## CHAPTER 4

### **Design of R C Beams**

#### **Learning Objectives**

- Identify the data, formulae and procedures for design of R C beams
- Design simply-supported and continuous R C beams by integrating the following processes
  - o determining design loads
  - o determining design forces by force coefficients
  - o determining reinforcement for bending and shear
  - o checking deflection by span-to-depth ratio

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### 4.1 Design Data, Formulae and Procedures

A beam is generally referred to a horizontal element designed to take up gravitational load, and, although it may be designed as an integral of a structural frame to resist lateral load, the predominant design forces for a beam are bending moment, M, and shear force, V.<sup>1</sup>

The design data, formulae and rules that you have learnt in Chapters 1 to 3, which are summarized in the "Annex - R C Design Formulae and Data", will be adopted in this Chapter. You will learn how to integrate and apply them to carry out the whole process of design calculations for a beam.

The basic steps in designing of R C beams are:

- I. Determine the design loads
- II. Determine the design forces
- III. Determine the reinforcement
- IV. Check deflection
- V. Detailing

In the each of the above step, we have to identify the necessary data or design parameters for inputting into appropriate formulae to get the required results and then to check for compliance with the design code. Some data may have to be assumed first and then to be verified when the result is available. The whole process always requires several iterations. For example, the effective depth of the section has to be assumed to calculate the steel area required, and if the steel area required



<sup>&</sup>lt;sup>1</sup> Beam may also be subjected to torsion and axial load. They are beyond the scope in this chapter. Refer to the design code for details.

is then found to be so large that it has exceeded the maximum limit as specified in the design code, the effective depth will has to be increased and the calculations are then re-iterated. Experience can help to make more realistic initial assumptions to reduce the number of iterations in the design process.

#### 4.1.1 Design Forces

In general, the following data are required to determine the design forces:

- (a) Design Loads<sup>2</sup>
  - Dead Load (G<sub>k</sub>, g<sub>k</sub>)
    - Principle dimensions and density of (i) the structural elements, and (ii) finishes, wall, etc. that are *permanent* in nature
  - Imposed Load (Q<sub>k</sub>, q<sub>k</sub>)
    - Load arises from the (i) usage of the floor and (ii) partition, heavy furniture or equipment, etc. that are *transient* in nature
  - Partial Factors of Safety for Load
- (b) Span
  - Center-to-center span
  - Support width, S<sub>w</sub>, and overall depth of the beam, h
  - Effective span, L
- (c) Force Coefficients

Under most circumstances, force coefficients from design code or design manuals can be used to obtain the design moments and shears without undergoing detailed structural analysis. If the configurations of the beam and/or the loading patterns do not meet the requirements of using these force coefficients, simplified sub-frame can be used for structural analysis.<sup>3</sup>

<sup>&</sup>lt;sup>2</sup> For the purposes of this course, we focus our discussion on gravitational loads only.

<sup>&</sup>lt;sup>3</sup> Beam may also be part of the structural frame to resist lateral load. Design forces, i.e. moment and shear, obtained from the lateral analysis have to be considered together with that due to gravitation loads. In order to ensure robustness of the whole structure in resisting lateral load, additional ductility requirements are imposed. It is beyond the scope of this chapter. Refer to the design code or other text for details.

#### 4.1.2 Force Coefficients

For *simply-supported* beam under uniformly distributed load (udl), the force coefficients are:

| Mid-span Moment, M =  | 0.125 F L | or | 0.125 w L <sup>2</sup> |
|-----------------------|-----------|----|------------------------|
| Shear at Support, V = | 0.5 F     | or | 0.5 w L                |

For *continuous beams* with *approximately equal spans* under udl, the following force coefficients are provided by HKCP-2013:

|        | At outer<br>support | Near middle<br>of end span | At first<br>interior<br>support | At middle of interior span | At interior<br>supports |
|--------|---------------------|----------------------------|---------------------------------|----------------------------|-------------------------|
| Moment | 0                   | 0.09FL                     | -0.11FL                         | 0.07FL                     | -0.08FL                 |
| Shear  | 0.45F               | -                          | 0.6F                            | -                          | 0.55F                   |

Notes:

1. No redistribution of the moments calculated from this table should be made.

2. Characteristic imposed load may not exceed the characteristic dead load.

3. Load should be substantially uniformly distributed over three or more spans.

4. Variation in span length should not exceed 15% of the longest.

# Table 4.1 – Force Coefficients for Continuous Beamswith Approximately Equal Span under udl

(Table 6.1 of HKCP-2013)

The values in the above table can be presented in the form of moment and shear force diagrams as shown below.



#### Figure 4.1 – Moment and Shear Force Diagrams for Continuous Beams with Approximately Equal Span under udl

In the above table, the design moment at the outer support is zero, i.e. simply supported. However, reinforced concrete beams are usually constructed monolithically with the support. In order to avoid unsightly cracks due to the moment arising from partial fixity at the support, a minimum design bending moment of at least 15% of the span moment is recommended by CI.9.2.1.5 of HKCP-2013.

#### 4.1.3 Design for Moment and Shear

In general, the procedures to design for *moment*, M, are:

- 1. Identify the effective sectional dimensions and design parameters.
- 2. Calculate the **K** value and check if K < K'.
- 3. Calculate the lever arm **z** and check its limits.
- 4. Calculate the amount of steel required, As and/or A's.
- 5. Determine the size and number of bars.
- 6. Check if the limits to steel area ratio are complied with.

In general, the procedures to design for *shear*, V, are:

- 1. Identify the effective sectional dimensions and design parameters.
- 2. Calculate the maximum shear at the faces of the supports,  $V_s$ .
- 3. Check if the shear stress  $v_s$  exceeds the maximum allowable shear stress, i.e.  $0.8\sqrt{f_{cu}}$ .

If  $v_s < 0.8 \sqrt{f_{cu}}$ , proceed to next step

If  $v_s > 0.8 \sqrt{f_{cu}}$ , increase the size of the section

- 4. Calculate the shear forces at d from the faces of the supports,  $V_d$ .
- 5. Calculate v<sub>c</sub>.
- 6. Check if the shear stress  $v_d$  exceeds  $v_c + 0.4$ .

If  $v_d < v_c + 0.4$ , provide nominal link.

If  $v_d > v_c + 0.4$ , provide shear link and determine the extent.

- 7. Determine the size and spacing of the links.
- 8. Check if the limits to spacing of links are complied with.

#### 4.1.4 Deflection Check by Span-to-depth Ratio

In general, the procedures to check deflection by span-to-depth ratio are:

- 1. Determine the basic L/d ratio.
- 2. Determine the modification factors,  $m_l$ ,  $m_t \& m_c$ .
- 3. Determine the allowable L/d ratio.
- 4. Check if the actual L/d ratio exceeds the allowable or not.

### 4.2 Simply-supported Beams

The whole design process of a simply-supported beam is illustrated by three examples in this section. They show you how to integrate what you have learnt in the previous chapters. The basic procedures of the design are:

- 1. Determine the effective span, L, a<sub>1</sub> & a<sub>2</sub>
- 2. Determine the design load and forces, M & V
- 3. Determine the effective dimensions, d, b<sub>eff</sub>, b<sub>w</sub>, b<sub>v</sub>.
- 4. Design the bars for mid-span moment
- 5. Design for the shear at supports
- 6. Check deflection by span-to-depth ratio

The example in 4.2.1 demonstrates the basic process to design a simple rectangular beam.

Two examples in 4.2.2 provide a more realistic illustration on the whole process of design. A beam from the framing plan in DWG-01 is used as a demonstration. It is a flanged section. Calculations of loading and determination of effective flange width are included in the design process. The beam is then re-designed with some changes: the beam depth is reduced and additional load is imposed. You can appreciate the implications of these changes on the design. The reinforcement details of the beams are presented as DWG-02 at the end of this Chapter.

#### **4.2.1 Example** – Simply-supported Rectangular Beam



#### Question

A rectangular beam simply supported at both ends as shown below. Design the reinforcement and check if the deflection is acceptable or not.



#### **Design parameters**

| Beam overall depth, h =          | 750 mm                  |
|----------------------------------|-------------------------|
| Beam breadth, b =                | 300 mm                  |
| c/c distance btw supports =      | 9 050 mm                |
| Width of LHS support, $S_{w1}$ = | 400 mm                  |
| Width of RHS support, $S_{w2}$ = | 850 mm                  |
| $f_{cu} =$                       | 35 MPa                  |
| $f_y =$                          | 500 MPa                 |
| $f_{yv} =$                       | 250 MPa                 |
| Cover =                          | 35 mm                   |
| Preferred bar size =             | 40                      |
| Preferred link size =            | 10                      |
| Design Load (udl), w =           | 60 0kN/m (S/W included) |

### Solution

#### Effective Span



#### **Design Forces**

(In this question, it is not necessary to determine the design load as it is given.)

