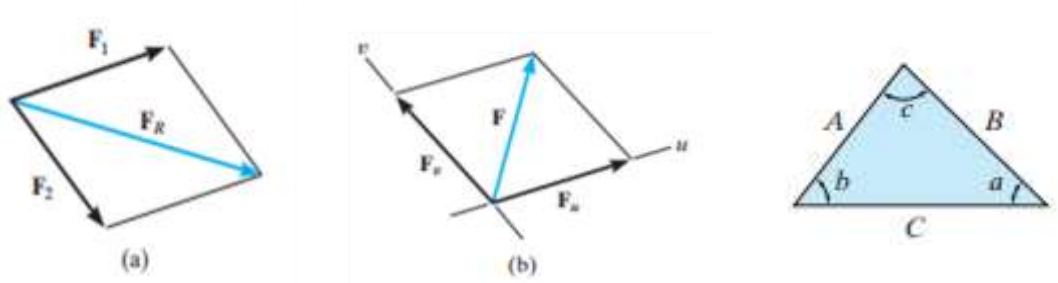
Parallelogram Law

The resultant of a pair of concurrent forces can be determined by means of the *Parallelogram Law*, which states that the resultant is proportional to the diagonal of the parallelogram whose sides are proportional to the two forces.

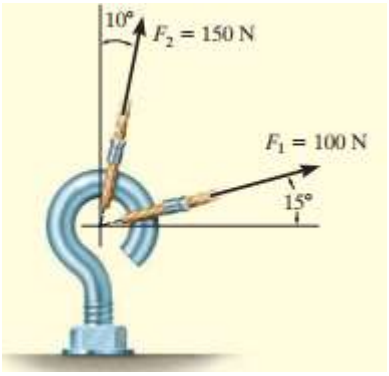
• From this triangle, the magnitude of the resultant force can be determined using the law of cosines, and its direction is determined from the law of sines. The magnitudes of two force components are determined from the law of sines.



Cosine Law: $$C = \sqrt{A^2 + B^2 - 2AB\cos c}$$

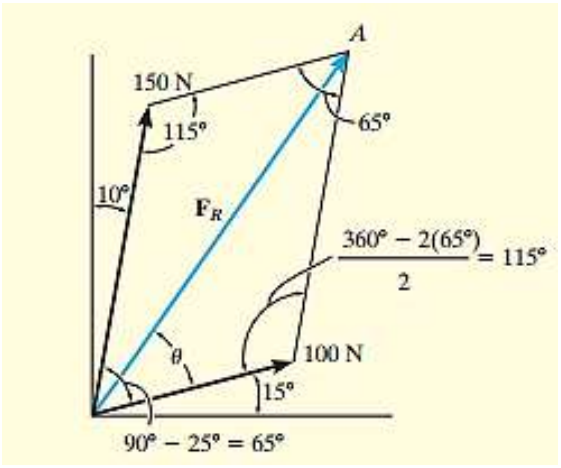
Sine Law: $$\frac{A}{\sin a} = \frac{B}{\sin b} = \frac{C}{\sin c}$$

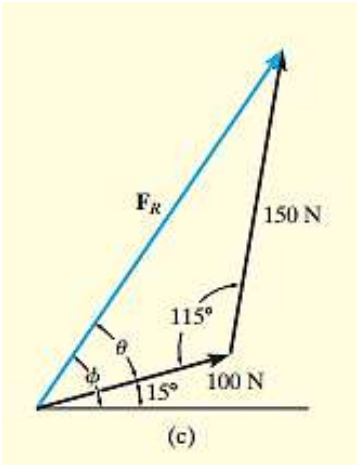
Example 1: Determine the magnitude and direction of the resultant force for the forces shown in the figure.



