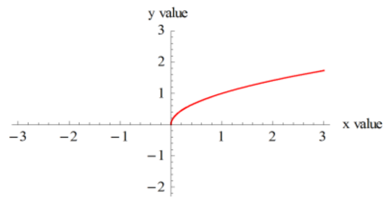
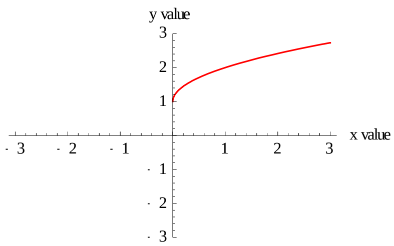
0.3 Shifting graphs

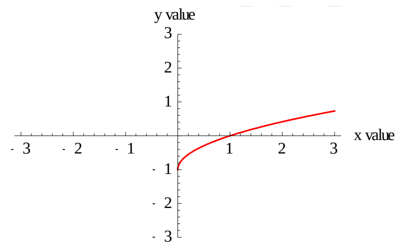
$f(x) = \sqrt{x}$



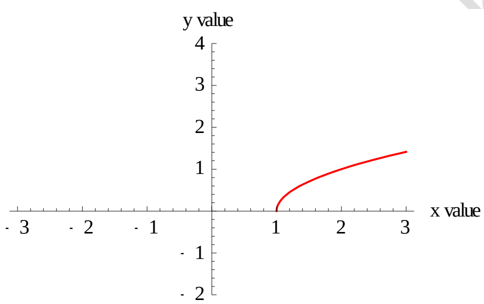
1) $f(x) = \sqrt{x} + 1$



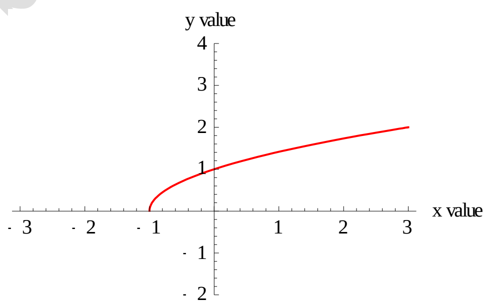
2) $f(x) = \sqrt{x} - 1$



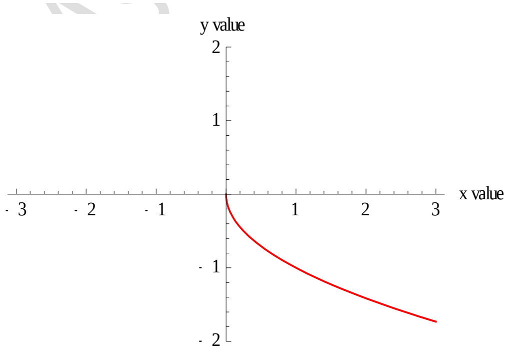
3) $f(x) = \sqrt{x-1}$



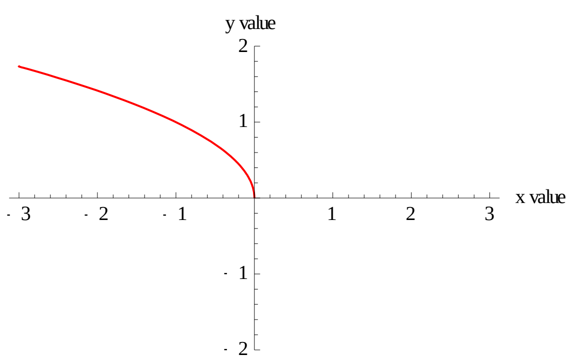
4) $f(x) = \sqrt{x+1}$



5) $f(x) = -\sqrt{x}$



6) $f(x) = \sqrt{-x}$



Ex: Sketch the graph of the function.

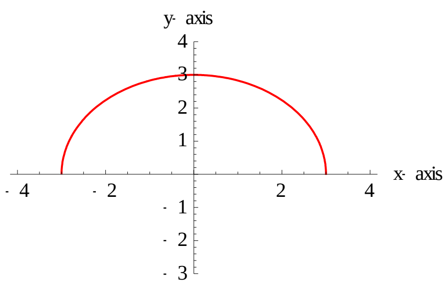
$f(x) = \sqrt{9 - x^2}$, ***find domain and range***