Design Load, w = 60.0 kN/m (given) Design Moment, M =  $0.125 \text{ w L}^2$ 

| :                                       | $= 0.125 \times 60.0 \times 9^2$                     |                     |
|---|--|---------------------|
| :                                       | = <u>607.5 kN-m</u>                                  |                     |
| Design Shear, V                         | = 0.5 w L  |                     |
| :                                       | = 0.5 x 60 x 9                                       |                     |
|   | = <u>270.0 kN</u>                                    |                     |
|   |  |                     |
| Effective Depth                         |  |                     |
| d                                       | = 750 - 35 - 10 - 40/2                               |                     |
| :                                       | = <u>685 mm</u>                                      |                     |
|   |  |                     |
| Design for Bending Moment               |  |                     |
| K                                       | = $M / (bd^2 f_{cu})$                                |                     |
|   | $= 607.5 \times 10^6 / (300 \times 685^2 \times 35)$ |                     |
|   | = 0.123  |                     |
| $\beta_b = 1.0$                         | < 0.156  | (Singly reinforced) |
|   |  |                     |
| Lever arm, z                            | = $[0.5 + (0.25 - K/0.9)^{0.5}] d$                   |                     |
| :                                       | $= [0.5 + (0.25 - 0.123/0.9)^{0.5}] \times 685$      |                     |
| :                                       | = 0.837 x 685  |                     |
|   | = 573 mm   |                     |
|   |  |                     |
| Tension steel req'd, A <sub>s,req</sub> | = M / (0.87 $f_y z$ )                                |                     |
| :                                       | $= 607.5 \times 10^6 / (0.87 \times 500 \times 573)$ |                     |
| :                                       | $= 2437 \text{ mm}^2$                                |                     |
|   |  | (Provide 2T40)      |
| A <sub>s,pro</sub>                      | = 2 x 1257   |                     |
|   | $= 2514 \text{ mm}^2$                                |                     |
| 100A <sub>s</sub> / bh                  | = 100 x 2514 / (300 x 750) = 1.117                   |                     |
| :                                       | > 0.13 and < 4.0                                     | (Steel ratio ok)    |
|   |  |                     |

#### Design for Shear at Support

Max shear at the face of LHS support

$$V_{s} = V - w a_{1}$$

$$= 270 - 60 \times 200 / 10^{3}$$

$$= 258 \text{ kN}$$

$$v_{max} = v_{s} = V / (b_{v}d)$$

$$= 258 \times 10^{3} / (300 \times 685)$$

$$= 1.26 \text{ MPa}$$

< 0.8 √ 35 = 4.73 MPa

(Concrete does not crush)

Shear at d from the face of LHS support

$$V_{d} = 258 - w d$$
  
= 258 - 60 x 685 / 10<sup>3</sup>  
= 217 kN  
$$v_{d} = 217 x 10^{3} / (300 x 685)$$
  
= 1.06 MPa

#### Calculate the design concrete shear stress, $v_c$ :

1.22 < 3

|                          | $100A_{s}/(b_{v}d) =$  | 100 x 2514 / (300 x 685) = 1.22 < 3                                  |
|--------------------------|------------------------|--|
|                          | $(400/d)^{1/4} =$      | (400 / 685) <sup>1/4</sup> < 1 (use 1.0)                             |
| As pro instead of As reg | <i>v<sub>c</sub></i> = | 0.79 x (1.22) <sup>1/3</sup> x (1.0) / 1.25 x (35/25) <sup>1/3</sup> |
| is used for $A_s$ in     | =                      | 0.675 x 1.12   |
| determining $v_c$        | =                      | 0.76 MPa   |

The tables/equations quoted in the calculations are referring to HKCP-2013.

(Table 6.3)

Nominal Links

 $A_{sv} / s_v = 0.4 \times b_v / (0.87 f_{yv})$  $= 0.4 \times 300 / (0.87 \times 250)$ = 0.552 Max  $s_v < 0.75 x d$ = 0.75 x 685 = 514 mm

 $v_c$  + 0.4 = 0.76 + 0.4 = 1.16 MPa

> 1.06 MPa

(Provide R10 - 275 - 2/legs)

(Provide nominal links)

 $A_{sv} / s_{v, pro} = 0.571 > 0.552$ 

#### Check Deflection by Span-to-depth Ratio

| by Span-to-de             | pth Ratio   |             |
|---------------------------|---|-------------|
| Basic L /d =              | 20 (Simply-supported Rectangular Beam)              | (Table 7.3) |
|                           |   |             |
| $M/(bd^2) =$              | 607.5 x 10 <sup>6</sup> / (300 x 685 <sup>2</sup> ) |             |
| =                         | 4.32 N/mm <sup>2</sup>                              |             |
| $A_{s,req} / A_{s,pro} =$ | 2437 / 2514 = 0.969                                 |             |
| $f_s =$                   | 2/3 x 500 x 0.969 = 323 MPa                         |             |
| $m_t =$                   | 0.55 + (477-323)/[120(0.9+4.32)]                    | (Table 7.4) |
| =                         | 0.55 + 0.246  |             |
| =                         | 0.796   |             |

Allowable L / d =  $0.796 \times 20 = 15.92$ Actual L / d = 9000 / 685=  $13.14 \le 15.92$ 

(Deflection ok)

### 4.2.2 Example – Simply-supported Flanged Beam



#### **Question A**

Design the R C beam, 5B2, shown on the framing plan in DWG-01 of Chapter 1. The following are the design parameters for the beam.

#### **Design parameters**

| Beam overall depth, h =           | 750 mm                           |
|-----------------------------------|----------------------------------|
| Beam breadth, b =                 | 300 mm                           |
| Slab thickness, h <sub>f</sub> =  | 160 mm                           |
| c/c distance btw supports =       | 9 000 mm                         |
| Width of support, $S_w$ =         | 500 mm (similar at both ends)    |
| c/c distance btw adjacent beams = | 3 300 mm (similar on both sides) |
| $f_{cu} =$                        | 35 MPa                           |
| $f_y =$                           | 500 MPa                          |
| $f_{yv} =$                        | 250 MPa                          |
| Cover =                           | 35 mm                            |
| Preferred bar size =              | 32                               |
| Preferred link size =             | 10                               |
| Density of concrete =             | 24.5 kN/m <sup>3</sup>           |
| Allowance for finishes =          | 2.0 kPa                          |
| Characteristic imposed load =     | 5.0 kPa                          |

#### Solution

#### Effective Span

| As h > S <sub>w</sub> | L =   | c/c distance between supports |
|-----------------------|---|-------------------------------|
|                       | =   | 9 000 mm                      |
|                       | <b>a</b> <sub>1</sub> = <b>a</b> <sub>2</sub> = | S <sub>w</sub> /2 = 250 mm    |
|                       |   |                               |

| Loading<br>Dead Load | Load width = | 3300 mm     | 2            | It is the c/c distance<br>between adjacent beams. |
|----------------------|--------------|-------------|--------------|---|
|                      | Finishes:    | 2.0         | x 3.3 = 6    | .6 kN/m   |
|                      | Slab S/W:    | 24.5 x 0.16 | 5 x 3.3 = 12 | .9 kN/m   |

b<sub>eff</sub>

|              | Beam S/W: | 24.5 x 0.3 x (0.75-0.16) = | 4.3 kN/m  |
|--------------|-----------|----------------------------|-----------|
|              |           | g <sub>k</sub> =           | 23.8 kN/m |
| Imposed Load |           |                            |           |
|              |           | 5.0 x 3.3 =                | 16.5 kN/m |
|              |           | q <sub>k</sub> =           | 16.5 kN/m |
|              |           |                            |           |

| The design load, w = | 1.4 x 23.8 + 1.6 x 16.5 |
|----------------------|-------------------------|
| =                    | <u>59.7 kN/m</u>        |

**Design Forces** 

| Design Moment, M = | 0.125 x 59.7 x 9 <sup>2</sup> |
|--------------------|-------------------------------|
| =                  | <u>604 kN-m</u>               |
| Design Shear, V =  | 0.5 x 59.7 x 9                |
| =                  | <u>269 kN</u>                 |

**Effective Depth** 

d = 750 - 35 - 10 - 32/2= 689 mm

#### Effective Flange Width

(Equal slab span on both sides of the beam)



