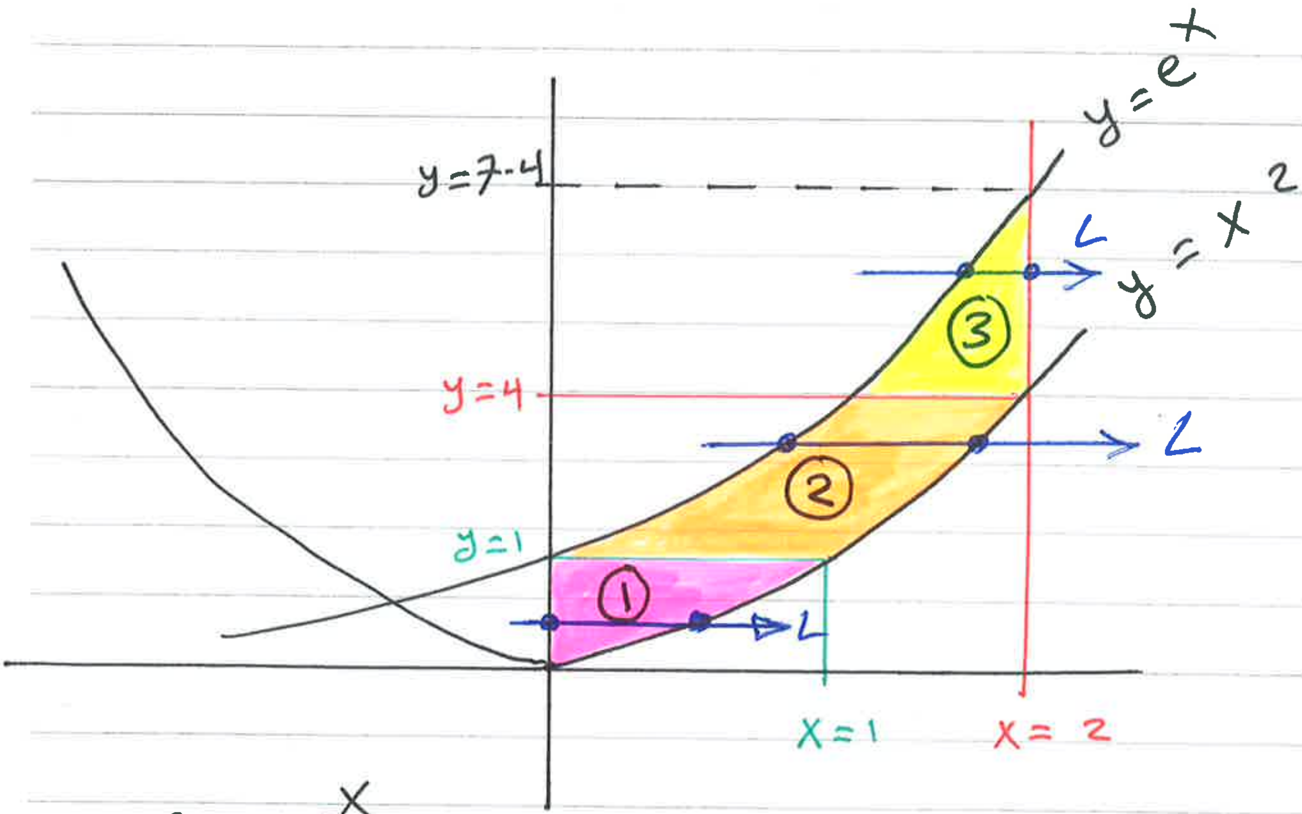
Q1/ Revers the order of the integral:

