Chapter One

Per unit System

Per unit System

The solution of an interconnected power system having several different voltage levels requires the cambersome transformation of all impedance to a single voltage level. However, power system engineers have devised the per-unit system such that the various physical quantities such a power, voltage, current and impedance are expressed as a decimal fraction or multiples of base quantities. In this system, the different voltage levels disappear, and a power network involving generators, transformer, and lines (of different voltage levels) reduces a system of simple impedance. The per unit value of any quantity is defined as

$$\text{Quantity in per unit} = \frac{\text{actual quantity}}{\text{base value quantity}} \quad \text{— (1)}$$

for example : $S_{pu} = \dfrac{S}{S_B}$; $V_{pu} = \dfrac{V}{V_B}$

$I_{pu} = \dfrac{I}{I_B}$; $Z_{pu} = \dfrac{Z}{Z_B}$

where S_{pu} : per unit apparent power

S : actual apparent power

S_B : Base " "

V_{pu} : per unit voltage

V : actual voltage

V_B : Base Voltage

I_{pu} : per unit current

I : actual current

I_B : Base current

Z_{pu} : per unit impedance

Z : actual impedance

Z_B : Base impedance

where the numerator (actual values) are phasor quantities or complex values and the denominators (base values) are always real number. A minimum of four base quantities are required to completely define a per unit system: volt amper, voltage, current and impedance.

Usually, the 3-φ base volt-ampere S_B or MVA_B and the line-to-line base voltage V_B or kV_B are selected.

Base current and base impedance are then dependent on S_B and V_B and must obey the circuit law.

These are given by:

$$I_B = \frac{S_B}{\sqrt{3}\, V_{B\,Line}}$$

$$Z_B = \frac{V_B/\sqrt{3}}{I_B} \Rightarrow Z_B = \frac{V_B/\sqrt{3}}{\frac{S_B}{\sqrt{3}\,V_B}}$$

$$Z_B = \frac{V_B^{\,2}}{S_B} = \frac{kV_B^{\,2}}{MVA_B}$$

Then $S_{pu} = V_{pu}\, I_{pu}^{*}$

$$V_{pu} = Z_{pu}\, I_{pu}$$

The complex Load power $S_{L(3\varphi)}$ can be defined as

$$\underset{(Load)}{S_{L(3\varphi)}} = 3 V_p I_p^{*}$$

I_p : Load current per phase
V_p : Load voltage per phase

$$\underset{(per\,phase)}{I_p} = \frac{V_p}{Z_p}$$

$$Z_p = \frac{V_p}{I_p} = \frac{V_p}{\left(\frac{S_{L(3\varphi)}}{3V_p}\right)^{*}}$$

$S_{L(3\varphi)}$: 3φ Load apparent power

Impedance are calculate per phase

$$Z_P = \frac{V_P}{\frac{S_L(3\varphi)^*}{3V_P^*}} = \frac{3V_P * V_P^*}{S_L(3\varphi)^*}$$

$$Z_P = \frac{3|V_P|^2}{S_L^*(3\varphi)} = \frac{V_{Line}^2}{S_L^*(3\varphi)} = \frac{V_{LL}^2}{S_L^*(3\varphi)}$$

$$Z_P = \frac{V_{LL}^2}{S_L^*(3\text{-}\varphi)}$$

$$Z_P = \frac{Z_P}{Z_B} = \frac{\frac{V_{L\text{-}L}^2}{S_L^*(3\text{-}\varphi)}}{\frac{V_B^2}{S_B}} = \frac{\text{actual}}{\text{Base}}$$

$$Z_{pu} = \frac{V_{L\text{-}L}^2}{V_B^2} * \frac{S_B}{S_L^*(3\text{-}\varphi)}$$

$$Z_{pu} = \frac{V_{pu}^2}{S_L^*(pu)}$$

S_L : Load apparent power (per unit)

$S_{pu} = V_{pu} * I_{pu}^{*}$ (per unit)

$S_B = \sqrt{3}\, V_{B\,Line}\, I_B$ (Base value)

$S_{L_{3\phi}} = 3 V_p I_p^{*}$ (actual value) per phase

$S_{L_{3\phi}} = \sqrt{3}\, V_{Line}\, I_{Line}^{*}$ (actual value) per line

$$Z_{pu} = \frac{V_{pu}}{I_{pu}}$$

$$Z_B = \frac{V_B}{I_B} = \frac{V_B^2}{S_B}$$

$$Z_{actual} = \frac{V_p}{I_p}$$ per phase

$$Z_{\substack{Load \\ actual}} = \frac{V_{Line}^2}{S^{*}_{Line\,3\phi}}$$

* Change of Base

For power system analysis, all impedances must be expressed in per unit on a common system base.
To accomplish this, an arbitrary base for apparent power is selected; for example, 100 MVA.
Then the voltage base must be selected.
Once a voltage base has been selected for a point in a system, the remaining voltage bases are no longer independent; they are determined by the various transformer turns ratios.
For example, if on a low-voltage side of $\frac{34.5 \text{ kV}}{115 \text{ kV}}$ transformer the base voltage of 36 kV is selected.
The base voltage on the high-voltage side must be

$$36 * \frac{115}{34.5} = 120 \text{ kV}$$

where $\frac{N_2}{N_1} = \frac{V_{B2}}{V_{B1}}$

Normaly we try to select the voltage bases that are the same as the nominal values.

Let Z_{pu}^{old} be the per-unit impedance on the power base S_B^{old} and the voltage V_B^{old}

$$Z_{pu}^{old} = \frac{Z_\Omega}{Z_B^{old}} = Z_\Omega \frac{S_B^{old}}{(V_B^{old})^2}$$

Expressing Z_Ω to a new power base and a new voltage base, results in the new per unit impedance

$$Z_{pu}^{new} = \frac{Z_\Omega}{Z_B^{new}} = Z_\Omega \frac{S_B^{new}}{(V_B^{new})^2}$$

$$Z_{pu}^{new} = \frac{Z_{pu}^{old}}{\frac{S_B^{old}}{(V_B^{old})^2}} * \frac{S_B^{new}}{(V_B^{new})^2} =$$

$$Z_{pu}^{new} = Z_{pu}^{old} \cdot \left(\frac{S_B^{new}}{S_B^{old}}\right)\left(\frac{V_B^{old}}{V_B^{new}}\right)^2$$

* The advantage of per-unit system.

