CHAPTER TWO: PRECIPITATION

The term precipitation denotes all forms of water that reach the earth from the atmosphere. The usual forms are rainfall, snowfall, hail, frost and dew. Of all these, only the first two contribute significant amounts of water. Rainfall being the predominant form of precipitation causing stream flow, especially the flood flow in a majority of rivers in India, unless otherwise stated the term rainfall is used in this book synonymously with precipitation. The magnitude of precipitation varies with time and space. Differences in the magnitude of rainfall in various parts of a country at a given time and variations of rainfall at a place in various seasons of the year are obvious and need no elaboration. It is this variation that is responsible for many hydrological problems, such as floods and droughts. The study of precipitation forms a major portion of the subject of hydrometeorology. In this chapter, a brief introduction is given to familiarize the engineer with important aspects of rainfall, and, in particular, with the collection and analysis of rainfall data. For precipitation to form:

(i) the atmosphere must have moisture,
(ii) there must be sufficient nuclei present to aid condensation,
(iii) weather conditions must be good to create condensation of water , and
(iv) the products of condensation must reach the earth.

FORMS OF PRECIPITATIO N

Some of common forms of precipitation are: rain, snow, drizzle, glaze, sleet and hail.

1. ***Rain*** : It is the principal form of precipitation in India. The term rainfall is used to describe precipitations in the form of water drops of sizes larger than 0.5 mm. The maximum size of a raindrop is about 6 mm. Any drop larger in size than this tends to break up into drops of smaller sizes during its fall from the clouds. On the basis of its intensity, rainfall is classified as:

TYPE	*Intensity*
1. Light rain	trace to 2.5 *mm/h*
2. Moderate rain	2.5 mm/h to 7.5 *mm/h*
3. Heavy rain	> 7.5 *mm/h*

2. ***Snow*** : It is another important form of precipitation. Snow consists of ice crystals which usually combine to form flakes. When fresh, snow has an initial density varying from 0.06 to 0.15 g/cm^3 and it is usual to assume an average density of 0.1 g/cm^3.

3. ***Drizzle*** : A fine sprinkle of numerous water droplets of size less than 0.5 mm and intensity less than 1 *mm/h* is known as drizzle. In this the drops are so small that they appear to float in the air.
4. ***Glaze*** : When *rain* or *drizzle* comes in contact with cold ground at around 0°C , the water drops freeze to form an ice coating called glaze or freezing rain.
5. **Sleet** : It is frozen raindrops of transparent grains which form when rain falls through air at subfreezing temperature. In Britain, sleet denotes precipitation of snow and rain simultaneously.
6. ***Hail*** : It is a showery precipitation in the form of irregular pellets or lumps of ice of size more than 8 mm. Hails occur in violent thunderstorms in which vertical currents are very strong.

MEASUREMENT OF PRECIPITATION

A. RAINFALL

Precipitation is expressed in terms of the depth to which rainfall water would stand on an area if all the rain were collected on it. Thus 1cm of rainfall over a catchment area of 1 km^2 represents a volume of water equal to 104 m^3. In the case of snowfall, an equivalent depth of water is used as the depth of precipitation. The precipitation is collected and measured in a raingauge. Terms such as pluviometer, ombrometer and Hyetometer are also sometimes used to designate a raingauge. A raingauge essentially consists of a cylindrical-vessel assembly kept in the open to collect rain. The rainfall catch of the raingauge is affected by its exposure conditions.

To enable the catch of raingauge to accurately represent the rainfall in the area surrounding the raingauge standard settings are adopted. For sitting a raingauge the following considerations are important:

- The ground must be level and in the open and the instrument must present a horizontal catch surface.
- The gauge must be set as near the ground as possible to reduce wind effects but it must be sufficiently high to prevent splashing, flooding, etc.
- The instrument must be surrounded by an open fenced area of at least 5.5 m × 5.5 m. No object should be nearer to the instrument than 30 m or twice the height of the obstruction.

Raingauges can be broadly classified into two categories as:

[1] Nonrecording raingauges

[2] Recording gauges.

[1] Nonrecording Gauges

The nonrecording gauge extensively used in Iraq Egypt and India is the Symons' gauge. It essential consists of a circular collecting area of 12.5 cm (5.0 inch) diameter connected to a funnel. The rim of the collector is set in a horizontal plane at a height of 30 cm above the ground level. The funnel discharges the rainfall catch into a receiving vessel. The funnel and receiving vessel are housed in a metallic container. Fig. 2.1 shows the details of the installation. Water contained in the receiving vessel is measured by a suitably graduated measuring glass, with an accuracy up to 0.1 mm.

[2] Recording Gauges

Recording gauges produce a continuous plot of rainfall against time and provide valuable data of intensity and duration of rainfall for hydrological analysis of storms. The following are some of the commonly used recording raingauges.

float Type: This type of recording raingauge is also known as Natural-Syphon type gauge, Fig. 2.2 . Here the rainfall collected by a funnel-shaped collector is led into a float chamber causing a float to rise. As the float rises, a pen attached to the float through a lever system records the elevation of the float on a rotating drum driven by a clockwork mechanism. A syphon arrangement empties the float chamber when the float has reached a pre-set maximum level. A typical chart from this type of raingauge is shown in Fig. 2.3.

Tipping-Bucket Type: This is a 30 cm size raingauge adopted for use by the *US Weather Bureau*. The catch from the funnel falls onto one of a pair of small buckets. These buckets are so balanced that when 0.25 mm of rainfall collects in one bucket, it tips and brings the other one in position. The water from the tipped bucket is collected in a storage can, Fig. 2.4. The tipping actuates an electrically driven pen to trace a record on clockwork-driven chart. The water collected in the storage can is measured at regular intervals to provide the total rainfall and also serve as a check. It may be noted that the record from the tipping bucket gives data for the intensity of rainfall. Further, the instrument is ideally suited for digitalizing of the output signal.

Weighting-Bucket Type : In this raingauge the catch from the funnel empties into a bucket mounted on a weighing scale, Fig. 2.5. The weight of the bucket and its contents are recorded on a clock-work-driven chart. The clockwork mechanism has the capacity to run for as long as one week. This instrument gives a plot of the accumulated rainfall against the elapsed time, i.e. the mass curve of rainfall. In some instruments of this type the recording unit is so constructed that the pen reverses its direction at every preset value, say 7.5 cm (3 in.) so that a continuous plot of storm is obtained.