$$\int_{0}^{2}\int_{x^2}^{e^x} f(x,y)\,dy\,dx$$

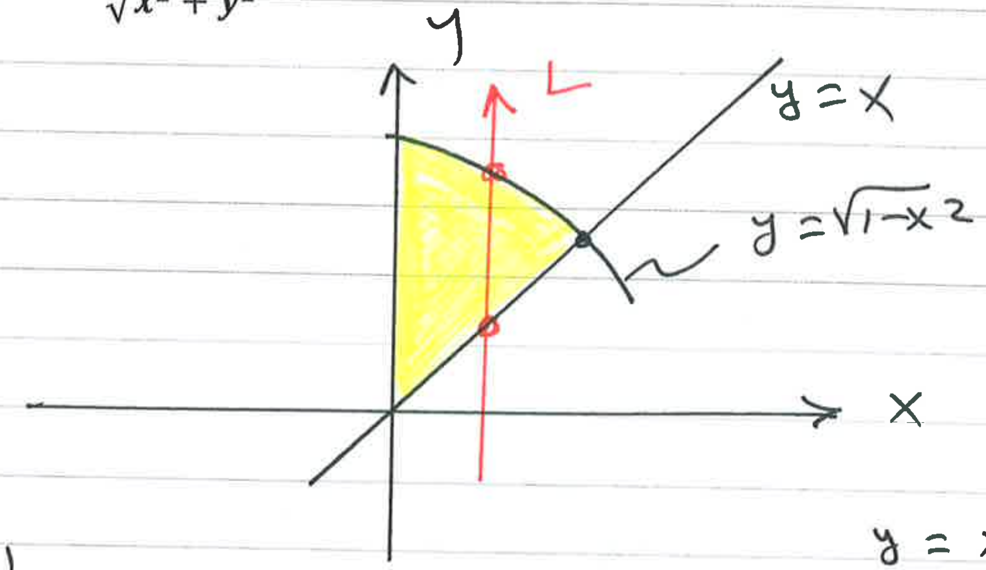


$$\int_{x=0}^{x=2}\int_{y=x^2}^{y=e^x} f(x,y)\,dy\,dx =$$

$$\int_{y=0}^{y=1}\int_{x=0}^{x=\sqrt{y}} f(x,y)\,dx\,dy+\int_{y=1}^{y=4}\int_{x=\ln y}^{x=\sqrt{y}} f(x,y)\,dx\,dy+\int_{y=4}^{y=7.4}\int_{x=\ln y}^{x=2} f(x,y)\,dx\,dy$$

Q2/ Evaluate the integral (by changing to polar coordinates):

$$\int_0^{0.707}\int_x^{\sqrt{1-x^2}} \frac{y^2}{\sqrt{x^2+y^2}}\,dy\,dx$$



$y = x$

$y = \sqrt{1-x^2}$

$x = \sqrt{1-x^2}$

$x^2 = 1 - x^2$

$2x^2 = 1$

$x^2 = \frac{1}{2}$

$x = 0.707$

$$\int_{\theta=\pi/4}^{\theta=\pi/2}\int_{r=0}^{r=1} \frac{r^2\sin^2\theta}{\cancel{r}}\cdot\cancel{r}\,dr\,d\theta$$

$$= \int_{\theta=\pi/4}^{\theta=\pi/2}\int_{r=0}^{r=1} r^2\sin^2\theta\,dr\,d\theta$$

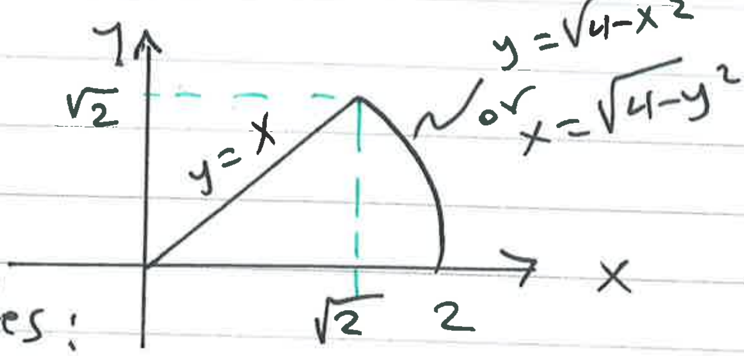
$$= \int_{\theta=\pi/4}^{\theta=\pi/2} \frac{r^3}{3}\sin^2\theta\,d\theta\Big|_0^1$$

$$= \frac{1}{3}\int_{\theta=\pi/4}^{\theta=\pi/2} \frac{1}{2}(1-\cos 2\theta)\,d\theta$$

$$= \frac{1}{3}\left[\frac{1}{2}\theta - \frac{1}{4}\sin 2\theta\right]_{\pi/4}^{\pi/2} =$$