Solution:

The parallelogram is formed by drawing a line from the head of F_1 that is parallel to F_2, and another line from the head of F_2 that is parallel to F_1. The resultant force F_R extends to where these lines intersect at point A. The two unknowns are the magnitude of F_R and the angle θ (theta).





$$F_R = \sqrt{(100)^2 + (150)^2 - 2 * 100 * 150 cos\ 115}$$

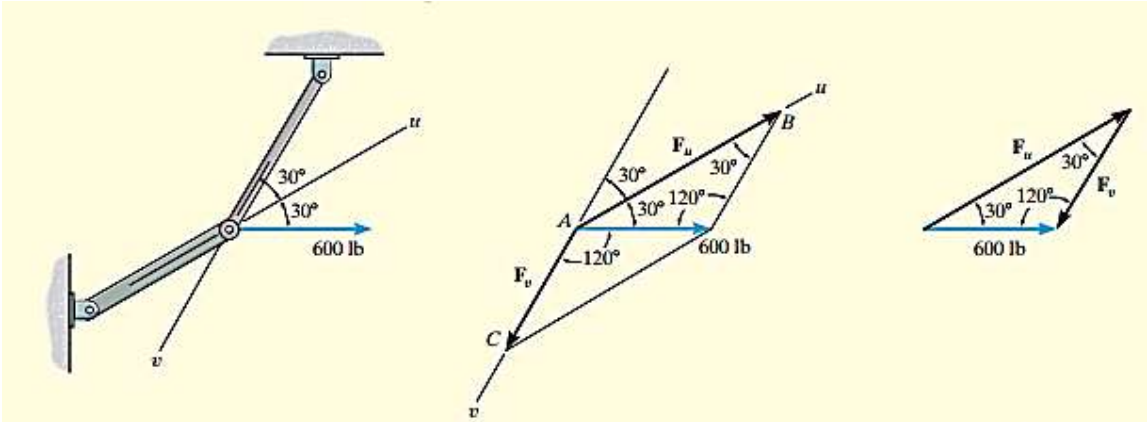
$$=212.6\ N$$

$$\frac{150}{\sin\theta} = \frac{212.6}{\sin 115} \quad \rightarrow \theta = 39.8^0$$

Thus, the direction Φ (phi) of F_R, measured from the horizontal, is

$$\Phi = 39.8^0 + 15.0^0 = 54.8^0$$

Example 2: Resolve the horizontal 600-lb force shown in the figure into components acting along the u and v axes and determine the magnitudes of these components.



Solution:

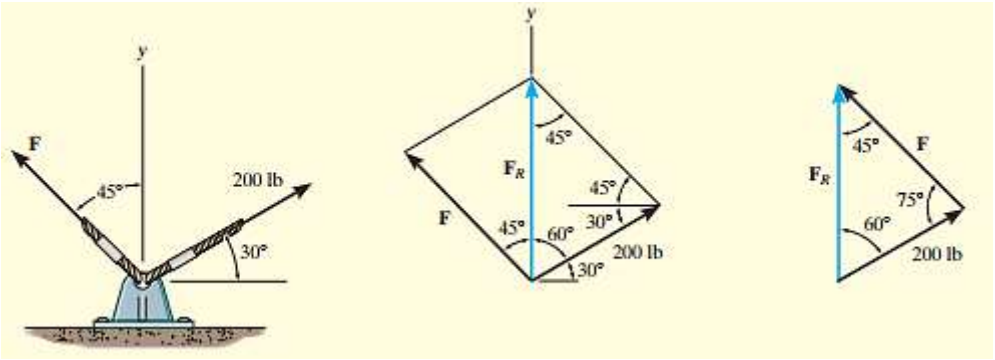
The parallelogram is constructed by extending a line from the head of the 600-lb force parallel to the v axis until it intersects the u axis at point B. The arrow from A to B represents $\boldsymbol{F_u}$. Similarly, the line extended from the head of the 600-lb force drawn parallel to the u axis intersects the v axis at point C, which gives $\boldsymbol{F_v}$.

$$\frac{F_u}{\sin 120} = \frac{600}{\sin 30} \quad \rightarrow F_u = 1039\ Ib$$

$$\frac{F_v}{\sin 30} = \frac{600}{\sin 30} \quad \rightarrow F_v = 600 Ib$$

Example 3: Determine the magnitude of the component force $\boldsymbol{F}$ and the magnitude of the resultant force $\boldsymbol{F_R}$ if $\boldsymbol{F_R}$ is directed along the positive $\boldsymbol{y}$ axis.