1- The per-unit system gives us a clear idea of relative magnitudes of various quantities, such as Voltage, Current, power and impedance.

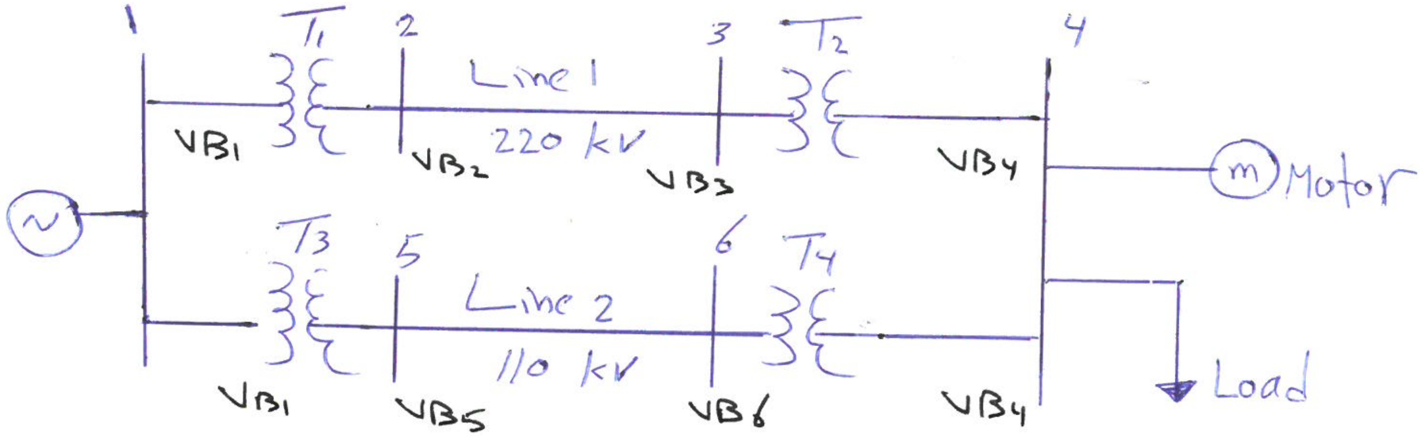
2- The per-unit impedance of equipment of the same general type based on their own ratings fall in a narrow range regardless of rating of the equibment. Whereas their impedeance in ohms vary greatly with the rating.

3- The per-unit values of impedance, voltage and current of a transformer are the same regardless of whether they are referred to the primary or the secondary side. This is a great advantage since the different voltage levels disappear and the entire system reduces to a system of simple impedance

4- The perunit system are ideal for the computerized analysis and simulation of complex power system problem.

5- The cct law are valied in per unit system and the power and the voltage equations are simplified since the factor $\sqrt{3}$ and 3 are eliminated in per unit system.

ex1) The one-line diagram of a 3-φ power system is shown in the fig. below:-



select a Common base of 100 MVA and 22 kV on the generator side.

Draw an impedance diagram in perunit.

The data for each device is given below:-

G :	90 MVA	22 kV	X = 18 %
T_1 :	50 MVA	22/220 kV	X = 10 %
T_2 :	40 MVA	220/11 kV	X = 6 %
T_3 :	40 MVA	22/110 kV	X = 6.4 %
T_4 :	40 MVA	110/11 kV	X = 8 %
Motor:	66.5 MVA	10.45 kV	X = 18.5 % 0.8 Lag. P.f

The 3-φ load at bus 4 absorbs 57 MVA, 0.6 P.f Lag. at 10.45 kV. Line 1 and Line 2 have a reactance of 48.4 and 65.43 Ω respectively.

Sol.

(1-9)

- The voltage bases must be determine for all sections of the network.

- The generator rated voltage is given at the base voltage at bus 1

$$V_{B1} = V_{G1} = 22\ kV$$

$$V_{B2} = 22 * \left(\frac{220}{22}\right) = 220\ kV$$

where

$$\frac{N_2}{N_1} = \frac{V_{B2}}{V_{B1}}$$

$$\frac{220}{22} = \frac{V_{B2}}{22} \Longrightarrow V_{B2} = \cancel{22} * \left(\frac{220}{\cancel{22}}\right) = 220\ kV$$

$$V_{B2} = V_{B3} = 220\ kV$$

$$V_{B4} = 220\left(\frac{11}{220}\right) = 11\ kV$$

$$V_{B5} = \cancel{22}\left(\frac{110}{\cancel{22}}\right) = 110\ kV$$

$$V_{B6} = V_{B5} = 110\ kV$$

$$V_{B4} = 110 * \left(\frac{11}{110}\right) = 11\ kV$$

$$X_{new} = X_{old}\left(\frac{S_B^{new}}{S_B^{old}}\right)\left(\frac{V_B^{old}}{V_{B\,new}}\right)^2$$

$$X_{G1} = 0.18\left(\frac{100}{90}\right)\left(\frac{\cancel{22}}{22}\right)^2 = 0.2\ pu$$

(1—10) (1—10)

$$X_{T_1} = 0.1 * \left(\frac{100}{50}\right)\left(\frac{\cancel{22}}{\cancel{22}}\right)^2 = 0.2\ pu$$
{ on primary side of Transformer 1 i.e: Low voltage side.

OR

$$X_{T_1} = 0.1 * \left(\frac{100}{50}\right)\left(\frac{\cancel{220}}{\cancel{220}}\right)^2 = 0.2\ pu$$
{ on secondary side of Transformer 1 i.e: High voltage side.

$$X_{T_2} = 0.06 * \left(\frac{100}{40}\right)\left(\frac{\cancel{220}}{\cancel{220}}\right)^2 = 0.15\ pu$$
{ on primary side of Transformer 2 i.e: Hig voltage side.