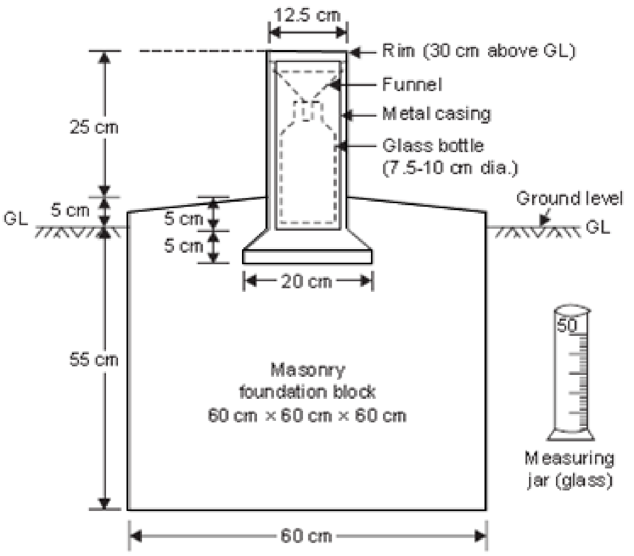


Fig. 2.1: Symons' type raingauge

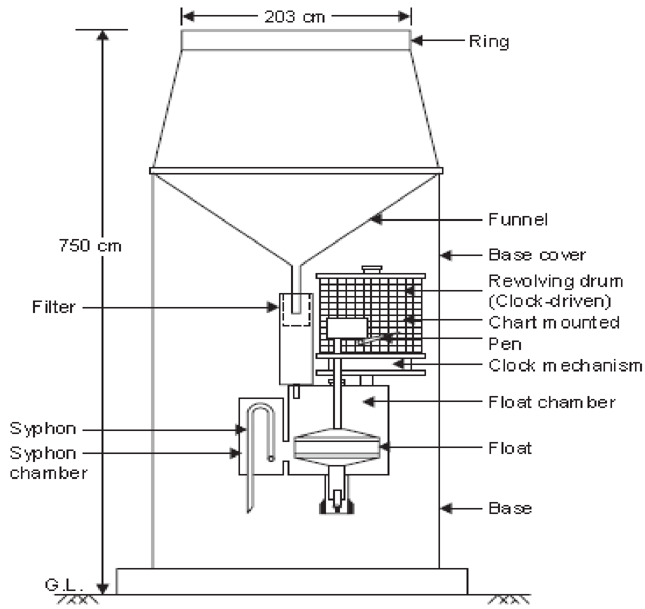


Fig.2.2: Float type raingauge

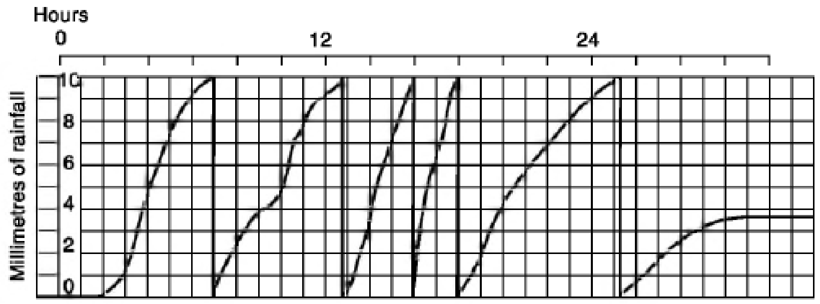


Fig. 2.3: Schematic Recording from Float type

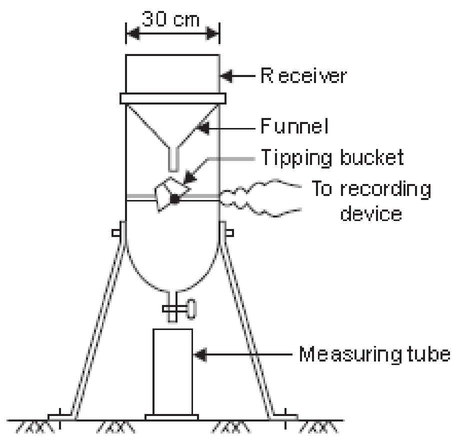


Fig.2.4: Tipping Bucket raingauge

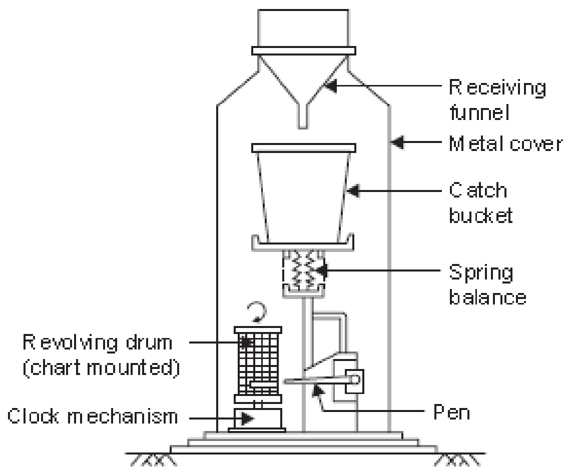


Fig.2.5: Weighing Bucket raingauge

B. SNOWFALL

Snowfall as a form of precipitation differs from rainfall in that it may accumulate over a surface for some time before it melts and causes runoff. Further, evaporation from the surface of accumulated snow surface is a factor to be considered in analysis dealing with snow. Water equivalent of snowfall is included in the total precipitation amounts of a station to prepare seasonal and annual precipitation records. Graduated stick or staff is used to measure the depth of snow at a selected place. Average of several measurements in an area is taken as the depth of snow in a snowfall event.
Snow boards are 40 cm side square boards used to collect snow samples. These boards are placed horizontally on a previous accumulation of snow and after a snowfall event the snow samples are cut off from the board and depth of snow and water equivalent of snow are then recorded. The *water equivalent of snow* ***is*** the amount of water present in a known depth of snow . It could be estimated if the density of snow is available.. Freshly fallen snow may have a density in the range of 0.07 to 0.15 with an average value of about 0.10. The accumulated snow however causes compaction and in regions of high accumulation densities as high as 0.4 to 0.6 is not uncommon. Where specific data is not available, it is usual to assume the density of fresh snow as 0.10. Water equivalent of snow is obtained in two ways.

RAINGAUGE NETWORK

For a storm over a catchment , the number of raingauges should be as large as possible, i.e. the catchment area per gauge should be small. The optimum density of gauges from accurate information about the storms can be considered. According to the **W**orld **M**eteorological **O**rganization (**WMO**) recommends the following densities:

1. In flat regions of temperate, Mediterranean and tropical zones
   الأراضي المنبسطة ، وحوض الأبيض المتوسط والمناطق الاستوائية :
   Ideal — 1 station for 600-900 km^2 المثالية
   Acceptable— 1 station for 900-3000 km^2 المقبولة
2. In mountainous regions of temperate, Mediterranean and topical zones
   الأراضي الجبلية في المناطق المعتدلة وحوض الأبيض المتوسط والمناطق الاستوائية :
   Ideal — 1 station for 100-250 km^2 المثالية
   Acceptable— 1 station for 25-1000 km^2 المقبولة
3. In arid and polar zones: المناطق الصحراوية والقطبية
   1 station for 1500-10,000 km^2 depending on the feasibility. يعتمد على ما يمكن إجراءه

Ten percent of raingauge stations should be self-recording gauges to know the intensities of rainfall.