#### **Design for Bending Moment**

|  | K =                 | $M / (bd^2 f_{cu})$  |                     |
|--|---------------------|--|---------------------|
|  | =                   | 604 x 10 <sup>6</sup> / (2700 x 689 <sup>2</sup> x 35)                                       |                     |
|  | =                   | 0.013  |                     |
| β <sub>b</sub> = 1.0                                   | <                   | 0.156  | (Singly reinforced) |
| Or, simply state:<br>"as K < 0.0428,<br>use z = 0.95d" | Lever arm, z =<br>= | [0.5 + (0.25 – K/0.9) <sup>0.5</sup> ] d<br>[0.5 + (0.25 – 0.013/0.9) <sup>0.5</sup> ] x 689 |                     |

$$= 0.985 \times 689 \text{ (use } 0.95 \times 689)$$

$$= 655 \text{ mm}$$
Check neutral axis, x = (689 - 655)/0.45  
= 76 < 160 mm (N.A. is within the flange)  
Tension steel req'd, A<sub>s,req</sub> = M / (0.87 f<sub>y</sub> z)  
= 604 × 10<sup>6</sup> / (0.87 × 500 × 655)  
= 2 120 mm<sup>2</sup>  
(Provide 3T32)  
A<sub>s,pro</sub> = 3 × 804  
= 2 412 mm<sup>2</sup>  
100A<sub>s</sub> / bh = 100 × 2412 / (300 × 750) = 1.072  
> 0.18 and < 4.0 (Steel ratio ok)

#### **Design for Shear at Support**

(As this beam is symmetrical, shears at both ends are the same.)

Max shear at the face of support

$$V_{s} = V - w a_{i}$$

$$= 269 - 59.7 \times 250 / 10^{3}$$

$$= 254 \text{ kN}$$

$$v_{max} = v_{s} = 254 \times 10^{3} / (300 \times 689)$$

$$= 1.23 \text{ MPa}$$

$$< 0.8 \sqrt{35} = 4.73 \text{ MPa} \qquad \text{(Concrete does not crush)}$$

Shear at d from the face of support

$$V_{d} = 254 - w d$$
  
= 254 - 59.7 x 689 / 10<sup>3</sup>  
= 213 kN  
$$v_{d} = 213 x 10^{3} / (300 x 689)$$
  
= 1.03 MPa

Calculate the design concrete shear stress,  $v_c$ :

(Table 6.3)

$$100A_{s}/(b_{v}d) = 100 \times 2412 / (300 \times 689) = 1.17 < 3$$

$$(400/d) = (400 / 689)^{1/4} < 1 \text{ (use 1.0)}$$

$$v_{c} = 0.79 \times (1.17)^{1/3} \times (1.0) / 1.25 \times (35/25)^{1/3}$$

$$= 0.666 \times 1.12$$

$$= 0.746 \text{ MPa}$$

| $v_c + 0.4 =$                      | 0.746 + 0.4 = 1.146 MPa                        |                              |
|------------------------------------|--|------------------------------|
| >                                  | 1.03 MPa                                       | (Provide nominal links)      |
| Nominal Links                      |  |                              |
| A <sub>sv</sub> / s <sub>v</sub> = | 0.4 x b <sub>v</sub> / (0.87 f <sub>yv</sub> ) |                              |
| =                                  | 0.4 x 300 / (0.87 x 250)                       |                              |
| =                                  | 0.552  |                              |
| Max s <sub>v</sub> <               | 0.75 x d                                       |                              |
| =                                  | 0.75 x 685 = 514 mm                            |                              |
|                                    |  | (Provide R10 – 275 – 2/legs) |
|                                    | 0 571 > 0 550                                  |                              |

 $A_{sv} / s_{v, prov} = 0.571 > 0.552$ 

#### **Check Deflection by Span-to-depth Ratio**

Basic L /d = 16 (Simply Supported Flanged Beam) (Table 7.3)

|                  |                         | $M/(bd^2) =$              | 604 x 10 <sup>6</sup> / (2700 x 689 <sup>2</sup> ) |                 |
|------------------|-------------------------|---------------------------|--|-----------------|
| $\left  \right $ | h ~instead of h         | =                         | 0.471 N/mm <sup>2</sup>                            |                 |
|                  | is used for <b>b</b> in | $A_{s,req} / A_{s,pro} =$ | 2120 / 2412  |                 |
|                  | L/d checking            | =                         | 0.879  |                 |
|                  |                         | $f_s =$                   | 2/3 x 500 x 0.879 = 293 MPa                        |                 |
|                  |                         | $m_t =$                   | 0.55 + (477-293)/[120(0.9+0.471)]                  | (Table 7.4)     |
|                  |                         | =                         | 0.55 + 1.118                                       |                 |
|                  |                         | =                         | 1.668  |                 |
|                  |                         |                           |  |                 |
|                  |                         | Allowable L / d =         | 1.668 x 16 = 26.69                                 |                 |
|                  |                         | Actual L / d =            | 9000 / 689   |                 |
|                  |                         | =                         | 13.06 ≤ 26.69                                      | (Deflection ok) |



#### **Question B**

Re-design the R C beam, 5B2, in Example A, as 5B2A with the following changes:

Beam overall depth, h = 550 mm (reduced)

Additional load = 100 mm thick brick wall with 15mm cement mortar on both sides, 3.0 m high seating directly on the beam over the whole span Density of brick is 21.7 kN/m<sup>3</sup> Density of cement mortar is 23 kN/m<sup>3</sup>

Q1 – Q2

#### Solution

**Design parameters** 

| Beam overall depth, h =           | 550 mm (revised)                   |  |
|-----------------------------------|------------------------------------|--|
| Beam breadth, b =                 | 300 mm                             |  |
| Slab thickness, h <sub>f</sub> =  | 160 mm                             |  |
| c/c distance btw supports =       | 9 000 mm                           |  |
| Width of support, $S_w$ =         | 500 mm (similar at both ends)      |  |
| c/c distance btw adjacent beams = | 3 300 mm (similar on both sides)   |  |
| $f_{cu} =$                        | 35 MPa                             |  |
| $f_y =$                           | 500 MPa                            |  |
| $f_{yv} =$                        | 250 MPa                            |  |
| Cover =                           | 35 mm                              |  |
| Preferred bar size =              | 40 (An increased value is assumed) |  |
| Preferred link size =             | 10                                 |  |
| Density of concrete =             | 24.5 kN/m <sup>3</sup>             |  |
| Allowance for finishes =          | 2.0 kPa                            |  |
| Characteristic imposed load =     | 5.0 kPa                            |  |

#### Effective Span

| As h > S <sub>w</sub> | L =   | c/c distance between supports |
|-----------------------|---|-------------------------------|
|                       | =   | 9 000 mm                      |
|                       | <b>a</b> <sub>1</sub> = <b>a</b> <sub>2</sub> = | S <sub>w</sub> /2 = 250 mm    |

#### Loading

|            | Load width = | 3.3 m                        |           |              |
|------------|--------------|------------------------------|-----------|--------------|
| Dead Load  |              |                              |           |              |
|            | Finishes:    | 2.0 x 3.3 =                  | 6.6 kN/m  |              |
|            | Wall:        | (21.7x0.1 + 23x0.03) x 3.0 = | 8.6 kN/m  | (additional) |
|            | Slab S/W:    | 24.5 x 0.16 x 3.3 =          | 12.9 kN/m |              |
|            | Beam S/W:    | 24.5 x 0.3 x (0.55-0.16) =   | 2.9 kN/m  |              |
|            |              | g <sub>k</sub> =             | 31.0 kN/m |              |
| Imposed Lo | ad           |                              |           |              |
|            |              | 5.0 x 3.3 =                  | 16.5 kN/m |              |
|            |              | q <sub>k</sub> =             | 16.5 kN/m | _            |
|            |              |                              |           |              |

The design load, w =  $1.4 \times 31.0 + 1.6 \times 16.5$ 

= <u>69.8 kN/m</u>

**Design Forces** 

Design Moment, M =  $0.125 \times 69.8 \times 9^{2}$ =  $\frac{707 \text{ kN-m}}{0.5 \times 69.8 \times 9}$ =  $\frac{314 \text{ kN}}{314 \text{ kN}}$ 

**Effective Depth** 

| d = | 550 - 35 - 10 - 40/2 |           |
|-----|----------------------|-----------|
| =   | <u>485 mm</u>        | (revised) |