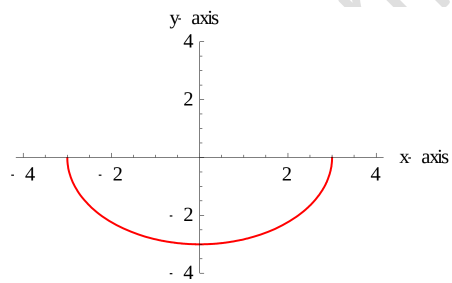
$$y = \sqrt{9 - x^2}$$

$$y^2 = 9 - x^2$$

$x^2 + y^2 = 9$ is a circle of center (0,0) and radius r = 3

$y = +\sqrt{9 - x^2}$ $y = -\sqrt{9 - x^2}$





D_f = [-3, 3], R_f = [0, 3]

Ex: sketch the graph of $y = \sqrt{7 - 6x - x^2}$

$$7 - 6x - x^2 = -[x^2 + 6x] + 7$$

$$= -[x^2 + 6x + 9 - 9] + 7$$

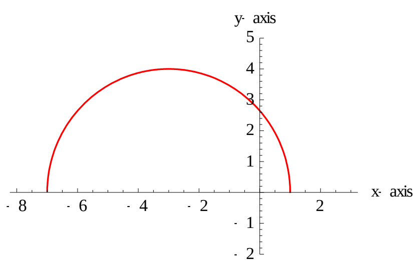
$$= -[(x+3)^2 - 9] + 7$$

$$= -(x+3)^2 + 9 + 7$$

$$= 16 - (x+3)^2$$

$\therefore$ $y = \sqrt{16 - (x+3)^2}$ or $y^2 = 16 - (x+3)^2$

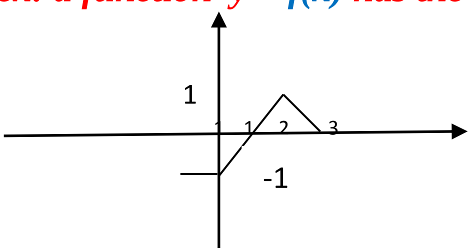
$(x+3)^2 + y^2 = 16$



Hw.:

1) use the graph of $y = |x|$ to sketch the graph the following
   a. $y = |x - 4|$
   b. $y = |x| + 4$
2) *use the graph of* $y = \sqrt{x}$ *to sketch the graph the following*
   a. $y = \sqrt{x - 3}$
   b. $y = \sqrt{x} + 3$
   c. $y = \sqrt{x - 3} + 3$
   d. $y = \sqrt{x + 1} - 2$

ex: a function $y = f(x)$ has the graph

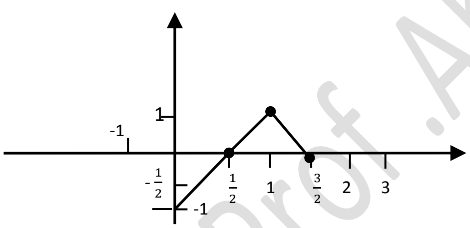


$D_f = [-1, 3]$

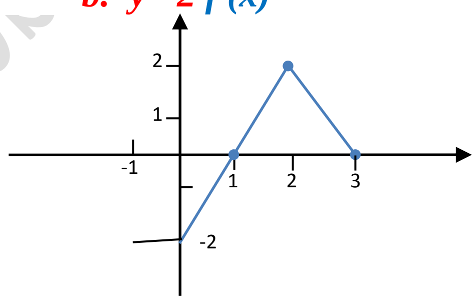
$R_f = [-1, 1]$

Use this graph to obtain the graphs of:

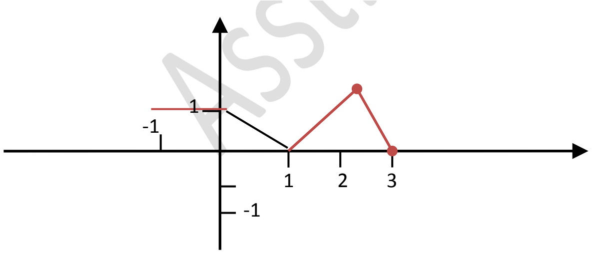
a. $y = f(2x)$



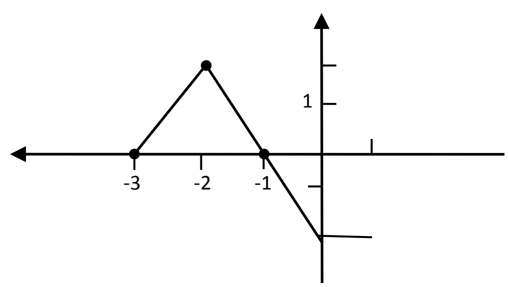
b. $y = 2f(x)$



c. $y = |f(x)|$



d. $y = 2f(-x)$



Def *: a function **f(x)** is said to be **even function** If **f(-x) = f(x)** and it is **symmetric about y – axis***

Ex: ***the function $f(x) = x^4 + x^2 - 3$ is even function.***

$$f(-x) = (-x)^4 + (-x)^2 - 3$$
$$= x^4 + x^2 - 3$$
$$= f(x)$$

Since $f(-x) = f(x)$ ∴ $f(x)$ is *even* fun.