ADEQUACY OF RAINGAUGE STATIONS

If there are already some raingauge stations in a catchment, the optimal number of stations that should exist to have an assigned percentage of error in the estimation of mean rainfall is obtained by statistical analysis as:

$$N = \left(\frac{C_v}{\epsilon}\right)^2 \quad \text{------------------------------------} \quad (2.1)$$

Where N = optimal number of stations,
ϵ = Allowable degree of error in the estimate of the mean rainfall.
C_v = coefficient of variation of the rainfall at the existing m stations (percent). If there are m stations in the catchment, each recording rainfall P1, P2, ... Pm , in a known time, the coefficient of variation C_v is calculated as:

$$C_v = \frac{\sigma_{m-1}}{\bar{P}} \times 100 \quad \text{------------------------------------} \quad (2.2)$$

In which σ_{m-1} is a standard deviation ;

$$\sigma_{m-1} = \sqrt{\frac{\sum_i^m (P_i - \bar{P})^2}{m-1}} \quad \text{------------------------------------} \quad (2.3)$$

P_i = Precipitation magnitude in the i^{th} station.
$\bar{P}$ = Mean precipitation

$$\bar{P} = \frac{\sum_i^m P_i}{m} \quad \text{------------------------------------} \quad (2.4)$$

In calculating N from Eq. (2.1) , it is usual to take ϵ = 10% . It is seen that if the value of ϵ is small, the number of raingauge stations will be more. According to **WMO** recommendations, at least 10% of the total raingauges should
be of self-recording type.

Example 2.1 : A catchment has six raingauge stations. In a year, the annual rainfall recorded by the gauges are as follows:

Station	A	B	C	D	E	F
Rainfall (cm)	82.6	102.9	180.3	110.3	98.8	136.7

For a 10% error in the estimation of the mean rainfall, calculate the optimum number of stations in the catchment.

Solution:

m = 6 $\bar{P}$ = 118.6 σ_{m-1} = 35.04 ϵ = 10%

$$C_v = \frac{35.04}{118.6} \times 100 = \mathbf{29.54}$$

$$N = \left(\frac{29.54}{10}\right)^2 = 8.7 \quad \text{.........} \quad \text{say 9 stations}$$

PREPARATION OF DATA

Before using the rainfall records of a station, it is necessary to first check the data for continuity and consistency. The continuity of a record may be broken with missing data due to many reasons such as damage or fault in a raingauge during a period. The missing data can be estimated by using the data of the neighbouring stations. In these calculations the normal rainfall is used as a standard of comparison. The normal rainfall is the average value of rainfall at a particular date, month or year over a specified 30-year period. The 30-year normals are recomputed every decade. Thus the term normal annual precipitation at station "A" means the average annual precipitation at "A" based on a specified 30-years of records.

Estimation of Missing Data

Given the annual precipitation values P1, P2, P3, ... Pm at neighbouring M stations 1,2, 3, ... m respectively, it is required to find the missing annual precipitation P_x at a station X not included in the above m stations. Further, the normal annual precipitations N1, N2, ……N at each of the above (m + 1) stations even station X are known.

- If the normal annual precipitations at various stations are within about 10% of the normal annual precipitation at station X , then the simple arithmetic average is used to estimate Px , if each magnitude $\left|\frac{Ni-Nx}{Nx}\right|$ *is less than* 0.1 *for* $i = 1,2,3, \ldots . m$

$$P_x = \frac{1}{M}[\mathrm{P1} + \mathrm{P2} + \cdots + \mathrm{Pm}] \quad \text{--(2.5)}$$

- If the normal precipitations vary considerably, then Px is estimated by weighing the precipitation at the various stations by the ratios of normal annual precipitations. This method , known as the normal ratio or Kohler method, and gives Px as;

$$P_x = \frac{N_x}{m}\left[\frac{\mathrm{P1}}{\mathrm{N1}} + \frac{\mathrm{P2}}{\mathrm{N2}} + \cdots + \frac{\mathrm{Pm}}{\mathrm{Nm}}\right] \quad \text{---(2.6)}$$

Example 2.2: The normal annual rainfall at stations A, B, C, and D in a basin are 80.97, 67.59, 76.28 and 92.01 cm respectively. In the year 1975, the station D was inoperative, and the stations A, B and C recorded annual precipitations of 91.11, 72.23 and 79.89 cm respectively. Estimate the rainfall at station D in that year.

Solution: As the normal rainfall values vary more than 10%, the normal ratio method is adopted using Eq. (2.7) as:

$$P_D = \frac{92.01}{3}\left[\frac{91.11}{80.97} + \frac{72.23}{67.59} + \frac{79.89}{76.28}\right] = 99.41\ cm$$

TEST OF CONSISTENCY OF RECORDS اختبار تجانس (اتساق) البيانات

If the conditions relevant to the recording of a raingauge station have undergone a significant change during the period of record, inconsistency would arise in the rainfall data of that station. This inconsistency would be felt from the time the significant change took place. Some of the common causes for inconsistency of record are :

(i) shifting of a raingauge station to a new location,
(ii) the neighbourhood of the station undergoing a marked change,
(iii) change in the ecosystem due to calamities, such as forest fires, landslides,
(iv) occurrence of observational error from a certain date.

The checking for inconsistency of a record is done by the *double-mass curve technique.* This technique is based on the principle that when each recorded data comes from the same parent population, they are consistent.

A group of 5 to 10 base stations in the neighbourhood of the problem station X is selected. The data of the annual (or monthly mean) rainfall of the station X and also the average rainfall of the group of base stations covering a long period is arranged in the reverse chronological order (i.e. the latest record as the first entry and the oldest record as the last entry in the list). The accumulated precipitation of the station X (i.e. ΣPx) and the accumulated values of the average of the group of base stations (i.e. ΣPav) are calculated starting from the latest record. Values of ΣPx are plotted against ΣPav for various consecutive time periods (Fig. 2.6). A break in the slope of the resulting plot indicates a change in the precipitation regime of station X. The precipitation values at station X beyond the period of change of regime (point 63 in Fig. 2.6) is corrected by using the relation:

$$p_{cx} = p_x \frac{Mc}{Ma}$$

Where;

P_{cx} = corrected precipitation at any time period t_1 at station x.
P_x = originated recorded precipitation at any time period t_1 at station x.
Mc = corrected slope double-mass curve.
Ma = originated slope double-mass curve.