#### Effective Flange Width

(Equal slab span on both sides of the beam)

$$b_{1} = b_{2} = 1500 \text{ mm}$$

$$b_{w} = b = 300 \text{ mm}$$

$$L_{pi} = L = 9000 \text{ mm}$$

$$b_{eff,1} = b_{eff,2} = Min (0.2x1500+0.1x9000 \text{ or } 0.2x9000 \text{ or } 1500)$$

$$= Min (1200 \text{ or } 1800 \text{ or } 1500)$$

$$= 1200 \text{ mm}$$

$$b_{eff} = 2 \times 1200 + 300$$

$$= 2700 \text{ mm}$$

#### **Design for Bending Moment**

| K =                                       | $M / (bd^2 f_{cu})$                                    |                             |
|---|--|-----------------------------|
| =   | 707 x 10 <sup>6</sup> / (2700 x 485 <sup>2</sup> x 35) | )                           |
| =   | 0.032  |                             |
| β <sub>b</sub> = 1.0 <                    | 0.156  | (Singly reinforced)         |
|   |  |                             |
| K < 0.0428 z =                            | 0.95 d   |                             |
| =   | 0.95 x 485   |                             |
| =   | 461 mm   |                             |
|   |  |                             |
| Check neutral axis, x =                   | (485 – 461)/0.45                                       |                             |
| =   | 53 < 160 mm  | (N.A. is within the flange) |
|   |  |                             |
| Tension steel req'd, A <sub>s,req</sub> = | M / (0.87 f <sub>y</sub> z)                            |                             |
| =   | 707 x 10 <sup>6</sup> / (0.87 x 500 x 461)             | )                           |
| =   | <u>3 526 mm<sup>2</sup></u>                            |                             |
|   |  | (Provide 3T40, as assumed)  |

$$A_{s,pro} = 3 \times 1257$$
  
=  $3771 \text{ mm}^2$   
100A<sub>s</sub> / bh = 100 x 3771 / (300 x 550) = 2.285  
> 0.18 and < 4.0 (Steel ratio ok)

#### **Design for Shear at Support**

(As this beam is symmetrical, shears at both ends are the same.)

Max shear at the face of support

$$V_{s} = V - w a_{i}$$
= 314 - 69.8 x 250 / 10<sup>3</sup>  
= 297 kN  

$$v_{max} = v_{s} = 297 x 10^{3} / (300 x 485)$$
= 2.04 MPa  
< 0.8 √ 35 = 4.73 MPa (Concrete does not crush)

Shear at d from the face of support

 $V_{d} = 297 - w d$ = 297 - 69.8 x 485 / 10<sup>3</sup> = 263 kN  $v_{d} = 263 x 10^{3} / (300 x 485)$ = 1.81 MPa

(Table 6.3)

Calculate the design concrete shear stress,  $v_c$ : Only 2T40 extends to the supports (see DWG-02)

$$A_{s} = 2 \times 1257 = 2514 \text{ mm}^{2}$$

$$100A_{s}/(b_{v}d) = 100 \times 2514 / (300 \times 485) = 1.73 < 3$$

$$(400/d)^{1/4} = (400 / 485)^{1/4} < 1 \text{ (use 1.0)}$$

$$v_{c} = 0.79 \times (1.73)^{1/3} \times (1.0) / 1.25 \times (35/25)^{1/3}$$

$$= 0.759 \times 1.12$$

$$= 0.85 \text{ MPa}$$

| $v_c + 0.4 =$ | 0.85 + 0.4 = 1.25 MPa |                       |
|---------------|-----------------------|-----------------------|
| <             | 1.81 MPa              | (Provide shear links) |

Shear Links

$$A_{sv} / s_v = b_v (v - v_c) / (0.87 f_{yv})$$
  
= 300 x (1.81 - 0.85) / (0.87 x 250)  
= 1.324

(Provide R10 – 100 -2/legs as shear links)

 $A_{sv} / S_{v, prov} = 1.570 > 1.324$ 



**Nominal Links** 

#### **Check Deflection by Span-to-depth Ratio**

Basic L /d = 16 (Simply Supported Flanged Beam) (Table 7.3)  $M/(bd^{2}) = 707 \times 10^{6} / (2700 \times 485^{2})$   $= 1.11 \text{ N/mm}^{2}$   $A_{s,req} / A_{s,pro} = 3526 / 3771 = 0.935$   $f_{s} = 2/3 \times 500 \times 0.935 = 312 \text{ MPa}$   $m_{t} = 0.55 + (477-312)/[120(0.9+1.11)]$  = 0.55 + 0.684 = 1.234(Table 7.4)

Allowable L / d = 
$$1.234 \times 16 = 19.74$$
  
Actual L / d =  $9000 / 485$   
=  $18.55 \le 19.74$  (Deflection ok)

#### 4.2.3 Notes on Detailing

The reinforcement details of the simply-supported beams of Questions B and C in 4.2.2 above are shown in DWG-02 at the end of the Chapter. Take note of the following in the details:<sup>4</sup>

- (a) In order to support the links, top bars are provided at the top corners of the links. They are called *carriers*, and should not be regarded as compression steel unless they are properly restrained from buckling.
- (b) In theory, there is no hogging moment at the supports and therefore top bar is not required. However, in order to avoid unsightly cracks due partial fixity to the supporting beams, a certain amount of rebars are provided at the top of the supports.
- (c) Provision of 3 number of bars on the tension face of the beam with 300mm width, the bar spacing deems appropriate for dispersing the cracks. (Refer to chapter 2 for details.)
- (d) The maximum moment at mid-span is adopted as the design moment to determine the reinforcement bars. The bending moment in the beam is decreasing, theoretically, to zero at the supports and therefore it is not necessary to have all the bottom bars extended to the supports. The design code requires at least half of the steel be extended to and anchored into the supports.

### 4.3 Continuous Beams

The design process of continuous beam is quite similar to that of simply-supported beam except that hogging moments at supports have to be designed for in continuous beam. For beam in building structures, sagging moment in the mid-span is usually resisted by *flanged section* while hogging moments in the supports are resisted by *rectangular section*.

<sup>&</sup>lt;sup>4</sup> The rules of reinforcement detailing are beyond the scope of this chapter. Refer to the design code for details.

Three examples are provided to demonstrate the whole design process of a continuous beam. The example in 4.3.1 demonstrates the basic principle of the design. It is a continuous beam of approximately equal span under uniformly distributed load, and therefore the force coefficients in 4.1.2 mentioned above can be used to determine the design forces. For demonstration purpose, only one of the spans of the continuous beam is designed in the example.

Questions A & B in 4.3.2 let you appreciate the design of a long-span beam by sub-frame analysis. It accentuates the heavily reinforced sections at the supports, and illustrates the effect of moment redistribution on the design.

#### 4.3.1 Example – Uniformly Loaded & Equal Span



#### Question

DWG-03 at the end of this Chapter shows the framing plan of a roof garden. There is a continuous beam RB21-RB22-RB23. Design the reinforcement and check the deflection for the *end span*, RB21 of this beam.

#### Solution

#### **Design parameters**

The following design parameters can be obtained from DWG-03:

| Beam overall depth, h =           | 600 mm                        |
|-----------------------------------|-------------------------------|
| Beam breadth, b =                 | 350 mm                        |
| Slab thickness, h <sub>f</sub> =  | 150 mm                        |
| c/c distance btw supports =       | 6 500 mm (same for all spans) |
| Width of LHS support, $S_{w1}$ =  | 200 mm                        |
| Width of RHS support, $S_{w2}$ =  | 350 mm                        |
| c/c distance btw adjacent beams = | 3 200 mm (same on both sides) |
| $f_{cu} =$                        | 35 MPa                        |
| $f_{\gamma} =$                    | 500 MPa                       |
| $f_{yv} =$                        | 250 MPa                       |
| Cover =                           | 40 mm                         |
| Preferred bar size =              | 32                            |
| Preferred link size =             | 10                            |
| Density of concrete =             | 24.5 kN/m <sup>3</sup>        |
|                                   |                               |

Allowance for roofing = 2.0 kPa Allowance for soil = 450 mm thick Characteristic imposed load = 5.0 kPa

#### **Effective Span**

| As $h > S_w$ , | L =              | center-to-center distance btw support |
|----------------|------------------|---------------------------------------|
|                | =                | <u>6 500 mm</u>                       |
|                | a <sub>1</sub> = | S <sub>w1</sub> /2 = 100 mm           |
|                | a <sub>2</sub> = | S <sub>w2</sub> /2 = 175 mm           |
|                |                  |                                       |

#### Loading

| Load width = | 3200 mm |
|--------------|---------|
|--------------|---------|

Dead Load

| Roofing:  | 2.0 x 3.2 =                 | 6.4 kN/m  |
|-----------|-----------------------------|-----------|
| Soil:     | 20 x 0.45 x 3.2 =           | 28.8 kN/m |
| Slab S/W: | 24.5 x 0.15 x 3.2 =         | 11.8 kN/m |
| Beam S/W: | 24.5 x 0.35 x (0.60-0.15) = | 3.9 kN/m  |
|           | g <sub>k</sub> =            | 50.9 kN/m |
|           |                             |           |