OR

$$X_{T_2} = 0.6 * \left(\frac{100}{40}\right)\left(\frac{\cancel{11}}{11}\right)^2 = 0.15\ pu$$
{ on secondary side of Transformer 2 i.e: Low voltage side.

$$X_{T_3} = 0.064 \left(\frac{100}{40}\right)\left(\frac{22}{22}\right)^2 = 0.16\ pu$$
{ on primary side of Transformer 3 i.e: Low voltage side }

OR

$$X_{T_3} = 0.064 \left(\frac{100}{40}\right)\left(\frac{110}{110}\right)^2 = 0.16\ pu$$
{ on secondary side of Transformer 3 i.e: High voltage side. }

(1—11) (1—1

$$X_{T4} = 0.08\left(\frac{100}{40}\right)\left(\frac{\cancel{110}}{\cancel{110}}\right)^2 = 0.2\ pu$$ { on primary side of Transformer 4 i.e: High Voltage side.

OR

$$X_{T4} = 0.08\left(\frac{100}{40}\right)\left(\frac{\cancel{11}}{\cancel{11}}\right)^2 = 0.2\ pu$$ { on secondary side of Transformer 4 i.e: Low Voltage side.

$$X_{motor} = 0.185\left(\frac{100}{65.5}\right)\left(\frac{10.45}{11}\right)^2 = 0.2\ pu$$

$$Z_{B_{Line1}} = \frac{V_{B\,Line1}^2}{S_B} = \frac{(V_{B2} \text{ or } V_{B3})^2}{S_B} = \frac{220^2}{100} = 484\ \Omega$$

$$Z_{B_{Line2}} = \frac{V_{B\,Line2}^2}{S_B} = \frac{(V_{B5} \text{ or } V_{B6})^2}{S_B} = \frac{110^2}{100} = 121\ \Omega$$

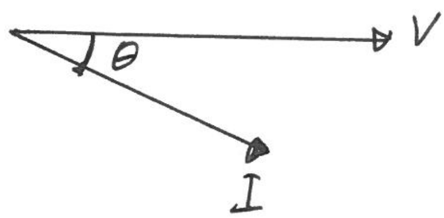
$$X_{Line1_{pu}} = \frac{\text{actual}}{\text{Base}} = \frac{48.4}{484} = 0.1\ pu = j0.1\ pu$$

$$X_{Line2_{pu}} = \frac{\text{actual}}{\text{Base}} = \frac{65.43}{121} = 0.54\ pu = j0.54\ pu$$

$$S_{Load_{3\phi}} = 57\angle 53.13^\circ\ MVA \quad \{0.6\ \text{Lag. P.f}\}$$

where :-

Lag. P.F {inductive Load}



$V = V\angle 0 \quad ; \quad I = I\angle -\theta$

$S_{3\phi} = \sqrt{3}\, V_{Line} I^{*}_{Line}$ or $3 V_P I_P^{*}$

$S_{3\phi} = \sqrt{3}\, V\,(I\angle -\theta)^{*}$

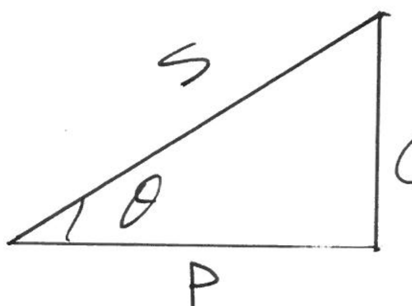
$= \sqrt{3}\, VI\angle\theta = \sqrt{3}\,(VI\angle\theta)$

$= \sqrt{3}\,(VI\cos\theta + j\,VI\sin\theta)$

$= \sqrt{3}\, VI\cos\theta + j\sqrt{3}\, VI\sin\theta$

$= P + jQ$

wher $P = \sqrt{3}\, VI\cos\theta$
$Q = \sqrt{3}\, VI\sin\theta$



Lag. P.F

$S = |S|\angle +\theta$

Lag p.f $\Rightarrow$

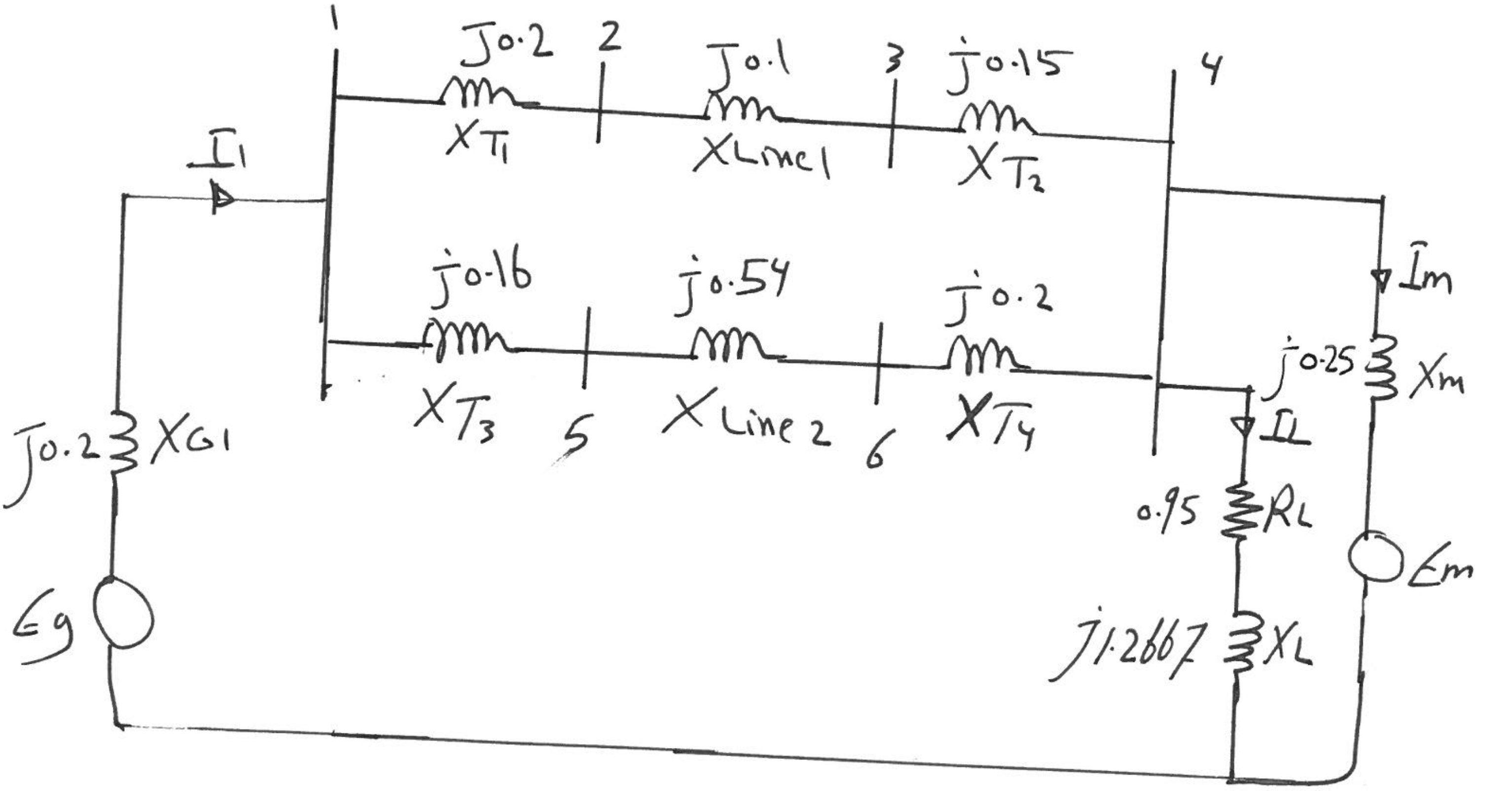
$$S_{3\phi\,Load} = 57\angle{+53.13^\circ}\ \ MVA$$