Q3/ Change the order of the integral to Cartesian coordinates:

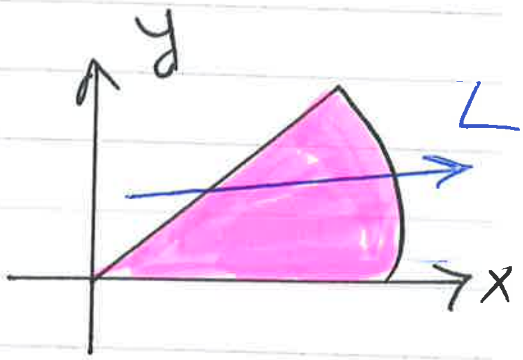
$$\int_0^{\frac{\pi}{4}}\int_0^2 \frac{r}{\sqrt{1+r^2}}\,dr\,d\theta$$



cartesian coordinates:

①

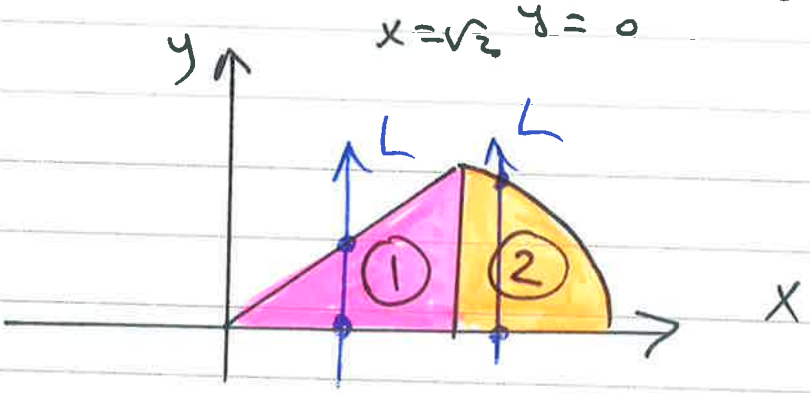
$$\int_{y=0}^{y=\sqrt{2}}\int_{x=y}^{x=\sqrt{4-y^2}} \frac{1}{\sqrt{1+x^2+y^2}}\,dx\,dy$$