Imposed Load

 $5.0 \times 3.2 = 16.0 \text{ kN/m}$  $q_k = 16.0 \text{ kN/m}$ 

The design load, w =  $1.4 \times 50.9 + 1.6 \times 16.0$ = 96.8 kN/mF =  $96.8 \times 6.5$ = 629 kN

#### **Design Forces**

**Design Moments** 

At LHS support, M = 0 At mid-span, M = 0.09 x 629 x 6.5 = 368 kN-m At RHS support, M = -0.11 x 629 x 6.5 = -450 kN-m Design Shears

At LHS support, V = 0.45 x 629

= 283 kN

(Table 6.1)

At RHS support, V = 0.6 x 629 = 377 kN

#### **Design for Mid-span Bending Moment**

M = 368 KN-m (sagging Mt resisted by flanged section)

Effective Depth, d = 600 - 40 - 10 - 32/2= <u>534 mm</u>

Effective Flange Width (equal slab span on both sides of the beam)

 $b_1 = b_2 = 1425 \text{ mm}$  $b_w = b = 350 \text{ mm}$  $L_{pi} = 0.85 \times 6500 \text{ mm}$ (End-span) = 5525 mm  $b_{eff,1} = b_{eff,2} = Min (0.2x1425+0.1x5525 \text{ or } 0.2x5525 \text{ or } 1425)$ = Min (837.5 or 1105 or 1425) = 837.5 mm  $b_{eff} = 2 \times 837.5 + 350$ = 2 025 mm 0  $K = M / (bd^2 f_{cu})$ b, =  $368 \times 10^6 / (2025 \times 534^2 \times 35)$ = 0.018  $\beta_{\rm b} = 1.0$ < 0.156 (Singly reinforced) K < 0.0428  $z = 0.95d = 0.95 \times 534$ = 507 mm Check neutral axis, x = (534 - 507)/0.45= 60 < 150 mm (N.A. is within the flange) Tension steel req'd,  $A_{s,reg} = M / (0.87 f_v z)$  $= 368 \times 10^6 / (0.87 \times 500 \times 507)$  $= 1.668 \text{ mm}^2$ (Provide 2T32+1T20 bottom)  $A_{s,pro} = 2 \times 804 + 314$  $= 1.922 \text{ mm}^2$  $100A_s$  / bh =  $100 \times 1922$  /  $(350 \times 600) = 0.92$ 

> 0.18 and < 4.0

(Steel ratio ok)

#### **Design for LHS Support Shear**

(The design moment is zero.

Therefore, A<sub>s</sub> and d of the bottom bars are used for shear design.)

Max shear at the face of support

$$V_{s} = V - w a_{1}$$

$$= 283 - 96.8 \times 100 / 10^{3}$$

$$= 273 \text{ kN}$$

$$v_{max} = v_{s} = 273 \times 10^{3} / (350 \times 534)$$

$$= 1.46 \text{ MPa}$$

$$< 0.8 √ 35 = 4.73 \text{ MPa}$$
 (Concrete does not crush)

Shear at d from the face of support

$$V_{d} = V_{s} - w d$$
  
= 273 - 96.8 x 534 / 10<sup>3</sup>  
= 221 kN  
$$v_{d} = 221 x 10^{3} / (350 x 534)$$
  
= 1.18 MPa

Calculate the design concrete shear stress,  $v_c$ :

(Table 6.3)

 $100A_{s}/(b_{v}d) = 100 \times 1922 / (350 \times 534) = 1.03 < 3$   $(400/d)^{1/4} = (400 / 534)^{1/4} < 1 \text{ (use } 1.0)$   $v_{c} = 0.79 \times (1.03)^{1/3} \times (1.0) / 1.25 \times (35/25)^{1/3}$   $= 0.64 \times 1.12$  = 0.72 MPa

| $v_c + 0.4 =$ | 0.72 + 0.4 = 1.12 MPa |                          |
|---------------|-----------------------|--------------------------|
| <             | 1.18 MPa              | (Shear link is required) |

Shear Link

 $A_{sv} / s_v = b_v (v - v_c) / (0.87 f_{yv})$ = 350 x (1.18 - 0.72) / (0.87 x 250) = 0.740

Nominal Link

 $A_{sv} / s_v = 0.4 \text{ x } b_v / (0.87 f_{yv})$ = 0.4 x 350 / (0.87 x 250) = 0.644 Max s\_v < 0.75 x d

= 0.75 x 685 = 514 mm (Provide R10 - 200 - 2/legs) $A_{sv} / S_{v, prov} = 0.785 > 0.644$  and 0.740 **Design for RHS Support Bending Moment** M = 450 kN-m (hogging Mt. resisted by rectangular section) Effective depth, d = 600 - 40 - 10 - 32/20 0 = 534 mm  $K = M / (bd^2 f_{cu})$  $= 450 \times 10^6 / (350 \times 534^2 \times 35)$ b = 0.129 (Singly reinforced)  $\beta_{\rm b} = 1.0$ < 0.156 Lever arm,  $z = [0.5 + (0.25 - K/0.9)^{0.5}] d$ =  $[0.5 + (0.25 - 0.129/0.9)^{0.5}] \times 534$  $= 0.827 \times 534$ = 442 mm Tension steel req'd,  $A_{s,req} = M / (0.87 f_y z)$  $= 450 \times 10^6 / (0.87 \times 500 \times 442)$  $= 2340 \text{ mm}^2$ (Provide 3T32 top)  $A_{s,pro} = 3 \times 804$  $= 2412 \text{ mm}^2$  $100A_s$  / bh =  $100 \times 2412$  /  $(350 \times 600) = 1.15$ > 0.13 and < 4.0 (Steel ratio ok)

#### **Design for RHS Support Shear**

Max shear at the face of support

$$V_{s} = V - w a_{2}$$

$$= 377 - 96.8 \times 175 / 10^{3}$$

$$= 360 \text{ kN}$$

$$v_{max} = v_{s} = 360 \times 10^{3} / (350 \times 534)$$

$$= 1.93 \text{ MPa}$$

$$< 0.8 \sqrt{35} = 4.73 \text{ MPa}$$
 (Concrete does not crush)

Shear at d from the face of support

 $V_{d} = V_{s} - w d$ = 360 - 96.8 x 534 / 10<sup>3</sup> = 308 kN  $v_{d}$  = 308 x 10<sup>3</sup> / (350 x 534) = 1.65 MPa

Calculate the design concrete shear stress,  $v_c$ : (Table 6.3)

$$100A_{s}/(b_{v}d) = 100 \times 2412 / (350 \times 534) = 1.29 < 3$$

$$(400/d)^{1/4} = (400 / 534)^{1/4} < 1 \text{ (use 1.0)}$$

$$v_{c} = 0.79 \times (1.29)^{1/3} \times (1.0) / 1.25 \times (35/25)^{1/3}$$

$$= 0.69 \times 1.12$$

$$= 0.77 \text{ MPa}$$

$$v_{c} + 0.4 = 0.77 + 0.4 = 1.17 \text{ MPa}$$

$$< 1.65 \text{ MPa} \qquad \text{(Shear link is required)}$$

Shear Link

 $A_{sv} / s_v = b_v (v - v_c) / (0.87 f_{yv})$ = 350 x (1.65 - 0.77) / (0.87 x 250) = 1.42

(Provide R10 – 200 -4/legs as shear link)

$$A_{sv} / S_{v, prov} = 2 \times 0.785 = 1.57 > 1.42$$

Extent of Shear Link

$$V_n = (v_c + 0.4) b_v d$$
  
= 1.17 x 350 x 534 / 10<sup>3</sup>  
= 219 kN  
Dist. btw V<sub>n</sub> and V<sub>s</sub> = (360 - 219) x 10<sup>3</sup> / 96.8  
= 1457 mm  
No. of link req'd = 1457 / 200 +1  
= 8.3 (Provide 9 no. of shear links)

#### Check Deflection by Span-to-depth Ratio

|   | Basic L /d =              | 18.5 (End Span of Flanged Beam)                    | (Table 7.3) |
|---|---------------------------|--|-------------|
|   | M/(bd <sup>2</sup> ) =    | 368 x 10 <sup>6</sup> / (2025 x 534 <sup>2</sup> ) |             |
|   | =                         | 0.637 N/mm <sup>2</sup>                            |             |
|   | $A_{s,req} / A_{s,pro} =$ | 1668 / 1922 = 0.868                                |             |
| -                                       | $f_s =$                   | 2/3 x 500 x 0.868 = 289 MPa                        |             |
| Use mid-span                            | $m_t =$                   | 0.55 + (477-289)/[120(0.9+0.637)]                  | (Table 7.4) |
| moment and steel<br>to check L/d ratio. | =                         | 0.55 + 1.019                                       |             |

= 1.569Allowable L / d =  $1.569 \times 18.5 = 29.03$ Actual L / d = 6500 / 534=  $12.17 \le 29.03$  (Deflection ok)

#### 4.3.2 Examples – Continuous Beam with Design Force Envelopes



#### **Question A**

A continuous beam 3B5-3B6-**3B7**-3B8 is analyzed by the method of sub-frame<sup>5</sup> according to Cl. 5.2.5.1 of HKCP-2013, as shown below.