$$Z_{Load\,actual} = \frac{V_{L-L\,kV}^2}{S^*_{Load\,3\phi\,MVA}} = \frac{10.45^2\ kV}{57\angle{-53.13}\ MVA} = 1.1495 + j1.1532\ \Omega$$

$$Z_{B\,Load} = \frac{V_{B\,Load}^2}{S_B} = \frac{11^2\ kV}{100\ MVA} = 1.21\ \Omega$$

$$Z_{Load\,pu} = \frac{actual}{Base} = \frac{1.1495 + j1.532}{1.21} = 0.95 + j1.2667\ pu$$

The per unit equivalent cct is shown in fig. below



ex2) If the motor in ex1 operate at full-Load 0.8 p.f leading at terminal voltage of 10.45 kV.

1- Determine the voltage at the generator busbar (bus 1)

2- Determine the generator and the motor internal emfs.

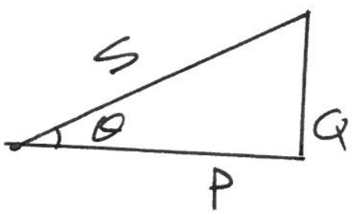
Sol)

The per-unit voltage at bus 4, taken as reference is

$$V_{4\,pu} = \frac{actual}{Base} = \frac{10.45}{11} = 0.95\angle 0 \ pu$$

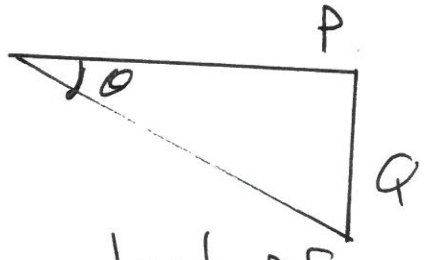
The apparent power of motor at 0.8 Lead p.f

$$S_{m\,pu} = \frac{actual}{Base} = \frac{66.5\angle -36.8}{100} = 0.665\angle -36.87 \ pu$$



Lag p.f

$S = S\angle +\theta$



lead p.f

$S = S\angle -\theta$

$$S_{pu} = V_{pu} \, I_{pu}^{*}$$

The current drawn by the motor is

$$I_m = \frac{S_m^{*}}{V_m^{*}} \quad \left\{ \text{where } S_m = V_m I_m^{*} \right.$$

$$I_m = \frac{0.665 \angle{+36.87}}{0.95 \angle 0} = 0.56 + j0.42 \text{ pu}$$

The current drawn by the Load is

$$I_L = \frac{V_4}{Z_L} = \frac{0.95 \angle 0}{0.95 + j1.2667} = 0.36 - j0.48 \text{ pu}$$

The current drawn from bus 4

$$I = I_m + I_L = I_1 \ \{\text{current of } G\}$$

$$= (0.56 + j0.42) + (0.36 - j0.48) = 0.92 - j0.06 \text{ pu}$$

The equivalent reactance of the parallel branches is

$$X = \frac{0.45 * 0.9}{0.45 + 0.9} = 0.3 \text{ pu}$$

The generator terminal voltage

$$V_1 = V_4 + ZI$$

$$= 0.95 \angle 0 + j0.3(0.92 - j0.06) = 0.968 + j0.276$$

$$= 1 \quad \angle 15.91^{\circ} \text{ pu}$$

actual Value $= 1 \quad * \underbrace{22 \text{ kV}}_{V_B} = 22 \quad \angle 15.91^{\circ} \text{ kV}$

$E_g = V_1 + I_1 X_g$ pu

$= 1\angle 15.91^\circ + (0.92 - j0.06) * (j0.2)$

$= 0.961 + j0.274 + j0.184 + 0.012$

$= 0.973 + j0.458 = 1.075\angle 25.17^\circ$

$E_{g\,actual} = 1.075 * V_B$ [V_B at the generator side]

$E_{g\,actual} = 1.075 * 22 = \underline{23.65 \text{ kV}}$

$E_m = V_4 - I_m Z_m$ [V_4 is terminal voltage at the motor]

$E_m = 0.95\angle 0 - (0.56 + j0.42) * j0.25$

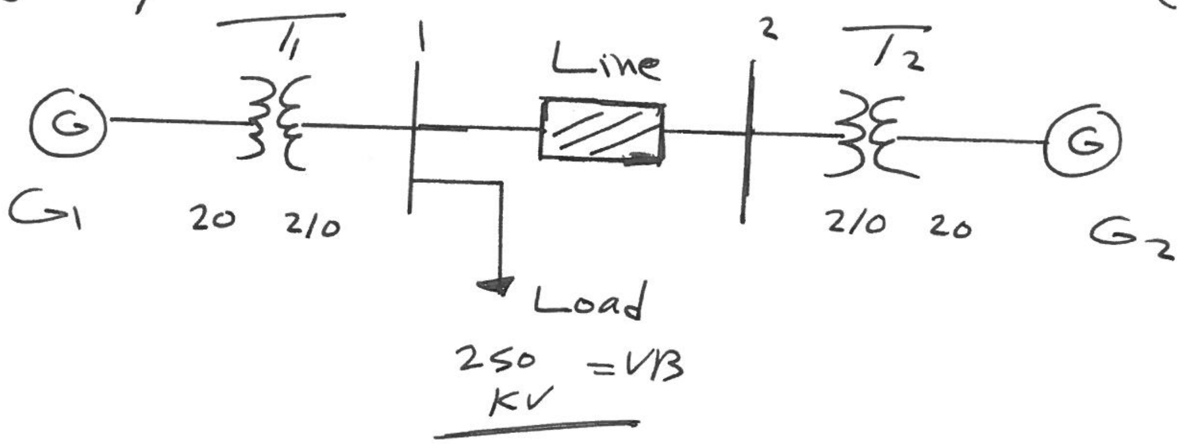
$= 0.95 - j0.14 + 0.105 = 1.055 - j0.14$

