0.1 Inequalities

Ex: Solve for x the inequality $2x - 3 < 7$

$$2x < 10$$

$$x < 5$$

∴ the set of sol. $= \{x : x \in R , x < 5\}$

$= (-\infty, 5)$

Ex: Solve for x $3+7x \leq 2x - 9$

$$7x - 2x \leq -9 - 3$$

$$5x \leq -12$$

$$x \leq -\frac{12}{5}$$

∴ the set of sol. $= \{x : x \in R, x \leq -\frac{12}{5}\} = (-\infty , -\frac{12}{5}]$

Ex: Solve for x $7 \leq 2-5x < 9$

$$5 \leq -5x < 7$$

$$-5 \geq 5x > -7$$

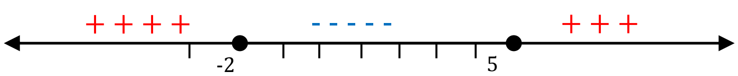
$$-1 \geq x > -\frac{7}{5}$$

∴ the set of sol. $= \{x : x \in R, -\frac{7}{5} < x \leq -1\} = (-\frac{7}{5} , -1]$

Ex: Solve for x $x^2 - 3x - 10 \geq 0$

$$(x+2)(x-5) \geq 0$$

equal to zero at $x = -2$ $x = 5$

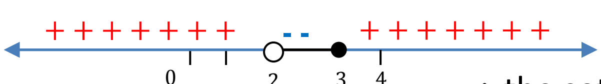


∴ set of sol. $= (-\infty, -2] \cup [5 , \infty)$

Ex: Solve for x $\frac{2x-5}{x-2} \leq 1$

$$\frac{2x-5}{x-2} - 1 \leq 0$$

$$\frac{(2x-5)-(x-2)}{(x-2)} \leq 0 \implies \frac{x-3}{x-2} \leq 0$$



∴ the set of sol. $= (2, 3]$

Ex: Solve for x the inequality $x^3 - 3x + 2 \leq 0$

$x = 1$ is a solution for the equation so $(x-1)$ is a factor.

$$x^3 - 3x + 2 \leq 0$$

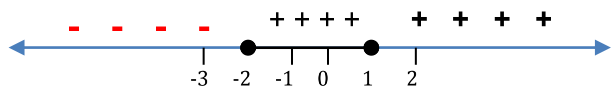
$$(x-1)(x^2 + x - 2) \leq 0$$

$$(x-1)(x-1)(x+2) \leq 0$$

equal to zero at $x=1$, $x=-2$

$$\begin{array}{r}
x^2 + x - 2 \\
(x-1)\overline{)\,x^3 - 3x + 2} \\
\mp x^3 \pm x^2 \\
\hline
x^2 - 3x + 2 \\
\underline{\mp x^2 \pm x} \\
-2x + 2 \\
\underline{\pm 2x \mp 2} \\
0 + 0
\end{array}$$

∴ the set of sol. $= (-\infty, -2]$



HW: Solve for x

1) $\frac{3x+1}{x-2} < 1$
2) $x^2 \leq 5$
3) $2 - 3x + x^2 \geq 0$
4) $\frac{1}{x+1} \geq \frac{3}{x-2}$
5) $x^3 - x^2 - x - 2 > 0$

Absolute Value

$$|x| = \begin{cases} x & x \geq 0 \\ -x & x < 0 \end{cases}$$

1) $|a| = \sqrt{a^2}$
2) $|a.b| = |a|\,|b|$
3) $\left|\frac{a}{b}\right| = \frac{|a|}{|b|}$
4) $|a + b| \leq |a| + |b|$
5) If $|x| \leq a$ then $-a \leq x \leq a$
6) If $|x| \geq a$ either $x \geq a$ or $x \leq -a$