The center-to-center distances between columns are used as the span length for analysis. The design moment and shear force diagrams of the interior span, **3B7**, are given below. There is no redistribution of moment, (i.e.  $\beta_{b} = 1.0$ ). The load cases are:

| Load Case | 3B6                                   | 3B7                                   | 3B8                                   |
|-----------|---------------------------------------|---------------------------------------|---------------------------------------|
| 1         | 1.0G <sub>k</sub>                     | 1.4G <sub>k</sub> + 1.6Q <sub>k</sub> | 1.0G <sub>k</sub>                     |
| 2         | 1.4G <sub>k</sub> + 1.6Q <sub>k</sub> | 1.4G <sub>k</sub> + 1.6Q <sub>k</sub> | 1.0G <sub>k</sub>                     |
| 3         | 1.0G <sub>k</sub>                     | 1.4G <sub>k</sub> + 1.6Q <sub>k</sub> | 1.4G <sub>k</sub> + 1.6Q <sub>k</sub> |

<sup>&</sup>lt;sup>5</sup> There are several approaches to simplify monolithic reinforced concrete frames for analysis. Details can be found in Cl. 5.2.4 of HKCP-2013. It is beyond the scope of this chapter.



**Design Bending Moment Diagram** 





Design the bending and shear reinforcement and check the deflection of the beam, 3B7, with the following design parameters.

#### **Design parameters**

| Beam overall depth, h =             | 650 mm                        |
|-------------------------------------|-------------------------------|
| Beam breadth, b =                   | 550 mm                        |
| Slab thickness, h <sub>f</sub> =    | 150 mm                        |
| c/c distance btw supports =         | 11 000 mm                     |
| Width of supports, $S_w$ =          | 500 mm                        |
| clear distance btw adjacent beams = | 3 300 mm (same on both sides) |
| $f_{cu} =$                          | 40 MPa                        |

 $f_y = 500 \text{ MPa}$  $f_{yv} = 500 \text{ MPa}$ Cover = 40 mm Max design load, w = 111 kN/m

#### Solution

Design for Mid-span Bending Moment

| М                                       | 748 KN-m (sagging Mt resisted by flanged section)     |                             |
|---|---|-----------------------------|
| Effective Depth, d                      | = 650 - 40 - 10 - 40/2                                | (assume bar size 40)        |
|   | = 580 mm  |                             |
| Effective Flange Width                  | (Equal slab span on both sic                          | les of the beam)            |
| $b_1 = b_2$                             | = 3 300 / 2 = 1 650 mm                                |                             |
| b <sub>w</sub> = b                      | = 550 mm  |                             |
| (Read from BMD) L <sub>pi</sub>         | = 9000 – 1400 = 7600 mm                               |                             |
| $b_{eff,1} = b_{eff,2}$                 | = Min (0.2x1650+0.1x7600 or                           | 0.2x6600 or 1650)           |
|   | = Min (1090 or 1320 or 1650)                          |                             |
|   | = 1090 mm   |                             |
| b <sub>eff</sub>                        | = 2 x 1090 + 550                                      |                             |
|   | = 2 730 mm  |                             |
| к                                       | = M / (bd <sup>2</sup> $f_{cu}$ )                     |                             |
|   | $= 748 \times 10^6 / (2730 \times 580^2 \times 40^3)$ | 0)                          |
|   | = 0.0204  |                             |
| $\beta_b = 1.0$                         | < 0.156   | (Singly reinforced)         |
| K < 0.0428 z                            | = 0.95d = 0.95 x 580                                  |                             |
|   | = 551 mm  |                             |
| Check neutral axis x                    | = (580 - 551)/0.45                                    |                             |
|   | = 64 < 150 mm   | (N.A. is within the flange) |
| Tension steel req'd, A <sub>s,req</sub> | = M / (0.87 $f_y z$ )                                 |                             |
|   | $= 748 \times 10^6 / (0.87 \times 500 \times 55^3)$   | 1)                          |
|   | = 3 120 mm <sup>2</sup>                               |                             |
|   | (   | (Provide 2T40+2T25 bottom)  |
| A <sub>s,pro</sub>                      | = 2 x 1257 + 2 x 491                                  |                             |

| =                 | 3 496 mm <sup>2</sup>           |                  |
|-------------------|---------------------------------|------------------|
| $100A_{s}$ / bh = | 100 x 3496 / (550 x 650) = 0.98 |                  |
| >                 | 0.18 and < 4.0                  | (Steel ratio ok) |

#### Design for LHS Support Bending Moment

Moment reduction due to support width<sup>6</sup>

|                       | $\Delta M_{Ed}$ =     | F <sub>Ed,sup</sub> S <sub>w</sub> /8              |                           |
|-----------------------|-----------------------|--|---------------------------|
|                       | =                     | 625 x 0.5 / 8 = 39 kN-m                            |                           |
| Design Moment         | M =                   | 1113 – 39 = 1 074 kN-m                             |                           |
|                       |                       | (hogging Mt. resisted by rectangular               | section)                  |
|                       |                       |  |                           |
| Effective Depth       | d =                   | 650 - 40 - 10 - 40/2 - 10                          | (Assume)                  |
|                       | =                     | 570 mm   |                           |
|                       | K =                   | $M/(bd^2f_{r_{r_{r_{r_{r_{r_{r_{r_{r_{r_{r_{r_{r_$ |                           |
|                       | =                     | $1074 \times 10^6 / (550 \times 570^2 \times 40)$  |                           |
|                       | =                     | 0.150  |                           |
| $\beta_{\rm b} = 1.0$ | <                     | 0.156  | (Singly reinforced)       |
| PD                    |                       |  | (eg.) . ee.eee,           |
| Lever                 | arm, z =              | [0.5 + (0.25 – K/0.9) <sup>0.5</sup> ] d           |                           |
|                       | =                     | [0.5 + (0.25 – 0.150/0.9) <sup>0.5</sup> ] x 570   |                           |
|                       | =                     | 0.788 x 570  |                           |
|                       | =                     | 449 mm   |                           |
| Tension steel rea'd   | Δ =                   | $M/(0.87 f_{-7})$                                  |                           |
|                       | , As,req –            | $1074 \times 10^6 / (0.87 \times 500 \times 449)$  |                           |
|                       | _                     | $5400 \text{ mm}^2$                                |                           |
|                       |                       | (Provic  | 4 = 4T40 + 2T25 ton       |
|                       | Δ =                   | 4 x 1257 + 2 x 491                                 | 10 + 1 + 0 + 2 1 2 3 top) |
|                       | As,pro –              | $6.010 \text{ mm}^2$                               |                           |
| Ch                    | eck d =               | (4×1257×580+2×401×523)/(4×1257+)                   | 2v/Q1)                    |
|                       |                       | 571 > 570  mm                                      | (Assumed d is ok)         |
|                       | _                     | 577 - 570 mm                                       |                           |
| 100/                  | A <sub>s</sub> / bh = | 100 x (6010 + 3496) / (550 x 650) = 2              | 2.66                      |

< 4.0 (Steel ratio ok)

<sup>&</sup>lt;sup>6</sup> Refer to CI.5.2.1.2(b) of HKCP-2013 for details. It is beyond the scope of this chapter.