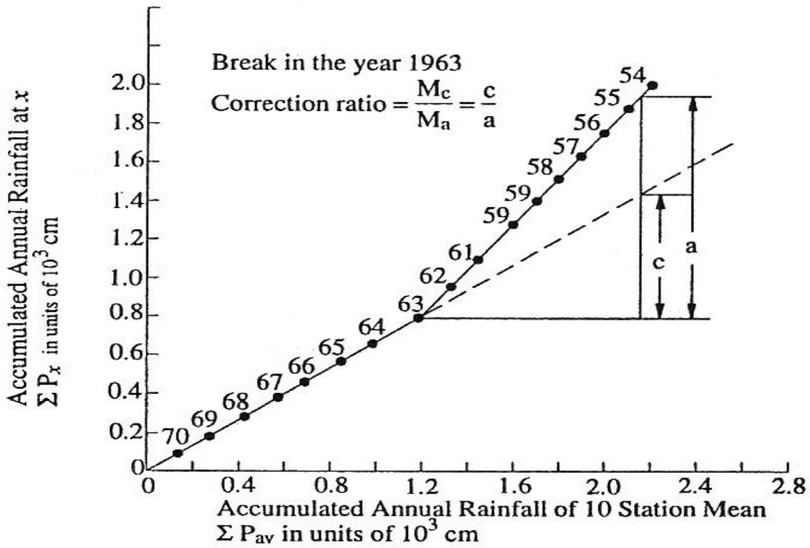


Fig. 2.6 : Double-mass curve

In this way the older records are brought up to the new regime of the station. It is apparent that the more homogenous the base station records are the more accurate will be the corrected values at station x. A change in the slope is normally taken as significant only where it persists for more than five years. The double- mass curve is also helpful in checking arithmetical errors in transferring rainfall data from one record to another.

<u>Example 2.3</u>: *Annual rainfall data for station* **M** *as well as the average annual rainfall values for a group of eight neighbouring stations located in a meteorologically homogeneous region are given below.*

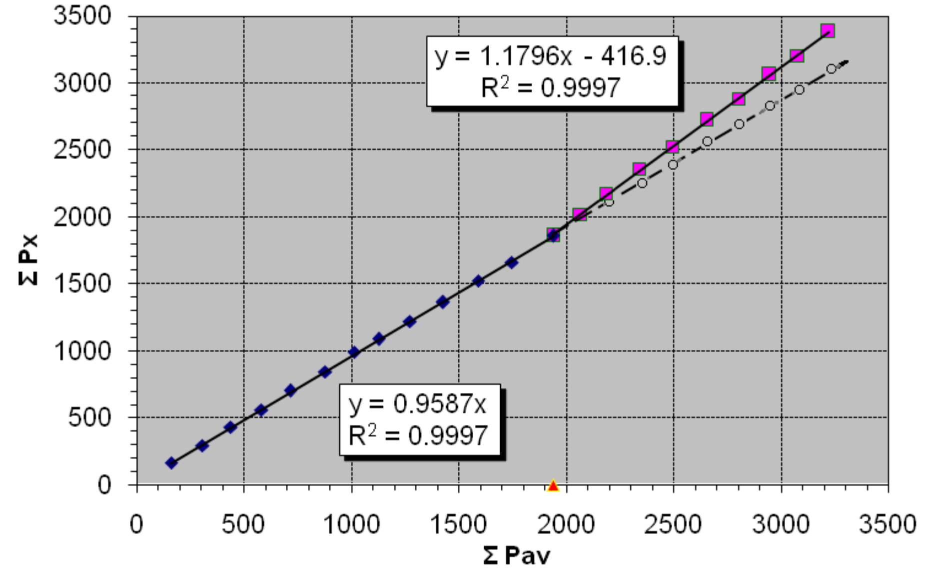
Year	*Annual Rainfall of Station* **M** *(mm)*	*Average Annual Rainfall of the group (mm)*	*Year*	*Annual Rainfall of Station* **M** *(mm)*	*Average Annual Rainfall of the group (mm)*
1946	*177*	*143*	*1957*	*158*	*164*
1947	*144*	*132*	*1958*	*145*	*155*
1948	*178*	*146*	*1959*	*132*	*143*
1949	*162*	*147*	*1960*	*95*	*115*
1950	*194*	*161*	*1961*	*148*	*135*
1951	*168*	*155*	*1962*	*142*	*163*
1952	*196*	*152*	*1963*	*140*	*135*
1953	*144*	*117*	*1964*	*130*	*143*
1954	*160*	*128*	*1965*	*137*	*130*
1955	*196*	*193*	*1966*	*130*	*146*
1956	*141*	*156*	*1967*	*163*	*161*

1) *In what year is the change in records of station* **M** *pointed out ?*

2) *Adjust the records and calculate the new mean annual precipitation of station* **M**.

Solution:

1	2	3	4	5	6
Year	P_M	Pav	Σ P_M	Σ Pav	Final P_M
1967	163	161	163	161	163
1966	130	146	293	307	130
1965	137	130	430	437	137
1964	130	143	560	580	130
1963	140	135	700	715	140
1962	142	163	842	878	142
1961	148	135	990	1013	148
1960	95	115	1085	1128	95
1959	132	143	1217	1271	132
1958	145	155	1362	1426	145
1957	158	164	1520	1590	158
1956	141	156	1661	1746	141
1955	196	193	1857	1939	196
1954	160	128	2017	2067	130.04
1953	144	117	2161	2184	117.03
1952	196	152	2357	2336	159.30
1951	168	155	2525	2491	136.54
1950	194	161	2719	2652	157.67
1949	162	147	2881	2799	131.66
1948	178	146	3059	2945	144.67
1947	144	132	3203	3077	117.03
1946	177	143	3380	3220	143.85
			New Mean =	140.672	mm
			Old Mean =	153.636	mm



Data Correction

PRESENTATION OF RAINFALL DATA

A few commonly used methods of presentation of rainfall data which have been found to be useful in interpretation and analysis of such data are given as follows:

Mass Curve of Rainfall

The mass curve of rainfall is a plot of the accumulated precipitation against time, plotted in chronological order (i.e., sorted by time). Records of float type and weighing bucket type gauges are of this form. A typical mass curve of rainfall at a station during a storm is shown in Fig.2.7. Mass curve is very useful to know the information on the duration , magnitude of a storm, and intensities at various time intervals in a storm that can be obtained by the slope of the curve.