$E_m = 1.0642\angle -7.559^\circ$

$E_{m\,actual} = 1.0642 * V_{B\,motor}$

$= 1.0642 * 11 = \underline{11.7062 \text{ kV}}$

ex3) (1-17)



G_1	90 MVA	20 kV	$X = 0.09$ pu
T_1	80 MVA	20/210	$X = 0.16$ pu
T_2	80 MVA	210/20	$X = 0.2$ pu
G_2	90 MVA	18 kV	$X = 0.06$ pu
Line	210 kV		$X = 120\ \Omega$
Load	210 kV		$S = 48 + j64$

Choose base MVA as 100 MVA and base voltage of 250 kV at the Load bus

Sol) The base voltage = 250 kV at the Load bus bar

$V_{B\,Load} = 250$ kV

V_{B2} at bus 2 = 250 kV

$$V_{B\,G_2} = 250 * \left(\frac{20}{210}\right) = 23.8 \text{ kV}$$

$$V_{BG1} = 250 * \left(\frac{20}{210}\right) = 23.8 \text{ kV}$$

$$X_{G1_{new}} = X_{old} \left(\frac{S_B^{new}}{S_B^{old}}\right)\left(\frac{V_B^{old}}{V_B^{new}}\right)^2$$

$$X_{G1} = 0.09\left(\frac{100}{90}\right)\left(\frac{20}{23.8}\right)^2 = 0.0706 \text{ pu}$$

$$X_{G2} = 0.06\left(\frac{100}{90}\right)\left(\frac{18}{23.8}\right)^2 = 0.038 \text{ pu}$$

$$X_{T1} = 0.16\left(\frac{100}{80}\right)\left(\frac{20}{23.8}\right)^2 = 0.141 \text{ pu} \quad \left\{ \text{on primary side} \right.$$

OR

$$X_{T1} = 0.16\left(\frac{100}{80}\right)\left(\frac{210}{250}\right)^2 = 0.141 \text{ pu} \quad \left\{ \text{on secondary side} \right.$$

$$X_{T2} = 0.2 * \left(\frac{100}{80}\right)\left(\frac{210}{250}\right)^2 = 0.176 \text{ pu} \quad \left\{ \text{on primary side} \right\}$$

OR

$$X_{T2} = 0.2 * \left(\frac{100}{80}\right)\left(\frac{20}{23.8}\right)^2 = 0.176 \text{ pu} \quad \left\{ \text{on secondary side} \right.$$

$$X_{B\,Line} = \frac{V_B^2\,Line}{S_B} = \frac{250^2}{100} = 625\ \Omega$$

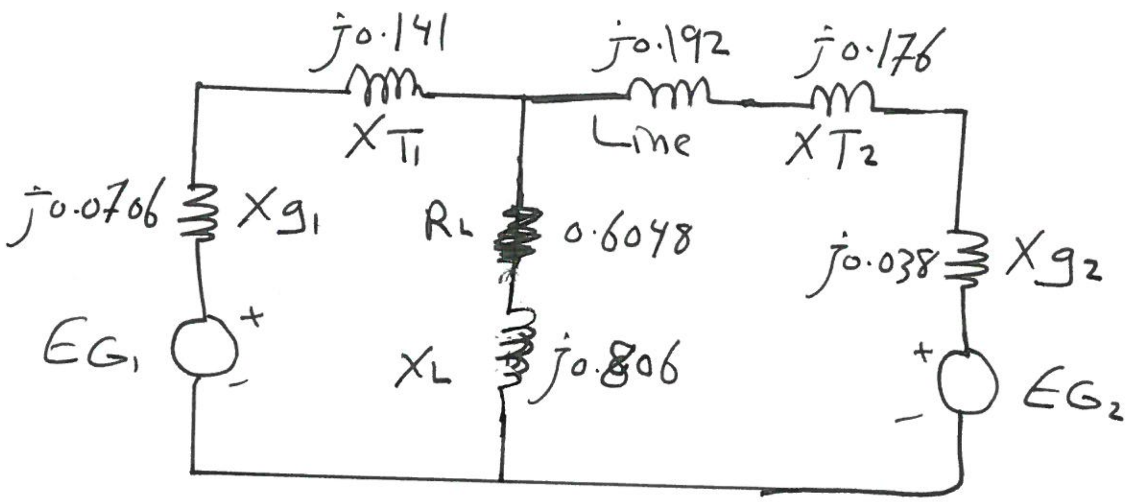
$$X_{pu\,Line} = \frac{\text{actual}}{\text{base}} = \frac{120}{625} = 0.192 \text{ pu}$$

(1-19)

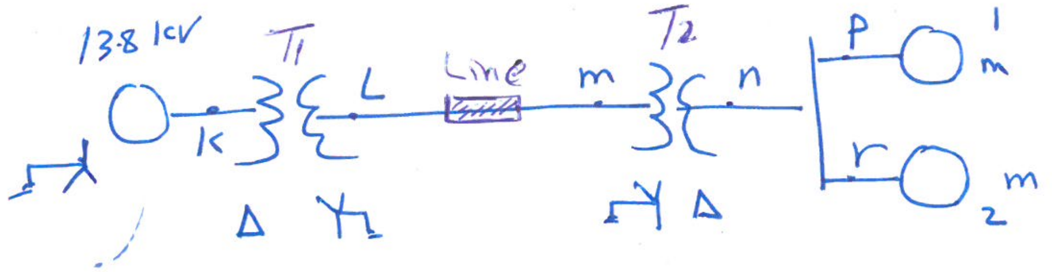
$$Z_{\text{Load actual}} = \frac{V_{LL}^2}{S_{3\phi}^*} = \frac{210^2}{48-j64} = 378+j504 \ \Omega$$

$$Z_{B\,\text{Load}} = \frac{V_{B\,\text{Load}}^2}{S_B} = \frac{250^2}{100} = 625$$

$$Z_{pu\,\text{Load}} = \frac{\text{actual}}{\text{Base}} = \frac{378+j504}{625} = 0.6048+j0.806 \text{ pu}$$



ex4 The system in the fig. below have :-



30,000 KVA, 13.8 kV, 3φ generator has subtransient reactance of 15%. The generator supplies two motors over a transmission line having transformers at both ends as shown in the fig. above.