#### **Design for LHS Support Shear**

Max shear at the face of support

$$V_{s} = V - w a_{1}$$

$$= 625 - 111 \times 250 / 10^{3}$$

$$= 597 \text{ kN}$$

$$v_{max} = v_{s} = 597 \times 10^{3} / (550 \times 571)$$

$$= 1.90 \text{ MPa}$$

$$< 0.8 √ 40 = 5.06 \text{ MPa} \qquad \text{(Concrete does not crush)}$$

Shear at d from the face of support

$$V_{d} = V_{s} - w d$$
  
= 597 - 111 x 571 / 10<sup>3</sup>  
= 534 kN  
$$v_{d} = 534 x 10^{3} / (550 x 571)$$
  
= 1.70 MPa

Calculate the design concrete shear stress,  $v_c$ : (Table 6.3)  $100A_{s}/(b_{v}d) = 100 \times 6010 / (550 \times 571) = 1.91 < 3$  $(400/d)^{1/4} = (400 / 571)^{1/4} < 1 \text{ (use 1.0)}$  $v_c = 0.79 \times (1.91)^{1/3} \times (1.0) / 1.25 \times (40/25)^{1/3}$ = 0.78 x 1.17 = 0.92 MPa  $v_c$  + 0.4 = 0.92 + 0.4 = 1.32 MPa < 1.70 MPa (Shear link is required) Shear Link  $A_{sv} / s_v = b_v (v - v_c) / (0.87 f_{yv})$  $= 550 \times (1.70 - 0.92) / (0.87 \times 500)$ = 0.986 (Provide T10-150-2/legs as shear link)  $A_{sv} / s_{v, pro} = 1.047 > 0.986$ Extent of shear link = (1.90 - 1.32) x 550 x 571 / 111 = 580 mm No. of shear link req'd = 580 / 150 + 1 = 4.9 (Provide 6 no. of shear link)

Nominal Link

 $A_{sv} / s_v = 0.4 \times b_v / (0.87 f_{yv})$ 

 $= 0.4 \times 550 / (0.87 \times 500)$ = 0.506  $Max s_v < 0.75 x d$ = 0.75 x 630 = 473 mm (Provide T10 – 300 – 2/legs)  $A_{sv} / s_{v, pro} = 0.524 > 0.506$ **Design for RHS Support Bending Moment**  $\Delta M_{Ed} = F_{Ed,sup} S_w / 8$ = 705 x 0.5 / 8 = 44 kN-m M = 1691 - 44 = 1 647 kN-m (hogging Mt. resisted by rectangular section) (Assume 2 layers of T40) Effective Depth d = 650 - 40 - 10 - 40 - 20= 540 mm  $K = M / (bd^2 f_{cu})$ =  $1647 \times 10^6 / (550 \times 540^2 \times 40)$ = 0.257  $\beta_{\rm b} = 1.0$ > 0.156 (Compression steel required) Lever arm, z = 0.775 d = 0.775 x 540= 418.5 mm Depth to neutral axis, x = 0.5 d = 0.5 x 540= 270 mm Check d' / x = 70 / 270= 0.26 < 0.38  $(f_{sc} = 0.87 f_v)$ Compression steel req'd, A'<sub>s</sub> =  $\frac{(K - K') f_{cu} bd^2}{0.87 f_y (d - d')}$  $= \frac{(0.257 - 0.156) \times 40 \times 550 \times 540^2}{0.87 \times 500 \times (540 - 70)}$ = 3 169 mm<sup>2</sup> (Provide 2T40 + 2T25 bottom)

 $A'_{s,pro} = 3496 \text{ mm}^2$ 

Tension steel req'd, 
$$A_s = \frac{K' f_{cu} bd^2}{0.87 f_y z} + A_s'$$
  

$$= \frac{0.156 \times 40 \times 550 \times 540^2}{0.87 \times 500 \times 418.5} + 3169$$

$$= 5497 + 3169$$

$$= 8666 \text{ mm}^2$$
(Provide 6T40 + 2T32 top)  
 $A_{s, \text{ pro}} = 6 \times 1257 + 2 \times 804$ 

= 
$$9150 \text{ mm}^2$$
  
100A<sub>s</sub> / bh =  $100 \times (9150 + 3496) / (550 \times 650) = 3.54$   
< 4.0 (Steel ratio ok)

#### **Design for RHS Support Shear**

Max shear at the face of support

$$V_{s} = V - w a_{2}$$

$$= 705 - 111 \times 250 / 10^{3}$$

$$= 677 \text{ kN}$$

$$v_{max} = v_{s} = 677 \times 10^{3} / (550 \times 540)$$

$$= 2.28 \text{ MPa}$$
< 0.8 √ 40 = 5.06 MPa (Concrete does not crush)

Shear at d from the face of support

$$V_{d} = V_{s} - w d$$
  
= 677 - 111 x 540 / 10<sup>3</sup>  
= 617 kN  
$$v_{d} = 617 x 10^{3} / (550 x 540)$$
  
= 2.08 MPa

Calculate the design concrete shear stress,  $v_c$ :

(Table 6.3)

$$100A_{s}/(b_{v}d) = 100 \times 9150 / (550 \times 540) = 3.08 \text{ (use } 3.0)$$

$$(400/d)^{1/4} = (400 / 540)^{1/4} < 1 \text{ (use } 1.0)$$

$$v_{c} = 0.79 \times (3.0)^{1/3} \times (1.0) / 1.25 \times (40/25)^{1/3}$$

$$= 0.91 \times 1.17$$

$$= 1.07 \text{ MPa}$$

$$v_{c} + 0.4 = 1.07 + 0.4 = 1.47 \text{ MPa}$$

$$< 2.08 \text{ MPa} \qquad \text{(Shear link is required)}$$

Shear Link

$$A_{sv} / s_v = b_v (v - v_c) / (0.87 f_{yv})$$

|                  | =                         | 550 x (2.08 - 1.07) / (0.87                        | x 500)           |                    |
|------------------|---------------------------|--|------------------|--------------------|
|                  | =                         | 1.277  |                  |                    |
|                  |                           | (Provide T   | 10 – 225 -4/leg  | s as shear link)   |
|                  | $A_{sv} / s_{v, pro} =$   | 2 x 0.698 = 1.396 > 1.277                          |                  |                    |
| Extent           | of shear link =           | (2.28 - 1.47) x 550 x 540 /                        | 111              |                    |
|                  | =                         | 2167 mm  |                  |                    |
| No.              | of link req'd =           | 2167 / 225 +1 = 10.6                               | (Provide 11 no   | o. of shear links) |
|                  | (These shea               | ar links also provide restrair                     | it to bottom cor | mpression bars)    |
|                  | ,                         |  |                  | . ,                |
| Nominal Link     |                           | (Same as above)                                    | (Provide T10     | – 300 – 2/legs)    |
| Check Deflection | by Span-to-de             | pth Ratio  |                  |                    |
|                  | Basic L /d =              | 21 (Continuous of Flangeo                          | l Beam)          | (Table 7.3)        |
| L > 10m          | <i>m</i> <sub>l</sub> =   | 10/11 = 0.91                                       |                  |                    |
|                  | M/(bd <sup>2</sup> ) =    | 748 x 10 <sup>6</sup> / (2530 x 580 <sup>2</sup> ) |                  |                    |
|                  | =                         | 0.879 N/mm <sup>2</sup>                            |                  |                    |
|                  | $A_{s,req} / A_{s,pro} =$ | 3120 / 3496 = 0.892                                |                  |                    |
|                  | <i>f</i> <sub>s</sub> =   | 2/3 x 500 x 0.892 = 297 M                          | Pa               |                    |
|                  | $m_t =$                   | 0.55 + (477-297)/[120(0.9-                         | +0.879)]         | (Table 7.4)        |
|                  | =                         | 0.55 + 0.843                                       |                  |                    |
|                  | =                         | 1.393  |                  |                    |
| All              | lowable L / d =           | 0.91 x 1.393 x 21 = 26.6                           |                  |                    |
|                  | Actual L / d =            | 11000 / 580  |                  |                    |
|                  | =                         | 19.0 ≤ 26.6  |                  | (Deflection ok)    |
|                  |                           |  |                  |                    |



#### **Question B**

Moment redistribution is applied to the design moments at the supports of the beam 3B7. The RHS support moment of load case 2 is redistributed by 25% and the LHS support moment of load case 3 is redistributed by 30% and the resultant design moment and shear diagrams are given below. The design parameters in Example A are still applicable. Re-design the beam 3B7.



**Design Bending Moment Diagram** 



#### **Design Shear Force Diagram**

#### Solution

There is no change to the mid-span design moment.

As a demonstration, only the RHS support bending moment is checked.