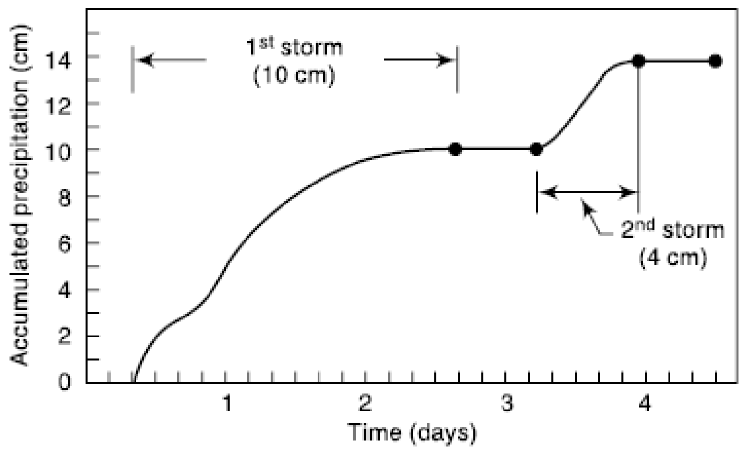


Fig. 2.7: Mass Curve of Rainfall

Hyetograph

A hyetograph is a plot of the intensity of rainfall against the time interval. The hyetograph is derived from the mass curve and is usually represented as a bar chart (Fig. 2.8). It is a very important in the development of design storms to predict extreme floods. Area under the hyetograph represents the total precipitation received in the period. The time interval used depends on the purpose, in urban-drainage problems small durations are used, while in flood-flow computations in larger catchments, the intervals are about 6h.

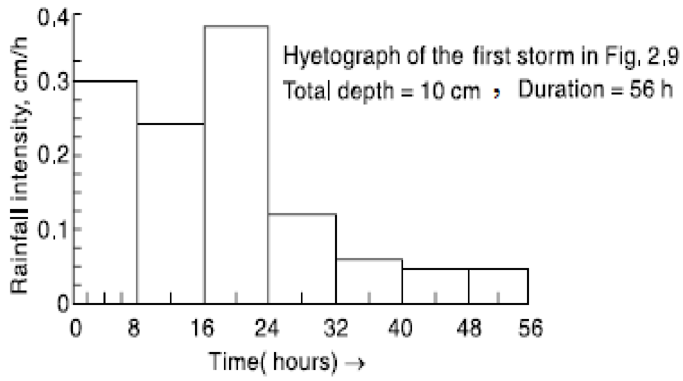


Fig. 2.8 Hyetograph of a Storm

MEAN PRECIPITATION OVER AN AREA

The raingauges represent only point sampling of the areal distribution of a storm. In practice, however, hydrological analysis requires knowledge of the rainfall over a catchment. To convert the point rainfall values at various stations into an average value over a catchment the following three methods are in use:

(i) Arithmetical-mean method,
(ii) Thiessen-polygon method, and
(iii) Isohyetal method.

Arithmetical Mean Method

When the rainfall measured at various stations in a catchment show little variation, the average precipitation over the catchment area is taken as the arithmetic mean of the station values. Thus if $P_1, P_2, \ldots P_i, \ldots P_n$ are the rainfall values in a given period in N stations within a catchment, then the value of the mean precipitation P over the catchment by the arithmetic-mean method is:

$$\bar{P} = \frac{P1 + P2 + \cdots + Pn}{N} = \frac{1}{N}\sum_{i}^{N} P_i$$

In practice, this method is used very rarely.

Thiessen Mean Method

In this method the rainfall recorded at each station is given a weightage on the basis of an area closest to the station. The procedure of determining the weighing area is as follows:

Consider a catchment area as in Fig. 2.9 containing three raingauge stations. There are three stations outside the catchment but in its neighbourhood. The catchment area is drawn to scale and the positions of the six stations marked on it. Stations 1 to 6 are joined to form a network of triangles. Perpendicular bisectors for each of the sides of the triangle are drawn.

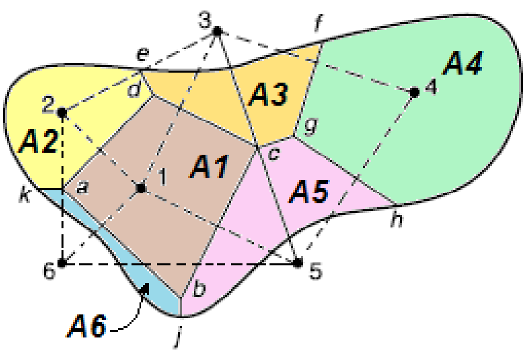


Fig. 2.9: Thiessen Polygons

Station	Bounded by	Area	Weighted
1	abcd	A1	A1/A
2	kade	A2	A2/A
3	edcgf	A3	A3/A
4	fgh	A4	A4/A
5	hgcbj	A5	A5/A
6	jbak	A6	A6/A
A = total catchment area			

These bisectors form a polygon around each station. The boundary of the catchment, if it cuts the bisectors is taken as the outer limit of the polygon. Thus for station 1, the bounding polygon is abcd. For station 2, kade is taken as the bounding polygon. These bounding polygons are called Thiessen polygons. The areas of these six Thiessen polygons are determined either with a planimeter or by using an overlay grid. Thus in general for N stations:

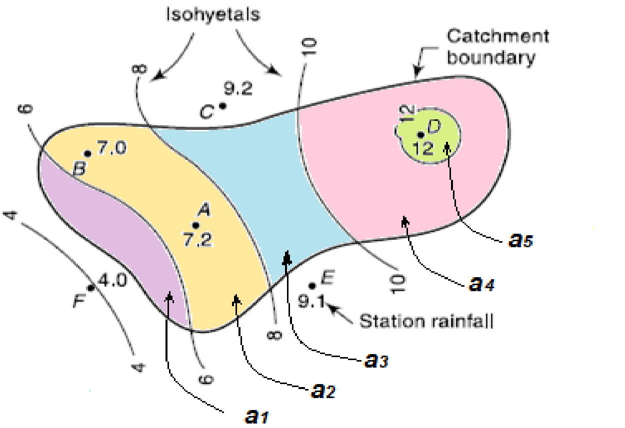
$$\bar{P} = \frac{P1\,A1 + P2\,A2 + P3\,A3 + P4\,A4 + P5A5 + P6A6}{A1 + A2 + A3 + A4 + A5 + A6}$$

Or ; $\bar{P} = \frac{\sum_{i=1}^{N} PiAi}{A}$ = $\sum_{i=1}^{N} pi\,\frac{Ai}{A}$

Thiessen polygons method is superior to the arithmetic-mean method. The raingauge stations outside the catchment are also used effectively, and its weightage factors are considered, whereas the arithmetic-mean method does not involve the stations outside the catchment area.