Def: *a function* ***f(x)*** *is said to be* ***odd function*** *If* ***$f(-x) = -f(x)$*** *and it is* ***symmetric about the origin***

Ex: The function $f(x) = \sin x - 3x^3 + x$ is odd function.

$$f(-x) = \sin(-x) - 3(-x)^3 + (-x)$$
$$= -\sin x + 3x^3 - x = -[\sin x - 3x^3 + x] = -f(x)$$

Since $f(-x) = -f(x)$ ∴ *f is* *odd* *fun.*

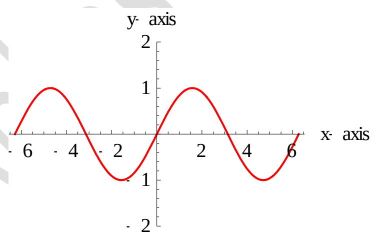
0.4 Trigonometric Functions

1) $y = f(x) = \sin x$

$D_f = R$

$R_f = [-1, 1]$

∵ $\sin(-x) = -\sin x$



∴ sin x is odd fun. and sym. about the origin

Def: a function ***f(x)*** is said to be **periodic** with period p if p is the smallest number such that ***$f(x + p) = f(x)$***.

Ex: $f(x) = \sin x$ is periodic function with period 2π.

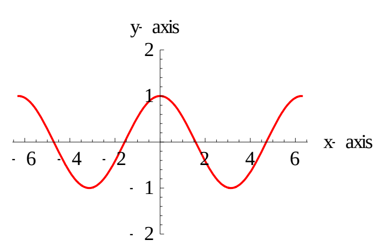
since $\sin(x + 2\pi) = \sin x$

2) $y = f(x) = \cos x$

$D_f = R$

$R_f = [-1, 1]$

∵ $\cos(-x) = \cos x$



cos x is even function and symmetric about the y- axis

3) y = f(x) = tan x $= \frac{\sin x}{\cos x}$

$D_f = R \setminus \{x : x = \frac{n\pi}{2}, n = \pm 1, \pm 3, \pm 5, \ldots\}$

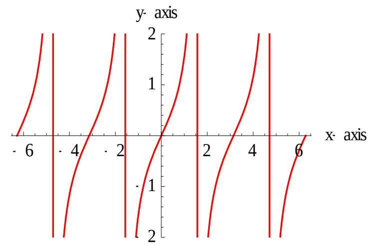
$R_f = R$

∵ tan (-x) = - tan x

∴ tan x is odd function and symmetric about the origin

∵ tan (x+ π) = tan x

∴ tan x is periodic with period π



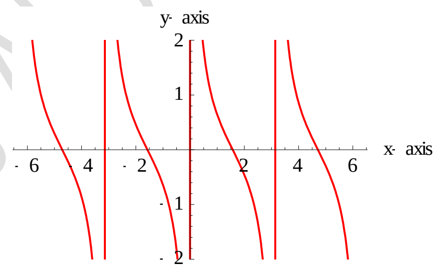
4) y= f(x) = cot x $= \frac{\cos x}{\sin x}$