***ex*: solve $|x-3| = 4$**

either: $(x-3) = 4$, $x = 7$, $x = 7$

Or: $-(x-3) = 4$, $-x = 1$, $x = -1$

$\therefore$ set of sol. $= \{-1, 7\}$

***Ex:* solve for x** $|x-3| < 4$

$-4 < x-3 < 4$

$-1 < x < 7$

$\therefore$ set of sol. $= \{x : -1 < x < 7\} = (-1, 7)$

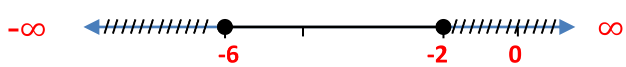
***Ex*: solve for x** $|x+4| \geq 2$

Either: $x+4 \geq 2$, $x \geq -2$

Or: $x+4 \leq -2$, $x \leq -6$

$\therefore$ set of sol. $= \{x : x \geq -2\} \cup \{x : x \leq -6\}$

$= (-\infty, -6] \cup [-2, \infty)$



***Ex*: solve for x** $\frac{2}{|x+3|} < 1$

$\frac{|x+3|}{2} > 1$

$|x+3| > 2$

Either: $x+3 > 2$, $x > -1$

Or: $x+3 < -2$, $x < -5$

$\therefore$ set of sol. $= \{x : x < -5\} \cup \{x : x > -1\}$

$= (-\infty, -5) \cup (-1, \infty)$

Ex: **solve for x** $|x+3| < |x-8|$

$$\sqrt{(x+3)^2} < \sqrt{(x-8)^2} \quad \text{using } |a| = \sqrt{a^2}$$

$$(x+3)^2 < (x-8)^2$$

$$x^2 + 6x + 9 < x^2 - 16x + 64$$

$$22x < 55$$

$$x < \frac{5}{2}$$

∴ set of sol. = $(-\infty, \frac{5}{2})$

HW: solve for x

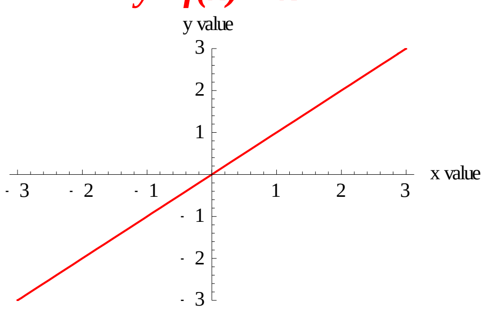
1) $|3x| \le |2x-5|$
2) $\left|\frac{3-2x}{1+x}\right| \le 4$
3) $\frac{1}{|x-3|} - \frac{1}{|x+4|} \ge 0$
4) $\frac{1}{|x-4|} < \frac{1}{|x+7|}$
5) Solve $|x-3|^2 - 4|x-3| = 12$

0.2 Function

Def: *is a rule that assigns to each element in a set A (domain) one and only one element in a set B (range)*

