \le 3.16 \times 10^{-2}$ m$^3$/s = 113 m$^3$/hr. ■

**EXAMPLE 4.8.2** It is required to dewater a construction site 80 m by 80 m. The bottom of the construction will be 1.5 m below the initial water surface elevation of 90 m. Four pumps are to be used in 0.5-m diameter wells at the four corners of the site. The transmissivity and the storage coefficient of the aquifer are 1,600 $m^2$/day and 0.16, respectively. The site needs to be ready after one month of pumping. Determine the required pumping rate.

**SOLUTION** To solve this problem, the least drawdown at the site should be greater than 1.5 m. It can be shown that the potential points of interest that may have the least drawdown are the center of the square (point *a*) and the midpoint on each side of the square (points *b*). Approximation is made using the Cooper-Jacob method.




At point *a* (the center of the square), $r = \sqrt{40^2 + 40^2} = 56.6\ \text{m}$, and

$$u = \frac{r^2 S}{4Tt} = \frac{(56.6\ \text{m})^2 (0.16)}{4(1600\ \text{m}^2/\text{day})(30\ \text{days})} = 0.00267$$

Since $u < 0.01$, we can use the approximate solution by Cooper-Jacob expressed by Equation 4.4.7:

$$s_a = \frac{Q}{4\pi T}(-0.5772 - \ln(u)) = \frac{Q}{4\pi(1600\ \text{m}^2/\text{day})}(-0.5772 - \ln(0.00267)) = 0.0002661Q$$

Using the principle of superposition and by symmetry, the drawdown caused by the four wells is $s_T = 4 \times s_a = 4 \times 0.0002661Q = 0.0010643Q$ and $s_T = 0.0010643Q = 1.5\ \text{m} \rightarrow Q = 1409\ \text{m}^3/\text{day}$.

At any of the four points represented by *b*, $r_1 = 40$ m for two of the wells and $r_2 = \sqrt{80^2 + 40^2} = 89.44$ m for the remaining two wells. Then

$$u_1 = \frac{r^2 S}{4Tt} = \frac{(40\ \text{m})^2 (0.16)}{4(1600\ \text{m}^2/\text{day})(30\ \text{days})} = 0.0013333$$

$$u_2 = \frac{r^2 S}{4Tt} = \frac{(89.44\ \text{m})^2 (0.16)}{4(1600\ \text{m}^2/\text{day})(30\ \text{days})} = 0.006666$$

Since $u_1 < 0.01$ and $u_2 < 0.01$, the Cooper–Jacob method of solution can be used again:

$$s_b = 2\left[\frac{Q}{4\pi T}\left(-0.5772 - \ln\left(u_1\right)\right)\right] + 2\left[\frac{Q}{4\pi T}\left(-0.5772 - \ln\left(u_2\right)\right)\right]$$

$$= 2\left[\frac{Q}{4\pi\left(1600 \text{ m}^2/\text{day}\right)}\left(-0.5772 - \ln(0.0013333)\right)\right] +$$

$$2\left[\frac{Q}{4\pi\left(1600 \text{ m}^2/\text{day}\right)}\left(-0.5772 - \ln(0.006666)\right)\right]$$

$$= 2 \times 0.0003Q + 2 \times 0.0002205Q$$

$$= 1.041 \times 10^{-3} Q = 1.5 \text{ m} \rightarrow Q = 1441 \text{ m}^3/\text{day}$$

Thus the points represented by $b$ are critical and a discharge of 1,441 $m^3$/day from each well is required.

■