#### **Design for RHS Support Bending Moment**

$$\Delta M_{Ed} = F_{Ed,sup} S_w / 8$$
  
= 659 x 0.5 / 8 = 41 kN-m  
M = 1183 - 41 = 1 142 kN-m  
(hogging Mt. resisted by rectangular section)

| Effective Depth       | d =                   | 650 - 40 - 10 - 30                    | (Assume)                          |
|-----------------------|-----------------------|---------------------------------------|-----------------------------------|
|                       | =                     | 570 mm                                |                                   |
|                       |                       |                                       |                                   |
|                       | K =                   | $M / (bd^2 f_{cu})$                   |                                   |
|                       | =                     | 1142 x 10 <sup>6</sup> / (550 x 570   | 0 <sup>2</sup> x 40)              |
|                       | =                     | 0.160                                 |                                   |
| β <sub>b</sub> = 0.7  | >                     | 0.104                                 | (Compression steel is required.)  |
|                       |                       |                                       |                                   |
| Lever                 | arm, z =              | 0.865 d = 0.865 x 570                 |                                   |
|                       | =                     | 493 mm                                |                                   |
|                       |                       |                                       |                                   |
| Depth to neutral a    | axis, x =             | 0.3 d = 0.3 x 570                     | (Eqn 2.1a of Chapter 2)           |
|                       | =                     | 171 mm                                |                                   |
|                       |                       |                                       |                                   |
| Check                 | k d' / x =            | 70 / 171                              |                                   |
|                       | =                     | 0.40 > 0.38                           |                                   |
|                       | $\mathcal{E}_{sc}$ =  | 0.0035 x (1- 0.40)                    |                                   |
|                       | =                     | 0.0021                                |                                   |
|                       |                       |                                       |                                   |
| Stress of comp'n st   | eel, $f_{sc} =$       | $E_{s} \ \mathcal{E}_{s}$             |                                   |
|                       | =                     | 200 000 x 0.0021                      |                                   |
|                       | =                     | 420 N/mm <sup>2</sup>                 |                                   |
|                       |                       |                                       |                                   |
| Compression steel rec | n'd A' -              | $(K - K') f_{cu} bd^2$                |                                   |
| Compression steer rec | μu, Λ <sub>s</sub> –  | <i>f<sub>sc</sub></i> (d - d')        |                                   |
|                       | _                     | (0.160 – 0.104) x 40 :                | x 550 x 570 <sup>2</sup>          |
|                       |                       | 420 x (570 –                          | 70)                               |
|                       | =                     | 1906 mm <sup>2</sup>                  |                                   |
|                       |                       |                                       | (Provide 2T40 + 2T25 bottom)      |
|                       | A' <sub>s,pro</sub> = | 3 496 mm <sup>2</sup>                 |                                   |
|                       |                       |                                       |                                   |
| Tension steel rea     | n'd A. =              | $\frac{K' f_{cu} bd^2}{H}$ + $f_{cu}$ | $A_{s'} = \frac{f_{sc}}{f_{sc}}$  |
|                       |                       | 0.87 f <sub>y</sub> z                 | $0.87 f_y$                        |
|                       | = -                   | 0.104 x 40 x 550 x 5                  | $\frac{170^2}{1906}$ + 1906 - 420 |
|                       |                       | 0.87 x 500 x 493                      | 0.87 x 500                        |
|                       | =                     | 3466 + 1840                           |                                   |
|                       | =                     | 5 306 mm <sup>2</sup>                 |                                   |
|                       |                       |                                       | (Provide 2T40 + 4T32 top)         |

$$A_{s, prov} = 2 \times 1257 + 4 \times 804$$
  
= 5306 mm<sup>2</sup>  
100A<sub>s</sub> / bh = 100 x (5306 + 3496) / (550 x 650) = 2.58  
< 4.0 (Steel ratio ok)

#### Comments:

- (a) As  $\beta_b = 0.7$ , the value of K', the upper limits to the depth of neutral axis, x, and the lever arm, z, are reduced.
- (b) As the upper limit to the neutral is reduced, d'/x becomes so large that the compression bars have not yielded at ultimate limit state.
- (c) As the compression bars have not yielded, the stress in the compression bars has to be determined from the strain, which is then used, instead of the yield stress, in the calculating the amount of compression bar.
- (d) The total amount of steel saved in this section is about 26%. The percentage of steel in this section is reduced from 3.37% to 2.58%.





HD in Civil Engineering (Aug 2014)

# Self-Assessment Questions

Q.1 Given the following design parameters:

Design Moment, M = 860 kN-m ( $\beta_b$  = 1) Concrete:  $f_{cu}$  = 40 MPa Reinforcement,  $f_{\gamma}$  = 500 NPa

Determine the areas of steel required for the following cases:

|                    | (a)         | (b)             | (c)         | (d)             |
|--------------------|-------------|-----------------|-------------|-----------------|
|                    | Rectangular | Flanged Section | Rectangular | Flanged Section |
|                    | Section     | Flanged Section | Section     | Flanged Section |
| b <sub>w</sub> =   | 400         | 400             | 750         | 750             |
| d =                | 680         | 680             | 380         | 380             |
| d' =               | 70          | 70              | 70          | 70              |
| b <sub>eff</sub> = |             | 2000            |             | 2000            |
| h <sub>f</sub> =   |             | 150             |             | 150             |
| K =                |             |                 |             |                 |
| z =                |             |                 |             |                 |
| A' <sub>s</sub> =  |             |                 |             |                 |
| A <sub>s</sub> =   |             |                 |             |                 |

Q.2 Calculate the *nominal shear reinforcement* ( $f_{yy}$  = 250MPa) for the following beam sections:

|                    | (a)                 | (b)             | (C)                 |
|--------------------|---------------------|-----------------|---------------------|
|                    | Rectangular Section | Flanged Section | Rectangular Section |
| b <sub>w</sub> =   | 400                 | 400             | 750                 |
| d =                | 680                 | 680             | 380                 |
| b <sub>eff</sub> = |                     | 2000            |                     |
| h <sub>f</sub> =   |                     | 150             |                     |
| $A_{sv}/s_v =$     |                     |                 |                     |
| Links:             |                     |                 |                     |

Q.3 Given the following information of a beam:

Distance from the end of effective span to the face of support,  $a_i = 200$  mm

Effective depth of the beam section, d = 573 mm

Design load (udl), w = 36 kN/m

Design shear force at the support, V = 250 kN

- (a) Calculate the design shear force at the face of support,  $V_s$ .
- (b) Calculate the design shear force at d from the face of the support,  $V_d$ .
- (c) What is the value of design shear force shall be used to check for crushing of concrete?
- Q.4 Calculate the *shear reinforcement* ( $f_{yy}$  = 250MPa) required for the following sections:

|                         | (a)  | (b)  | (C)  |
|-------------------------|------|------|------|
| b <sub>v</sub> =        | 400  | 400  | 750  |
| d =                     | 680  | 680  | 380  |
| <i>v</i> <sub>c</sub> = | 0.92 | 0.78 | 0.92 |
| <i>v</i> <sub>d</sub> = | 1.82 | 1.45 | 1.28 |
| $A_{sv}/s_v =$          |      |      |      |
| Links:                  |      |      |      |

Answers:

Q1a=0+3430; Q1b=0+3060; Q1c=1366+6641; Q1d=0+5723

Q2a=0.736(R10-200-2/legs); Q2b=0.736(R10-200-2/legs); Q2c=1.379(R10-225-4/legs)

Q3a=242.8; Q3b=222.2; Q3c=242.8;

Q4a=1.655(R12-125-2/legs); Q4b=1.232(R10-125-2/legs); Q4c=1.379(R10-225-4/legs)

### Tutorial Questions

- AQ1 The design parameters, including loading, span, beam size, etc. of the beams in the examples in 4.2.1 and 4.2.2 are quite similar.
  - (a) Identify their similarities.
  - (b) Identify the differences in their design process.
  - (c) Identify and discuss the differences in the result.
- AQ2 Re-design the reinforcement and check the deflection for the beam 5B2 shown in DWG-01 of Chapter 1 with the following changes (refer to Question A of 4.2.2 for the original design):
  - The center-to-center distance between adjacent beams is changed from 3300 mm to 3500 mm, i.e. the distance between gridlines 6 and 7 is changed to 10 500 mm.
  - ii. An additional allowance for 300 mm thick soil is required.
  - iii. The width of the beam is increased to 400mm.

The design parameters are as follows:

| Beam overall depth, h =           | 750 mm                           |  |
|-----------------------------------|----------------------------------|--|
| Beam breadth, b =                 | 400 mm                           |  |
| Slab thickness, h <sub>f</sub> =  | 160 mm                           |  |
| c/c distance btw supports =       | 9 000 mm                         |  |
| Width of support, $S_w$ =         | 500 mm (similar at both ends)    |  |
| c/c distance btw adjacent beams = | 3 500 mm (similar on both sides) |  |
| $f_{cu} =$                        | 35 MPa                           |  |
| $f_y =$                           | 500 MPa                          |  |
| $f_{yv} =$                        | 250 MPa                          |  |
| Cover =                           | 35 mm                            |  |
| Preferred bar size =              | 40                               |  |
| Preferred link size =             | 10                               |  |

Allowance for finishes = 2.0 kPa Soil thickness = 300 mm Characteristic imposed load = 5.0 kPa

AQ3 Design the reinforcement and check the deflection for the *interior span*, RB22, of the continuous beam in 4.3.1, and as shown in DWG-03. Adopt the design parameters in 4.3.1.