Isohyetal Method

An isohyet is a line joining points of equal rainfall magnitude. In the isohyetal method, the catchment area is drawn to scale and the raingauge stations are marked. The recorded values for which areal average P is to be determined are then marked on the plot at appropriate stations. Neighbouring stations outside the catchment are also considered. The isohyets of various values are then drawn by considering point rain- falls as guides and interpolating between them by the eye (Fig. 2.10).



$$\bar{P} = \frac{a_1\left(\frac{4+6}{2}\right) + a_2\left(\frac{6+8}{2}\right) + \ldots + a_4\left(\frac{10+12}{2}\right) + a_5\,(12)}{a_1 + a_2 + \ldots + a_4 + a_5}$$

Fig. 2.10: Isohyetals of a Storm

The procedure is similar to the drawing of elevation contours based on spot levels. The area between two adjacent isohyets are then determined with a planimeter. If the isohyets go out of catchment, the catchment boundary is used as the bounding line. The average value of the rainfall indicated by two isohyets is assumed to be acting over the inter-isohyet area. Thus $P_1, P_2, \ldots, P_n$ are the values of isohyets and if $a_1, a_2, \ldots, a_{n-1}$ are the inter-isohyet areas respectively, then the mean precipitation over the catchment of area A is given by;

$$\bar{P} = \frac{a_1\left(\frac{P_1 + P_2}{2}\right) + a_2\left(\frac{P_2 + P_3}{2}\right) + \ldots + a_{n-1}\left(\frac{P_{n-1} + P_n}{2}\right)}{A}$$

The isohyet method is superior to the other two methods especially when the stations are large in number.

DEPTH-AREA-DURATION RELATIONSHIPS

Some aspects of the interdependency of depth, area and duration of storms are discussed below.

Depth-Area Relation

For a rainfall of a given duration, the average depth decreases with the area in an exponential mode given by:

$$P = P_o \exp(-kA^n)$$

Where; P = average depth in cm over an area A km^2,

P_0 = highest amount of rainfall in cm at the storm centre.

K and **n** are constants for a given region. **K** is ranged from 0.0008526 to 0.001745 and **n** is ranged from 0.6614 to 0.5961 related to storm duration ranged from 1day to 3 days. This equation is useful in extrapolating an existing storm data over an area. It is often that the storm centre does not coincide over a raingauge station, the exact determination of P_o is not possible. Hence the highest station rainfall is taken as the average depth over an area of 25 km^2.

Maximum Depth-Area-Duration Curves

It is necessary to have information on the maximum amount of rainfall of various durations occurring over various sizes of areas. The development of relationship, between maximum depth area-duration for a region is known as **DAD** analysis and forms an important aspect of hydro-meteorological study. Isohyetal maps and mass curves of the storm are compiled. A depth-area curve of a given duration of the storm is prepared. Then from a study of the mass curve of rainfall, various durations and the maximum depth of rainfall in these durations are noted. The maximum depth-area curve for a given duration **D** is prepared by assuming the area distribution of rainfall for smaller duration to be similar to the total storm. The procedure is then repeated for different storms and the envelope curve of maximum depth-area for duration **D** is obtained. A similar procedure for various values of **D** results in a family of envelope curves of maximum depth vs. area, with duration as the third parameter. These curves are called **DAD** curves , Fig. 2.11. Figure 2.16 shows typical **DAD** curves for a catchment. It may be seen that the maximum depth for a given storm decreases with the area; for a given area the maximum depth increases with the duration.

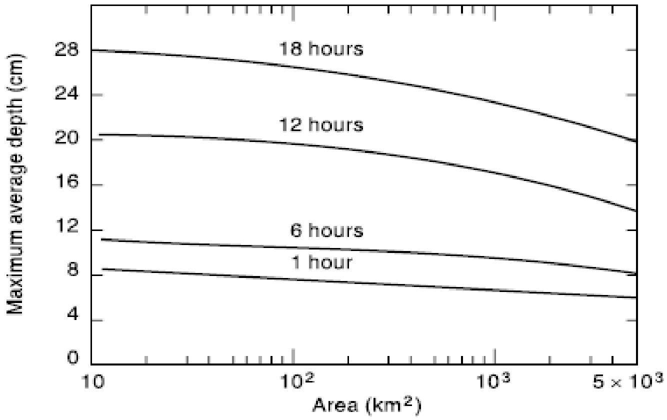


Fig. 2.11: Typical DAD Curves

Preparation of DAD curves involves considerable computational effort and requires meteorological and topographical information of the region. Detailed data on severemost storms in the past are needed. DAD curves are essential to develop design storms for use in computing the design flood in the hydrological design of major structures such as dams.

FREQUENCY OF POINT RAINFALL

In many hydraulic-engineering applications such as those concerned with floods, the probability of occurrence of a particular extreme rainfall, e.g., 24-h maximum rainfall, will be of importance. The probability of occurrence of an event of a random variable ,such as rainfall whose magnitude is equal to or in excess of a specified magnitude such as X is denoted by P .The recurrence interval (also known as return period) is defined as $T = 1/P$ .This represents the average interval between the occurrence of a rainfall of magnitude equal to or greater than X. Thus if it is stated that the return period of rainfall of 20 cm in 24 h is 10 years at a certain station A, it implies that on average rainfall magnitudes equal to or greater than 20 cm in 24 h occur once in 10 years. However, it does not mean that every 10 years one such event is likely, i.e. periodicity is not implied. The probability of a rainfall of 20 cm in 24 h occurring in anyone year at station A is $1/T = 1/10 = 0.1$.

If the probability of an event occurring is P the probability of the event not occurring in a given year is $q = (1 - P)$. The binomial distribution can be used to find the probability of occurrence of the event *r* times in *n* successive years. Thus;

$$p_{r,n} = q^n = [n!/(n-r)!r!] \times P^r q^{n-r}$$

where $p_{r,n}$ = probability of a random hydrologic event (rainfall) of given magnitude and exceedence probability P occurring *r* times in *n* successive years. Thus, for example:

(a) The probability of an event of exceedence probability P occurring 2 times in *n* successive years is;

$p_{2,n} = q^n = [n!/(n-2)!2!] \times P^2 q^{n-2}$

(b) The probability of the event not occurring at all in n successive years is:

$p_{o,n} = q^n = (1 - P)^n$

(c) The probability of the event occurring at least once in *n* successive years is:

$p_1 = 1 - q^n = 1 - (1 - P)^n$

Example 2.4: Analysis of data on maximum one-day rainfall depth , indicated that a depth of 280 mm had a return period of 50 years. Determine the probability of a one-day rainfall depth equal to or greater than 280 mm occurring **(a)** once in 20 successive years, **(b)** two times in 15 successive years, **(c)** at least once in 20 successive years, and **(d)** not occurring in 20 successive years .