The motors have rated inputs of 20,000 and 10,000 KVA, both 12.5 kV with $X'' = 20\%$.

The 3-φ transformer T_1 is rated 35,000 KVA, 13.2 Δ − 115 Y kV with leakage reactance of 10%.

Transformer T_2 is composed of three single phase transformers each rated 10,000 KVA, 12.5 − 67 kV Δ − Y with leakage reactance of 10%.

Series reactance of the transmission line is 80 Ω.

- Draw the reactance diagram with all reactances marked in per unit.

Select the generator rating as base in the generator circuit.

(1-21)

Sol) - The 3-φ rating of T_2

$$3 * 10{,}000 = 30{,}000 \text{ kVA} = 30 \text{ MVA}$$

- The line-to-line voltage ration $\frac{N_2}{N_1} = \frac{V_2}{V_1} =$

$$\frac{12.5}{\sqrt{3} * 67} = \frac{12.5}{116} \quad \left\{ \frac{\text{secondary}}{\text{primary}} = \frac{\Delta}{\curlywedge} \right\}$$

* The base of 30,000 kVA = 30 MVA
  13.8 kV in the generator cct

∴ Base MVA = 100 MVA

Bases voltage :-

$$V_K = 13.8 \text{ kV}.$$

$$V_L = 13.8 * \left(\frac{N_2}{N_1} = \frac{V_2}{V_1} = \text{turns ratio of } T_1 \text{ Line to line 3-φ} \right)$$

$$V_L = 13.8 * \frac{115}{13.2} = 120 \text{ kV}$$

= voltage of Transmission Line

$$V_m = V_L = 120 \text{ kV}$$

$$V_n = 120 * \frac{12.5}{116} = 12.9 \text{ kV}$$

$$V_P = V_r = 12.9 \text{ kV}$$

$$X_{new} = X_{old} \cdot \left(\frac{S_{B\,new}}{S_{B\,old}}\right)\left(\frac{V_{B\,old}}{V_{B\,new}}\right)^2$$

Low side

$$X_{T_1} = 0.1 \left(\frac{30}{35}\right)\left(\frac{13.2}{13.8}\right)^2 = 0.0784 \text{ pu}$$ {on L.s of T_1 i.e primary of T_1}

OR

High side

$$X_{T_1} = 0.1 \left(\frac{30}{35}\right)\left(\frac{115}{120}\right)^2 = 0.078 \text{ pu}$$ {on H.s of T_1 i.e secondary of T_1}

$$X_{T_2} = 0.1 \left(\frac{30}{30}\right)\left(\frac{115}{120}\right)^2 = 0.094 \text{ pu}$$ {on H.S of T_2 i.e primary of T_2}

OR

$$X_{T_2} = 0.1 \left(\frac{30}{30}\right)\left(\frac{12.5}{12.9}\right)^2 = 0.094 \text{ pu}$$ {on L.S of T_2 i.e secondary of T_1}

$$Z_{B\,Line} = \frac{V_B^2}{S_B} = \frac{120^2}{30} = 480\,\Omega$$

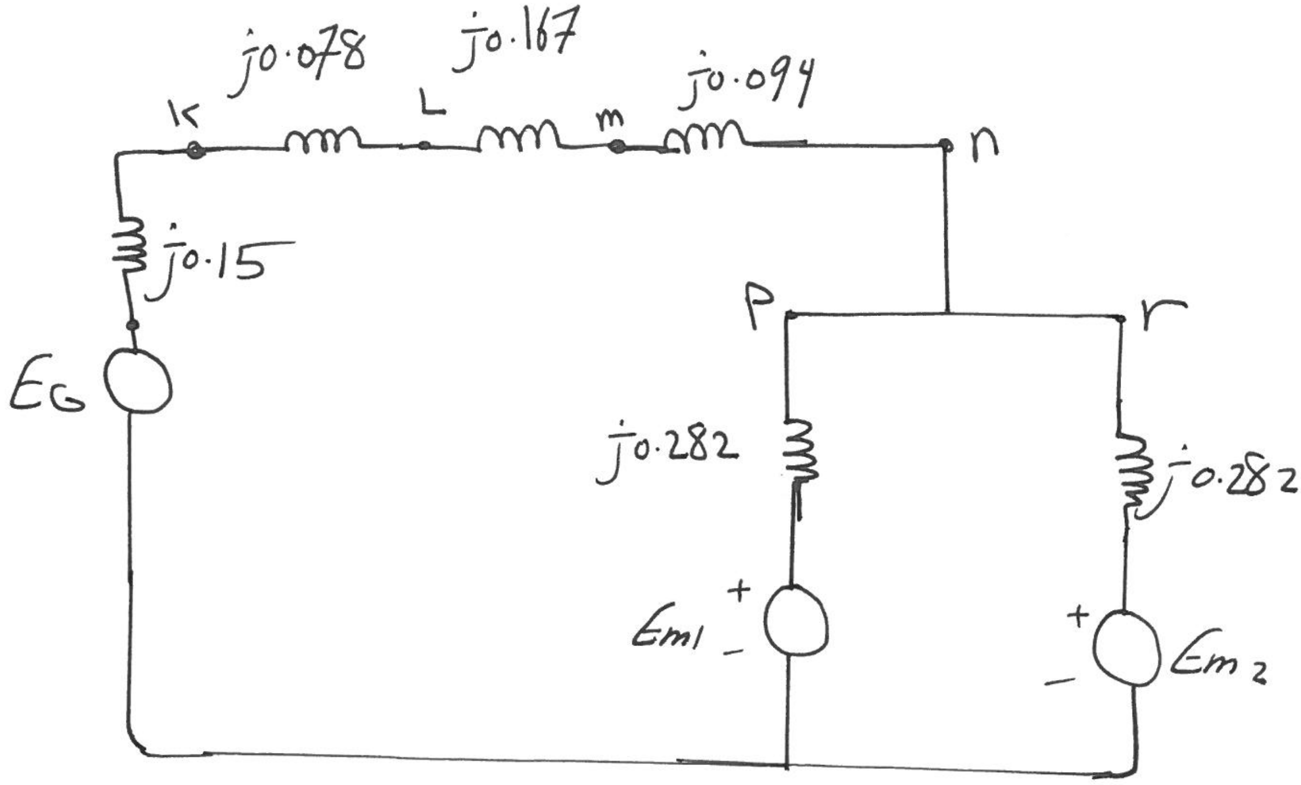
$$Z_{Line\,pu} = \frac{\text{actual}}{\text{Base}} = \frac{80}{480} = 0.167 \text{ pu}$$

