2.1.1 Uniformly Accelerated Rectilinear Motion.

When the acceleration is constant $a=a_c$, each of the following three kinematic equations

$$a_c = \frac{dv}{dt} \qquad v = \frac{dS}{dt} \qquad a_c = v\frac{dv}{dS}$$

can be integrated to obtain formula that relates a_c,v,S, and t.

Velocity as a Function of Time.

Assume $v=v_0$ when t=0

$$a_c = \frac{dv}{dt}$$

$$\int_{v_0}^{v} dv = \int_0^t a_c dt$$

$$\overset{+}{\rightarrow} \quad \boldsymbol{v = v_0 + a_c t} \qquad (\mathbf{1})$$

Position as a Function of Time

Assume $S=S_0$ when t=0

$$v = \frac{dS}{dt} = v_0 + a_c t$$

$$\int_{S_0}^{S} dS = \int_0^t (v_0 + a_c t) dt$$

$$\overset{+}{\rightarrow} \quad \boldsymbol{S = S_0 + v_0 t + \frac{1}{2} a_c t^2} \qquad (\mathbf{2})$$

Velocity as a Function of Position

Assume $v=v_0$ at $S=S_0$

$$a_c = v\frac{dv}{dS}$$

$$\int_{v_0}^{v} v dv = \int_{S_0}^{S} a_c dS$$

$$\overset{+}{\rightarrow} \quad \boldsymbol{v^2 = v_0^2 + 2a_c(S - S_0)} \qquad (\mathbf{3})$$

Highway and Transport. Engineering Department
College of Engineering
Mustansiriyah University
Lec.Rana Hashim

A typical example of constant accelerated motion occurs when a body falls freely toward the earth, in this case the downward acceleration is $9.81m/s^2$ or $32.2ft/s^2$.

Remember that these equations are useful only the acceleration is constant and when $t=0, S=S_0, v=v_0$.

Examples

Example(1):-

During a test a rocket travels upward at 75m/s, and when it is 40m from the ground its **engine fails**. Determine the maximum height S_B reached by the rocket and its speed just before it hits the ground. While in motion the rocket is subjected to a constant downward acceleration of $9.81m/s^2$ due to gravity. Neglect the effect of air resistance.

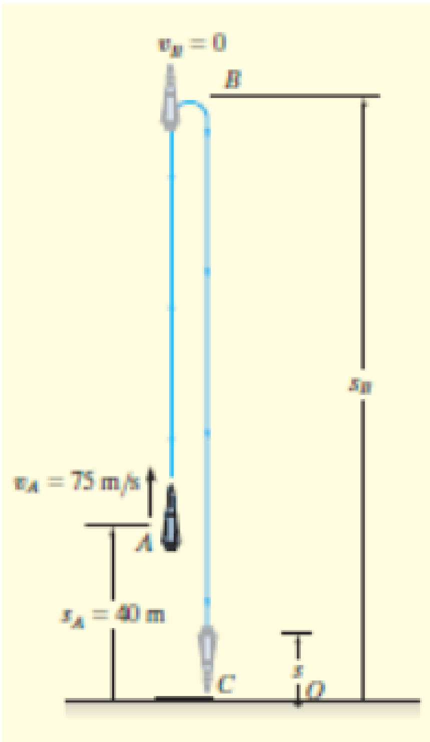
Solution:-

Maximum height

$+\uparrow \quad v_B^2 = v_A^2 + 2a_c(S_B - S_A)$

$0 = (75)^2 + 2(-9.81)(S_B - 40)$

$S_B = 327\text{m}$



Velocity

$+\uparrow \quad v_C^2 = v_B^2 + 2a_c(S_C - S_B)$

$= 0 + 2(-9.81)(0 - 327) = -80.1\frac{m}{s} = 80.1\frac{m}{s}\downarrow$

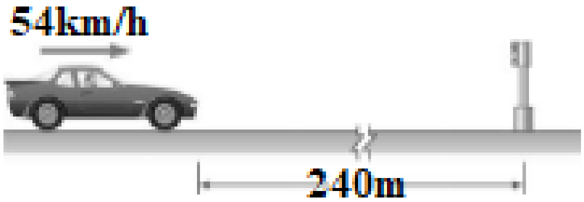
م.رنا هاشم
قسم الطرق والنقل
كلية الهندسة .الجامعة المستنصرية

Highway and Transport. Engineering Department
College of Engineering
Mustansiriyah University
Lec.Rana Hashim

Example(2):-

A motorist is travelling at 54km/h when she observes that a traffic light 240m ahead of her turns red. The traffic light is timed to stay red for 24s. If the motorist wishes to pass the light without stopping just as it turns green again, determine (**a**) the required uniform deceleration of the car, (**b**) the speed of the car as it passes the light.