or

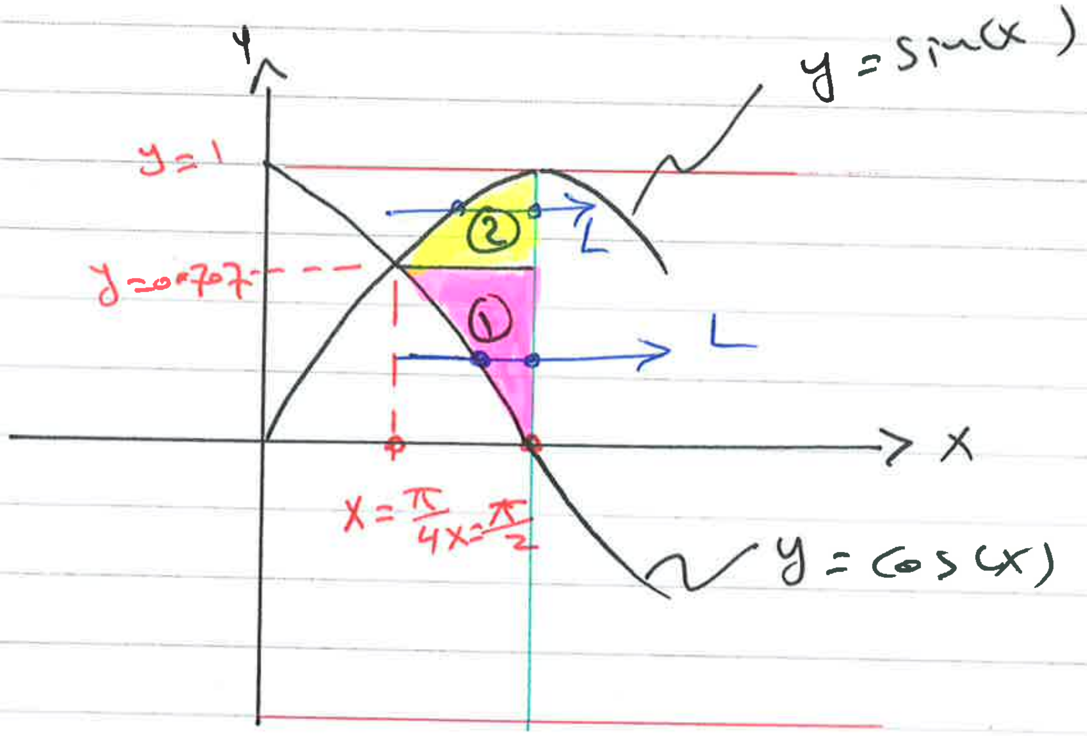
②

$$\int_{x=0}^{x=\sqrt{2}}\int_{y=0}^{y=x} \frac{1}{\sqrt{1+x^2+y^2}}\,dy\,dx + \int_{x=\sqrt{2}}^{x=2}\int_{y=0}^{y=\sqrt{4-x^2}} \frac{1}{\sqrt{1+x^2+y^2}}\,dy\,dx$$



Q4/ Revers the order of integral:

$$\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \int_{\cos(x)}^{\sin(x)} f(x,y)\, dy\, dx$$

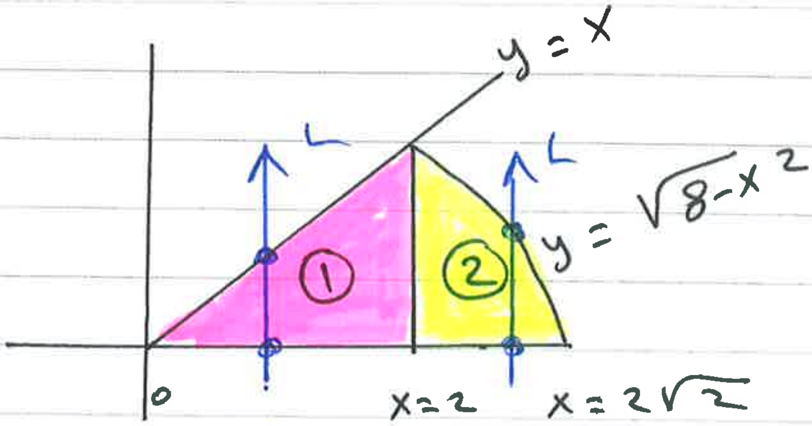


$$\int_{x=\frac{\pi}{4}}^{x=\frac{\pi}{2}} \int_{y=\cos(x)}^{y=\sin(x)} f(x,y)\, dy\, dx =$$

$$\int_{y=0}^{y=0.707} \int_{x=\cos^{-1}y}^{x=\frac{\pi}{2}} f(x,y)\, dx\, dy + \int_{y=0.707}^{y=1} \int_{x=\sin^{-1}y}^{x=\frac{\pi}{2}} f(x,y)\, dx\, dy$$

Q5/ Evaluate the integral (by changing to polar coordinates):

$$\int_0^2 \int_0^x \sqrt{x^2+y^2}\,dydx + \int_2^{2\sqrt{2}} \int_0^{\sqrt{8-x^2}} \sqrt{x^2+y^2}\,dy\,dx$$



in polar coordinates:

$$\int_{\theta=0}^{\theta=\pi/4} \int_{r=0}^{r=2\sqrt{2}} \sqrt{r^2}\cdot r\,dr\,d\theta$$

$$= \int_0^{\pi/4}\int_0^{2\sqrt{2}} r^2\,dr\,d\theta = \int_0^{\pi/4} \frac{r^3}{3}\,d\theta \Big|_0^{2\sqrt{2}}$$

$$= \int_0^{\pi/4} \frac{16\sqrt{2}}{3}\,d\theta$$

$$= \frac{16\sqrt{2}}{3}\,\frac{\pi}{4} = \frac{16\sqrt{2}\,\pi}{12} = \frac{4\sqrt{2}}{3}\pi$$