Solution:



$$\frac{F}{\sin 60} = \frac{200}{\sin 45} \quad \rightarrow F_u = 245\ Ib$$

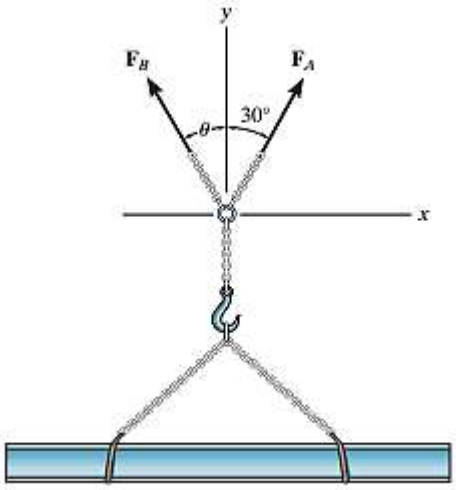
$$\frac{F_R}{\sin 75} = \frac{200}{\sin 45} \quad \rightarrow F_R = 273\ Ib$$

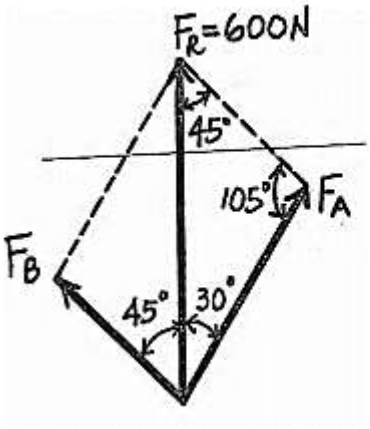
Example 4: The beam is to be hoisted using two chains. Determine the magnitudes of forces $\boldsymbol{F_A}$ and $\boldsymbol{F_B}$ acting on each chain in order to develop a resultant force of 600 N directed along the positive $\boldsymbol{y}$ axis. Set $\boldsymbol{\theta}$ = 45°.

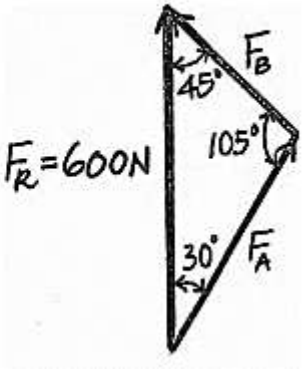
Solution:

$$\frac{F_A}{\sin 45} = \frac{600}{\sin 105} \quad \rightarrow F_A = 439\ Ib$$

$$\frac{F_B}{\sin 30} = \frac{600}{\sin 105} \quad \rightarrow F_B = 311\ Ib$$

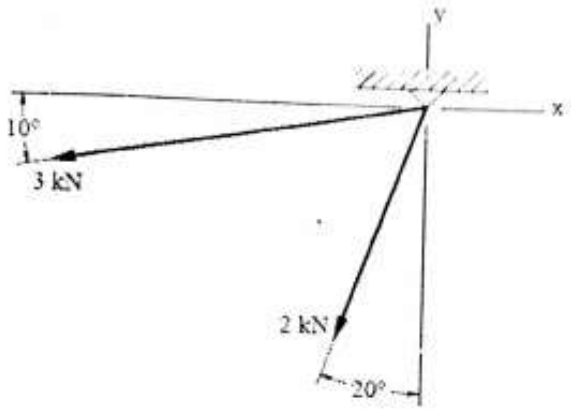






Homework:

1- Determine the magnitude and direction of the resultant of the forces shown in the figure below.



2- Determine the magnitude and direction of the resultant of the forces shown in the figure below.