$$X_{m_1} = 0.2 \left(\frac{30}{20}\right)\left(\frac{12.5}{12.9}\right)^2 = 0.282 \text{ pu}$$

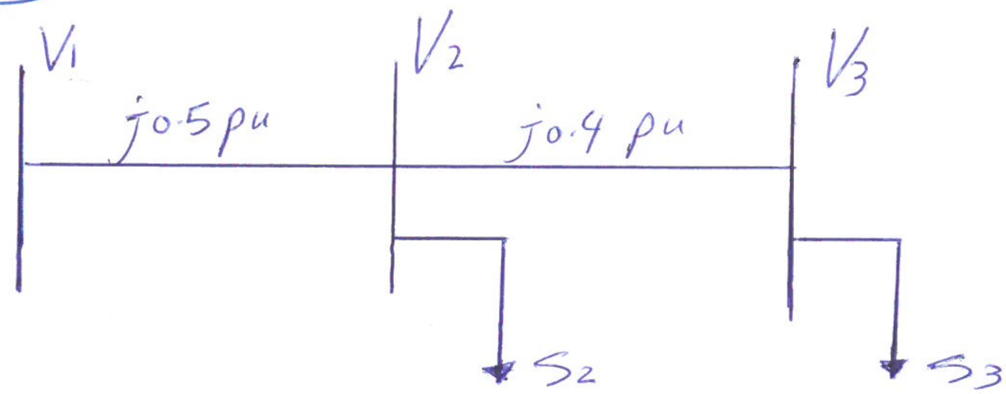
$$X_{m_2} = 0.2 \left(\frac{30}{10}\right)\left(\frac{12.5}{12.9}\right)^2 = 0.563 \text{ pu}$$

$$X_G = 0.15 \left(\frac{30}{30}\right)\left(\frac{13.8}{13.8}\right)^2 = 0.15 \text{ pu}$$

(1-23)



ex9) The one-line diagram of a 3-φ system in the fig. below



Impedance are marked in per unit on 100 MVA, 400 kV base. The Load at bus 2 is $S_2 = 15.93 - j33.4$ (15.93 MW, 33.4 Mvar) and at bus 3 $S_3 = 77 + j14$ (77 MW, 14 MVar). It is required to hold the voltage at bus 3 at $400\angle 0$ kV

working in per unit, determin the voltage at bus 2 and 1

Sol) $V_B = 400$ (in all point because there is no Transformer)

$S_B = 100$ MVA

$$S_2 = \frac{15.9 - j33.4}{100} = 0.159 - j0.334 \text{ pu}$$

$$S_3 = \frac{77 + j14}{100} = 0.77 + j0.14 \text{ pu}$$

$$S_{3\,pu} = V_{3\,pu} \, I_{3\,pu}^{*}$$

$$I_3 = \frac{S_3^{*}}{V_3^{*}}$$

$$V_{3\,pu} = \frac{\text{actual}}{\text{Base}} = \frac{400}{400}$$

$$V_{3\,pu} = 1\angle 0 \text{ pu}$$

$$\underset{pu}{I_3} = \frac{S_3^*}{V_3^*} = \frac{0.77 - j0.14}{1\angle 0} = 0.77 - j0.14 \text{ pu}$$

$$V_2 = V_3 + I_3 (j0.4)$$

$$= 1\angle 0 + (0.77 - j0.14) * (j0.4)$$

$$= 1 + j0.308 + 0.056 = 1.056 + j0.308 \text{ pu}$$

$$= 1.1 \angle 16.26^\circ$$

$$\underset{actual}{V_2} = 1.1\,\text{pu} * \underset{Base}{400\,k} = \underline{\underline{440\,kV}}$$

Line to Line

$$\underset{pu}{I_2} = \frac{S_2^*}{V_2^*} = \frac{0.1593 + j0.334}{1.1\angle -16.26^\circ} = 0.054 + j0.332 \text{ pu}$$