Solution:-



Uniformly accelerated motion

$S_o=0$, $v_o=54km/h=15m/s$

a) $S = S_0 + v_0 t + \frac{1}{2} a_c t^2$

when $t = 24s, S = 240m$

$$240m = 0 + (15\ m/s)(24s) + \frac{1}{2} a(24s)^2$$

$.4167m/s^2$

b) $v = v_0 + a_c t$

when t=24s

v=(15m/s)+(-0.4167 m/s)(24s)

v=5 m/s

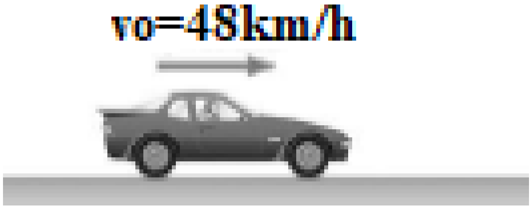
v=18km/h

م.رنا هاشم
قسم الطرق والنقل
كلية الهندسة .الجامعة المستنصرية

Highway and Transport. Engineering Department
College of Engineering
Mustansiriyah University
Lec.Rana Hashim

Example(4):-

A motorist enters a freeway at 48km/h and accelerates uniformly to 96km/h. from the odometer in the car. The motorist knows that she traveled 167m while accelerating .determine (**a**) the acceleration of the car, (**b**) the time required to reach 96km/h.



Solution:-

a)Acceleration of the car

$$v^2 = v_0^2 + 2a_c(S - S_o)$$

$$a_c = \frac{v^2 - v_0^2}{2(S - S_0)}$$

v_o=48km/h=48 $\frac{km}{h} \times \frac{1000m}{km} \times \frac{h}{3600s}$

v_o=13.33m/s

v=96km/h=26.66 m/s

S_o=0

S=167m

$$a_c = \frac{26.66^2 - 13.33^2}{2(167 - 0)} = 1.6m/s^2$$

b)Time to reach 96 km/h

$$v = v_0 + a_c t$$

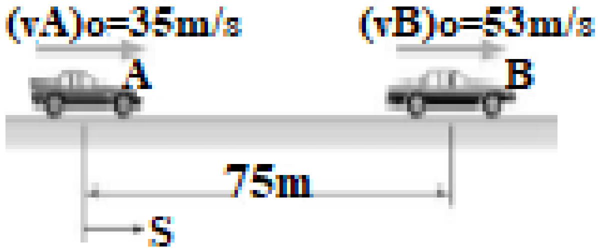
$$t = \frac{v - v_o}{a_c} = \frac{26.66 - 13.33}{1.6} = 8.33s$$

م.رنا هاشم
قسم الطرق والنقل
كلية الهندسة .الجامعة المستنصرية

Example(5):-

Automobiles A and B are travelling in adjacent highway lanes and at t=o have the positions and speeds shown. Knowing that automobile A has a constant acceleration of 1.8 m/s^2 and that B has a constant deceleration of 1.2 m/s^2, determine (**a**) when and where A will overtake B (**b**) the speed of each automobile at that time(***H.W4***).

Solution:-



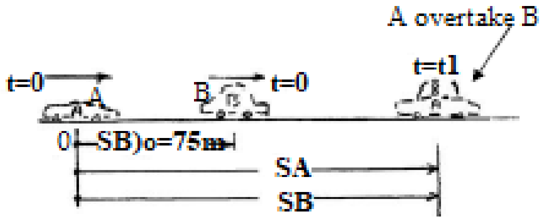
Motion of A

$v_A = (v_A)_o + a_A t$

=35+1.8t

$$S_A = (S_A)_0 + (v_A)_O t + \frac{1}{2} a_A t^2$$

$$S_A = 0 + 35t + \frac{1}{2}(1.8)t^2 \qquad (1)$$



Motion of B

$v_B = (v_B)_o + a_B t$

=53-1.2t

$$S_B = (S_B)_0 + (v_B)_O t + \frac{1}{2} a_B t^2$$

$$S_B = 75 + 53t + \frac{1}{2}(-1.2)t^2$$

(**a**) A overtake B at t=t1

$S_A = S_B$

$35t + 0.9t_1^2 = 75 + 53t_1 - 0.6t_1^2$

t1=-3.22 s and t1=15.2s

subs in (1)

SA=35(15.2)+0.9(15.2)^2=740m