Stress Limitations for Prestress Concrete

AASHTO specifications limit the allowable stresses in prestressed concrete. So, only service loading is used [service limit state]. Also, at any section in prestressed member, there are checks required at two loading stages:

Transfer Stage

It is a temporary stage (before losses) begins when the initial prestress force (P_i) transfers to the concrete. At this stage, the concrete is in initial compressive strength (f'_{ci}), but the stresses are only due to prestress force and self-weight.

The stresses at this stage must be complied to the following:

- $f_i = -\frac{P_i}{A} \mp \frac{P_i.e}{S}$ [concrete stresses due to initial prestress force only]
- $f_i = -\frac{P_i}{A} \mp \frac{P_i.e}{S} \pm \frac{M_D}{S}$ [concrete stresses due to initial prestress force and its self-weight]
- $f_i \leq 0.60 f'_{ci}$ [compressive stress limit for concrete in both prestressing methods]
- $f_i \leq 0.63\sqrt{f'_{ci}}$ [tensile stress limit for concrete with bonded reinforcement]
- $f_i \leq 0.25\sqrt{f'_{ci}} \leq 1.38$ [tensile stress limit for concrete without bonded reinforcement]
- $f_{pi} \leq 0.75 f_{pu}$ [prestressing steel stress for pretensioning method]
  $\leq 0.90 f_{py}$ [prestressing steel stress for post-tensioning method]

where:

f_i: initial stress at top or bottom fiber of concrete (MPa)
A: area of prestressed concrete (mm^2)
e: distance (eccentricity) between the centroids of prestressing tendons and prestressed concrete structure (mm)
S: top or bottom section modulus of concrete (mm^3)
M_D: dead load moment due to self-weight of prestressed concrete structure (N.mm)
f_{pi}: tensile stress of tendon immediately before initial prestress force transfer (MPa)
f_{pu}: specified tensile strength of tendon (MPa)
f_{py}: yield strength of tendon (MPa).

Service Stage

It is the design life stage of the member to carry all service dead and live loads in addition to the prestress force and self-weight. Also, the concrete has its full strength (f'_c), but prestress force is reduced due to losses into effective value (P_e).

So, the stresses in prestressed concrete at this stage must be complied to the following:

- $f = -\frac{P_e}{A} \mp \frac{P_e.e}{S}$ [due to effective prestress force only]
- $f = -\frac{P_e}{A} \mp \frac{P_e.e}{S} \pm \frac{M_S}{S}$ [due to effective prestress force and service loads]
- $f \leq 0.45 f'_c$ [extreme fiber in compression for both prestressing methods]

- $f \leq 0.50\sqrt{f_c'}$ [extreme fibers in tension for area with bonded reinforcement]
- $f_{pe} \leq 0.80 f_{py}$ [prestressing steel stress for both prestressing methods]

where:

f: stress at top or bottom fiber of concrete (MPa)

M_S: service load moment due to self-weight of prestressed structure, superimposed dead and live loads (N.mm)

f_{pe}: effective tensile stress of tendon after losses depletion (MPa)

Ex.: Simply supported beam has a symmetrical section (A) = 114x10^3 mm^2 with total height (h) = 610 mm and moment of inertia (I) = 5x10^9 mm^4. The beam carries uniformly distributed load $(D + L)$ = 8 kN/m in addition to own weight of 2.7 kN/m over a span (L) = 12 m. Also, the eccentricity (e) = 130 mm for initial prestress force (P_i) = 750 kN and effective prestress force (P_e) = 640 kN. Check the permissibility of concrete stresses if initial compressive strength (f_{ci}') = 25 MPa and design compressive strength (f_c') = 35 MPa.

Sol:

Always the checking of concrete stresses must be done at midspan of the structure where the effect of applied loads is maximum and at ends where only the prestress force is effective.