Some important functions

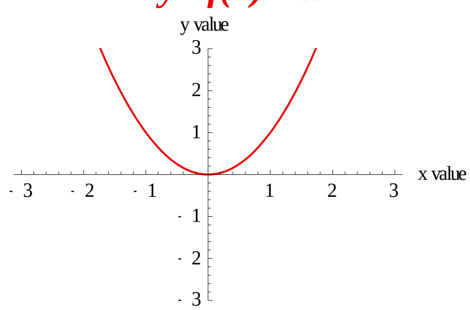
$y = f(x) = x$



$D_f = R$

$R_f = R$

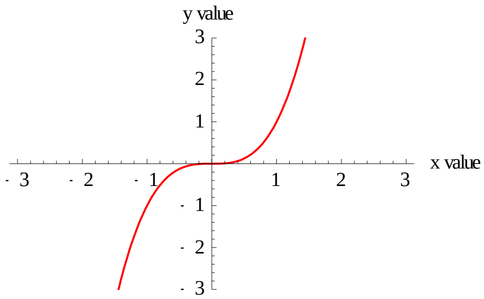
$y = f(x) = x^2$



$D_f = R$

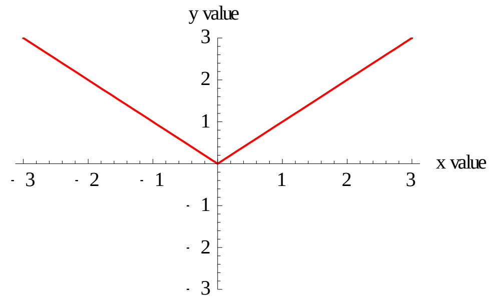
$R_f = \{y : y \ge 0\} = [0, \infty)$

$y = f(x) = x^3$



$D_f = R_f = R$

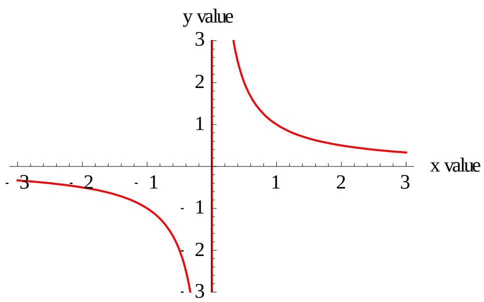
$y = f(x) = |x|$



$D_f = R$

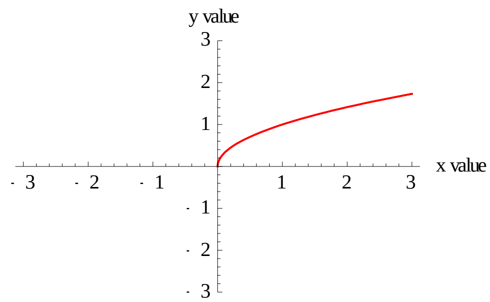
$R_f = \{y : y \geq 0\}$

$y = f(x) = \frac{1}{x}$



$D_f = R_f = R \setminus \{0\}$

$y = f(x) = \sqrt{x}$



$D_f = \{x : x \in R, x \geq 0\}$

$R_f = \{y : y \in R, y \geq 0\}$

Note:* any polynomial of the following forms have the domain *R

Ex:

$$\left.\begin{aligned} f(x) &= \frac{1}{2}x^3 + 3x^2 - x + \pi \\ f(x) &= 5x^2 - 2x - \sqrt{2} \\ f(x) &= \frac{3}{2}x^5 + x^3 - x + 1 \end{aligned}\right\} D_f = R$$

ex : Find the domain & range of $f(x) = \sqrt{x - 3}$

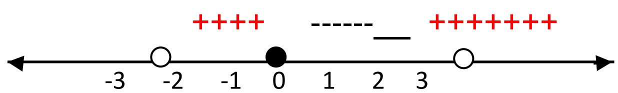
$$x - 3 \geq 0$$

$$x \geq 3$$

$\therefore \quad D_f = \{x : x \in R, x \geq 3\} \quad , \quad R_f = \{y : y \in R, y \geq 0\}$

Ex: Find the domain of $f(x) = \sqrt{\frac{x+1}{x^2-9}}$

$$\frac{x+1}{x^2-9} \geq 0$$



$$D_{f} = (-3,-1] \cup (3,\infty)$$

Ex : Find domain & range of $y = f(x) = \frac{x+1}{x-3}$

$$D_f = R \setminus \{3\}$$

$y = \frac{x+1}{x-3} \rightarrow$ $x+1 = xy - 3y$

$$x - xy = -3y - 1$$

$$x(1-y) = -(3y+1)$$

$$x = \frac{-(3y+1)}{1-y}$$

$$x = \frac{3y+1}{y-1}$$

$$\therefore \quad R_f = R \setminus \{1\}$$

1) $D_{f \pm \cdot g} = D_f \cap D_g$

2) $D_{\frac{f}{g}} = D_f \cap D_g \setminus \{x : g(x) = 0\}$

Ex: find domain of the function.

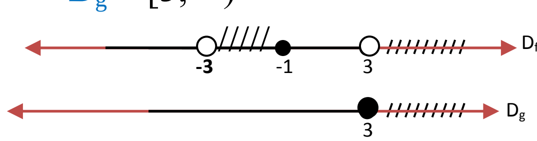
$$h(x) = \sqrt{\frac{x+1}{x^2-9}} + \sqrt{x-3}$$

$D_f = (-3, -1] \cup (3, \infty)$

$D_g = [3, \infty)$

$D_h = D_f \cap D_g$

$= (3, \infty)$



Ex: find domain of the function.

$$h(x) = \sqrt{\frac{x+1}{x^2-9}} - \sqrt{x-3}$$

or $$h(x) = \sqrt{\frac{x+1}{x^2-9}} \cdot \sqrt{x-3}$$

$D_h = (3, \infty)$