Solution :

Here P = 1/50=0.02

(a) *n* = 20, *r* = 1

$P_{1,20}$ = 20!/19! x 0.02 x $(0.98)^{19}$ = 20 x 0.02 x 0.68123 = 0.272

(b) *n* = 15, *r* = 2

$P_{2,15}$ = 15!/(13!x2!) x $(0.02)^2$ x $(0.98)^{13}$ = 15 x 14/2 x 0.0004 x 0.769 = 0.323

(c) at least once in 20 successive years.

$P_1 = 1 - q^n = 1 - (1 - P)^n$

P_1= 1 -(l - 0.02)^{20} = 0.332

(d) not occurring in n=20 ,

$P_{0,20}$= (l - 0.02)^{20}=0.667

PLOTTING POSITION

The purpose of the frequency analysis of an annual series is to obtain a relation between the magnitude of the event and its probability of exceedence. The probability analysis may be made either by empirical or by analytical methods. A simple empirical technique is to arrange the given annual extreme series in descending order of magnitude and to assign an order number *m.* Thus for the first entry *m* = 1, for the second entry *m* = 2, and so on, till the last event for which *m* = *N* =Number of years of record. The probability **P** of an event equals to or exceeds, is given by the *Weibull formula:* $\mathbf{P = m / (N + 1)}$, and the recurrence interval $\mathbf{T = 1/P = (N + 1)/m}$.

The Weibull formula is an empirical equation, while there are several other such empirical formulae available to calculate **P** *shown in table below.* The exceedence probability of the event obtained by using an empirical formula, is called plotting position . Weibull Equation is the most popular plotting position formula in hydrology, and hence only this formula is considered herein.

Method	P
California	m / N
Hazen	$(m - 0.5) / N$
Weibull	$m /(N + 1)$
Chegodayev	$(m - 0.3) / (N + 0.4)$
Blom	$(m - 0.44) / (N + 0.12)$
Gringorten	$(m - 3/8) / (N + 1/4)$

Having calculated **P** (and hence T) for all the events in the series, the variation of the rainfall magnitude is plotted against the corresponding T on a semi-log paper or log-log paper. By suitable extrapolation of this plot, the rainfall magnitude of specific duration for any recurrence interval can be estimated. For accurate work, various analytical methods calculation using frequency factors are available. Gumbel's extreme value distribution and Log Pearson Type III method are two commonly used analytical methods and are described later.

Example 2.5 : The record of annual rainfall at Station A covering a period of 22 years is given below:

Year	*Annual Rainfall (mm)*	*year*	*Annual Rainfall (mm)*
1960	130.0	1971	90.0
1961	84.0	1972	102.0
1962	76.0	1973	108.0
1963	89.0	1974	60.0
1964	112.0	1975	75.0
1965	96.0	1976	120.0
1966	80.0	1977	160.0
1967	125.0	1978	85.0
1968	143.0	1979	106.0
1969	89.0	1980	83.0
1970	78.0	1981	95.0

(a) Estimate the annual rainfall with return periods of 10 years and 40 years.
(b) What would be the probability of an annual rainfall of magnitude equal to or exceeding 100 cm occurring at Station A?
(c) What is the 75% dependable annual rainfall at station A?

Solution:
The data are arranged in descending order and the rank number assigned to the recorded events. The probability P of the event being equaled to or exceeded is calculated by using Weibull formula. Calculations are shown in the following Table. It may be noted that when two or more events have the same magnitude (as for m = 13

and 14 in the table) the probability P is calculated for the largest m value of the set. The return period T is calculated as T = 1 /P.

N = 22 years Table of Calculation of Return Periods

m	***Annual Rainfall (mm)***	***Probability = m / (N + 1)***	***Return Period T = 1/P (years)***	***m***	***Annual Rainfall (mm)***	***Probability = m / (N + 1)***	***Return Period T = 1/P (years)***
1	160	0.043	23.00	12	90.0	0.522	1.917
2	143.0	0.087	11.500	13	89.0	0.565	-----
3	130.0	0.130	7.667	14	89.0	0.609	1.643
4	125.0	0.174	5.750	15	85.0	0.652	1.533
5	120.0	0.217	4.600	16	84.0	0.696	1.438
6	112.0	0.261	3.833	17	83.0	0.739	1.353
7	108.0	0.304	3.286	18	80.0	0.783	1.278
8	106.0	0.348	2.875	19	78.0	0.826	1.211
9	102.0	0.391	2.556	20	76.0	0.870	1.150
10	96.0	0.435	2.300	21	75.0	0.913	1.095
11	95.0	0.478	2.091	22	60.0	0.957	1.045

A graph is plotted between the annual rainfall magnitude as the ordinate (on arithmetic scale) and the return period *T* as the abscissa (on logarithmic scale) as shown in figure 2.12 below.

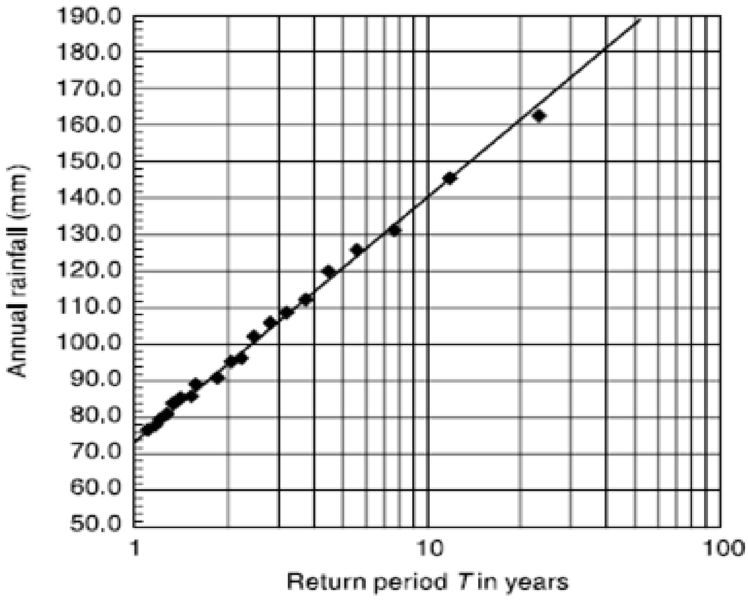


Fig. 2.12: Return Periods of Annual Rainfall at Station A

It can be seen that a straight line could represent the trend of the rest of data.