At-release Stage

$$f_{ci} = 0.60 f_{ci}' = 0.6\ x\ 25 = 15\ MPa$$

$$f_{ti} = 0.63\sqrt{f_{ci}'} = 0.63\ x\ \sqrt{25} = 3.15\ MPa$$

At midspan P_i and self-weight loads.

$$M_D = w_D L^2/8 = 2.7\ x\ 12^2/8 = 48.6\ kN.m$$

$$S_{top} = S_{bot} = I/y = 5x10^9/305 = 16.39x10^6\ mm^3$$

$$f_{i,top} = -\frac{P_i}{A} + \frac{P_i.e}{S_{top}} - \frac{M_D}{S_{top}} = -\frac{750x10^3}{114x10^3} + \frac{750x10^3\ x\ 130}{16.39x10^6} - \frac{48.6x10^6}{16.39x10^6}$$

$$= -6.58 + 5.95 - 2.97 = -3.6\ MPa < 15\ MPa \quad \therefore OK$$

$$f_{i,bot} = -\frac{P_i}{A} - \frac{P_i.e}{S_{bot}} + \frac{M_D}{S_{bot}}$$

$$= -6.58 - 5.95 + 2.97 = -9.56\ MPa < 15\ MPa \quad \therefore OK$$

At ends P_i load only.

$$f_{i,top} = -\frac{P_i}{A} + \frac{P_i.e}{S_{top}} = -6.58 + 5.95 = -0.63\ MPa < 15\ MPa \quad \therefore OK$$

$$f_{i,bot} = -\frac{P_i}{A} - \frac{P_i.e}{S_{bot}} = -6.58 - 5.95 = -12.53\ MPa < 15\ MPa \quad \therefore OK$$

In-service Stage

$$f_c = 0.45 f_c' = 0.45\ x\ 35 = 15.75\ MPa$$

$$f_t = 0.50\sqrt{f_c'} = 0.5\ x\ \sqrt{35} = 2.96\ MPa$$

At midspan P_e, self-weight and superimposed loads.

$$M_S = w_S L^2/8 = 10.7\ x\ 12^2/8\ = 192.6\ kN.m$$

$$f_{top} = -\frac{P_e}{A} + \frac{P_e.e}{S_{top}} - \frac{M_S}{S_{top}} = -\frac{640x10^3}{114x10^3} + \frac{640x10^3\ x\ 130}{16.39x10^6} - \frac{192.6x10^6}{16.39x10^6}$$

$$= -5.61 + 5.08 - 11.75 = -12.29\ MPa\ < 15.75\ MPa\ \ \therefore OK$$

$$f_{bot} = -\frac{P_e}{A} - \frac{P_e.e}{S_{bot}} + \frac{M_S}{S_{bot}}$$

$$= -5.61 - 5.08 + 11.75 = 1.06\ MPa < 2.96\ MPa\ \ \therefore OK$$

∴ the stresses in concrete due prestressing force is within permissible limits.

Design Procedure for Prestressed Beams

In bridges, the prestressed beams are designed to support the deck slab, and all act together as a composite section along the lifecycle of the structure. The required prestress force is determined for the composite section within service limit state. Whereas, according to strength limit state, flexural and shear design can be done. Due to composite section, there are interior and exterior T-section shapes. The main design is basically for the interior beams and then exterior beams need to be checked. In general case, the beam dimensions are given because it is a precast.

The main steps of the design procedure can be listed as:

- Determine the composite section properties.
- Determine the service loads.
- Determine the required effective prestress load (P_e) according to given (estimated) eccentricity (e_c) and total losses.
- Calculate the number of stands (N_p) from the maximum initial prestress force (P_i).
- Distribute the strands in symmetry about vertical axis and check its eccentricity (e_c).
- Calculate the accurate value of prestress losses (Δf_{pT}).
- Check concrete stresses at transfer and service stages.
- Check the flexural strength of the composite section [strength limit state].
- Check the provision of minimum reinforcement to resist cracking moment (M_{cr}).
- Design the section for shear stresses.

Eccentricity of Prestressing Steel

At midspan of beam, the typical distance of the strands center (y_{bp}) from the beam bottom fibers is about (5 - 15) % of the beam height (h_g). A value of 5% is appropriate for modern efficient sections like the bulb-tee beams while 15% for is used less efficient AASHTO standard shapes.

- $y_{bp} = (0.05 \sim 0.15)h_g$ [typical value]
- $e_c = y_{bg} - y_{bp}$