$D_f = R \setminus \{x : x = n\pi, n = 0, \pm 1, \pm 2, \ldots\}$

$R_f = R$

∵ cot (-x) = - cot x

∴ *cot x is **odd** function and Symmetric about the origin*



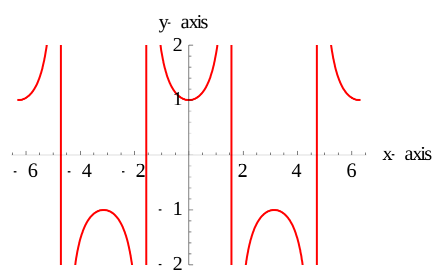
∵ *cot (x+ π) = cot x*

∴ *cot x is **periodic** with period π.*

5) y = f (x) = sec x $= \frac{1}{\cos x}$

$D_f = R \setminus \{x : x = \frac{n\pi}{2}, n = \pm 1, \pm 3, \ldots\}$

$R_f = R \setminus (-1, 1)$



∵ *sec (-x) = sec x*

∴ *sec x is **even** function.*

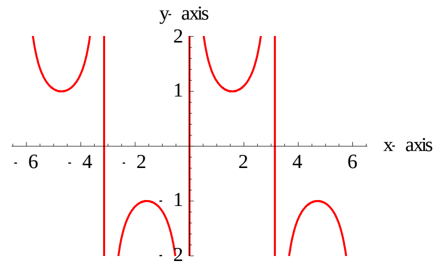
∵ *sec (x+2π) = sec x*

∴ *sec x is **periodic** function with period 2π*

6) y = f(x) = csc x = $\frac{1}{\sin x}$

D_f = R\ {x : x = n π, n= 0, ± 1, ∓ 2, ..}

R_f = R\ (-1, 1)



∵ *csc (-x) = - csc x*

∴ *csc x is* ***odd*** *function.*

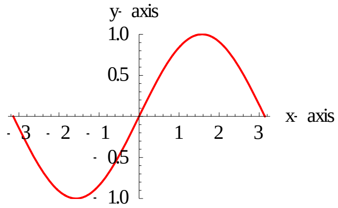
∵ *csc (x +* 2π *) = csc x*

∴ *csc x is* ***periodic*** *function with period* 2π

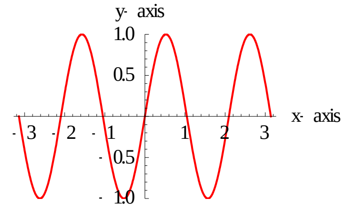
Trigonometric identities:

1. $\cos^2 x + \sin^2 x = 1$
2. $\cot^2 x + 1 = \csc^2 x$
3. $1 + \tan^2 x = \sec^2 x$
4. $\sin 2x = 2 \sin x \cos x$
5. $\cos 2x = \cos^2 x - \sin^2 x$
   $= 2\cos^2 x - 1$
   $= 1 - 2\sin^2 x$
6. $\sin (x \pm y) = \sin x \cos y \pm \cos x \sin y$
7. $\cos (x \pm y) = \cos x \cos y \mp \sin x \sin y$
8. $\tan (x \pm y) = \frac{\tan x \pm \tan y}{1 \mp \tan x \tan y}$
9. $\sin^2 x = \frac{1}{2}(1 - \cos 2x)$
10. $\cos^2 x = \frac{1}{2}(1 + \cos 2x)$
11. $\sin x \cos y = \frac{1}{2}[\sin (x-y) + \sin (x+y)]$
12. $\sin x \sin y = \frac{1}{2}[\cos (x-y) - \cos (x+y)]$
13. $\cos x \cos y = \frac{1}{2}[\cos (x-y) + \cos (x+y)]$
14. $\cos (\frac{\pi}{2} - x) = \sin x$
15. $\sin (\frac{\pi}{2} - x) = \cos x$
16. $\sin (\pi - x) = \sin x$

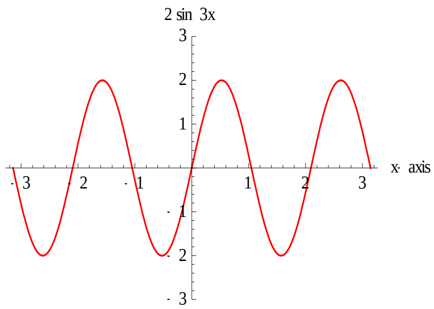
Ex: ***Sketch the graph of*** ***y= 2 sin (3x) +1***



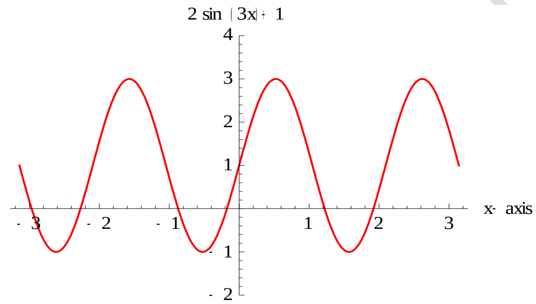
y = sin (x)



y = sin (3x)



y = 2 sin (3x)



y = 2 sin (3x) +1