(**A**) (i) For T = 10 years, the corresponding rainfall magnitude is obtained not by interpolation between two appropriate successive values of the table, but directly from figure, or from equation of line [$d=a+b\ ln(T)$], as 140 mm.(why ?)

(ii) for T = 40 years the corresponding rainfall magnitude, by extrapolation of the best fit straight line, is 180.0 mm.

(**B**) Return period of an annual rainfall of magnitude equal to or exceeding 100 mm, from figure , is 2.4 years. As such the exceedence probability **P** = 1/2.4 = 0.417.

(**C**) 75% dependable annual rainfall at Station *A* = Annual rainfall with probability **P** = 0.75, i.e. T = 1/0.75 = 1.33 years. From figure2.12, the 75% dependable annual rainfall at Station *A* , corresponding to return period T = 1.33 years is 81.8 mm.

MAXIMUM INTENSITY-DURATION–FREQUENCY RELATIONSHIP

The maximum intensity of rainfall of specified return period and of duration equal to the critical time of concentration is of considerable practical importance in evaluating peak flows related to hydraulic structures. Briefly, the procedure to calculate the intensity-duration-frequency relationship for a given station is as follows:

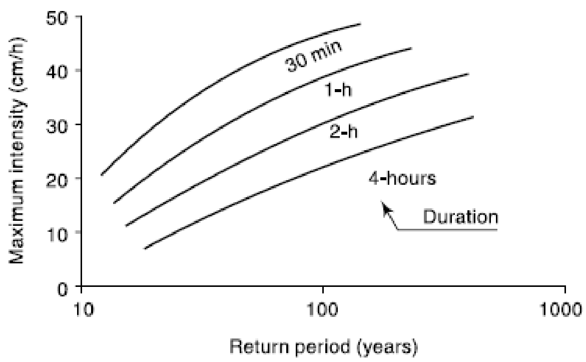
• M numbers of significant and heavy storms in a particular year Y_1 are selected for analysis. Each of these storms are analyzed for maximum intensity-duration relationship by the equation $I_m=c/(t+a)^b$.

• This gives set of maximum intensity I_m as a function of duration for the year Y_1.

• The procedure is repeated for all the N years of record to obtain the maximum Intensity $Im\ (Dj)k$, for all j = 1 to M and k = 1 to N.

• Each record of $Im\ (Dj)k$ for k = 1 to N constitutes a time series which can be analyzed to obtain frequencies of occurrence of various $Im\ (Dj)$ values. Thus there will be M time series generated.

• The results are plotted as maximum intensity (*Im*) vs return period(*T*) with the Duration (*D*) as the third parameter (Fig. 2.13). Alternatively, maximum intensity vs duration with frequency as the third variable can also be adopted (Fig. 2.14).



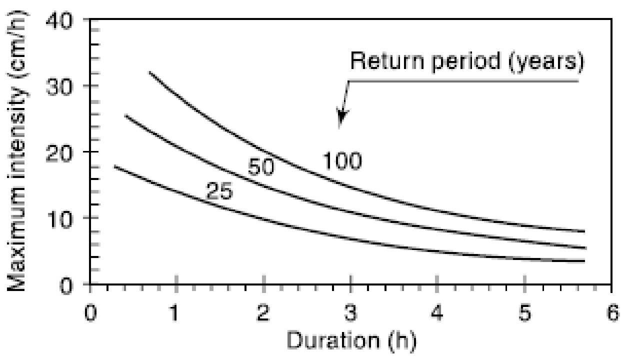


Fig. 2.13 Maximum Intensity-Return Period-Duration Curves

Fig. 2.14 Maximum Intensity-Duration-Frequency Curves

Analytically, these relationships are commonly expressed in a condensed form by general form:

$$i = \frac{KT^x}{(D+a)^n}$$

where i = maximum intensity (cm/h), T = return period (years), D = duration (hours) K, x, a and n are coefficients for the area represented by the station.

Sometimes, instead of maximum intensity, maximum depth is used as a parameter and the results are represented as a plot of maximum depth vs duration with return period as the third variable (Fig. 2.15).

In a similar way of the referring to the intensity – duration - frequency relationship, the depth-duration-frequency relationship deals with maximum depth for a given duration was also considered.

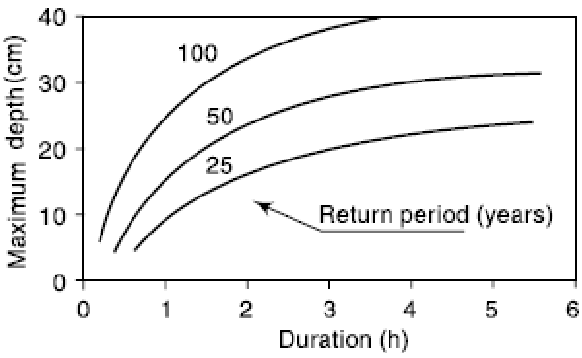


Fig. 2.15 Maximum Depth-Duration-Frequency Curves

Note that Typical values of Coefficients K, x, a an d n in equation above are ranged as; x = 0.1-0.25 K= 3-15 a= 0.15-0.5 n=0.6-1.2.

PROBABLE MAXIMUM PRECIPITATION (PMP)

In the design of major hydraulic structures such as spillways in large dams, the hydrologist and hydraulic engineer would like to keep the failure probability as low as possible, i.e. virtually zero. This is because the failure of such a major structure will cause very heavy damages to life, property, economy and national morale. In the design and analysis of such structures, the maximum possible precipitation that can reasonably be expected at a given location is used. This originate from the recognition that there is a physical upper limit to the amount of precipitation that can fall over a specified area in a given time.

The probable maximum precipitation (PMP) is defined as the greatest or extreme rainfall for a given duration that is physically possible over a station or basin. From the operational point of view, PMP can be defined as that rainfall over a basin which would produce a flood flow with virtually no risk of being exceeded. The development of PMP for a given region is an involved procedure and requires the knowledge of an experienced hydro-meteorologist. Basically two approaches are used:

(i) Meteorological methods and (ii) the statistical study of rainfall data.
The statistical studies indicate that PMP can be estimated as:

$$PMP = P + K\sigma$$

where P = mean of annual maximum rainfall series, σ = standard deviation of the series and K = a frequency factor which depends upon the statistical distribution of the series, number of years of record and the return period.

An equation based upon the rainfall records available all over the world was constructed. World's greatest recorded rainfalls of various duration were plotted on a log-log paper, an enveloping straight line drawn to the plotted points obeys the equation:

$$Pm = 42.16\ D^{0.475}$$

where Pm = extreme rainfall depth in cm and D = duration in hours.
The values obtained from this equation are of use in PMP estimations in case of data loss.