$$I_1 = I_2 + I_3$$

$$= (0.054 + j0.332) + (0.77 - j0.14)$$

$$I_1 = 0.824 + j0.192 \text{ pu}$$

$$V_1 = V_2 + I_1 (j0.5)$$

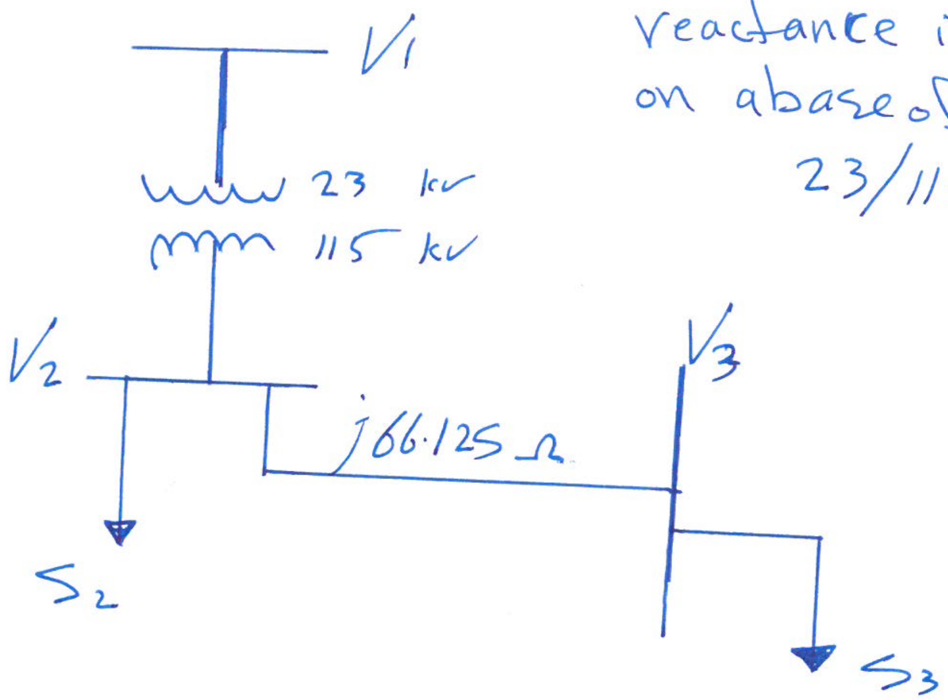
$$= 1.1\angle 16.26^\circ + (0.824 + j0.192)(j0.5)$$

$$= 1.2 \angle 36.87^\circ$$

$$V_1 = 400 * 1.2 = \underline{\underline{480\,kV}}$$

Line to Line

ex 10) The one-Line diagram of a 3-φ power system is shown in the fig. below. The Transformer reactance is 20 percent on a base of 100 MVA, 23/115 kV.



The line impedance $Z = j66.125\ \Omega$.

The load $S_2 = 184.8\ MW + j6.6\ Mvar$

$S_3 = 0\ MW + j20\ Mvar$

It is required to hold the voltage at bus 3 at $115\angle 0$ kV. Working in per unit, find the Voltage V_2, V_1

Sol)

$S_B = 100\ MVA$

$V_{B1} = 23\ kV$

$V_{B2} = 115\ kV$

$$S_2 = \frac{184.8 + j6.6}{100} = 1.848 + j0.066 \text{ pu}$$

$$S_3 = \frac{0 + j20}{100} = 0 + j0.2 \text{ pu}$$

$$V_3 = \frac{\text{actual}}{\text{Base}} = \frac{115\angle 0}{115} = 1\angle 0 \text{ pu}$$

$$V_2 = V_3 + I_3 * Z$$

$$I_{3\,pu} = \frac{S_3^*}{V_3^*} = \frac{-j0.2}{1\angle 0} = -j0.2 \text{ pu}$$

$$Z_{pu} = \frac{\text{actual}}{\text{Base}}$$

$$Z_B = \frac{V_B^2}{S_B} = \frac{115^2}{100}$$

$$Z_B = 132.25\ \Omega$$

$$Z = \frac{66.125}{132.25} = j0.5 \text{ pu}$$

$$V_2 = 1\angle 0 + (-j0.2)(j0.5)$$

$$= 1 + 0.1 = 1.1\angle 0 \text{ pu}$$

$$V_2 = 1.1 * 115 = 126.5 \text{ kV}$$ line to line

$$I_2 = \frac{S_2^*}{V_2^*} = \frac{1.848 - j0.066}{1.1\angle 0} = 1.68 - j0.26$$

$$I_1 = I_3 + I_2$$

$$I_1 = (-j0.2) + (1.68 - j0.06) = 1.68 - j0.26 \text{ pu}$$

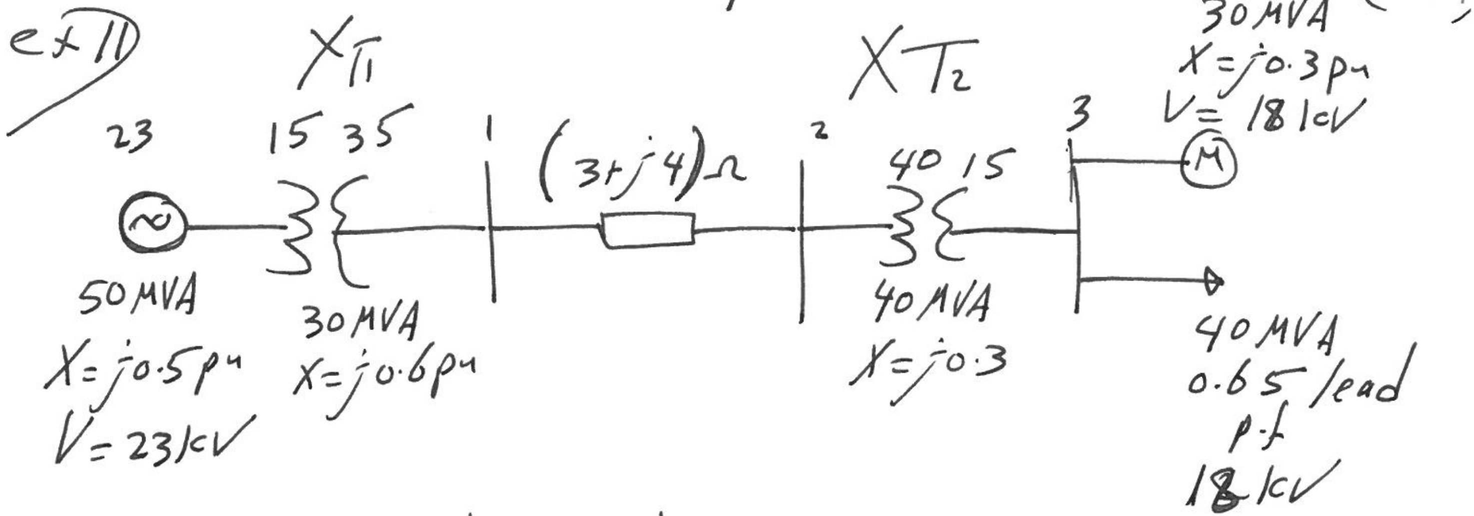
$$V_1 = I_1 X_{Trans} + V_2$$

$$= (1.68 - j0.26) * (j0.2) + 1.1\angle 0$$

$$= 1.2\angle 16.26^\circ$$

$$V_1 = 1.2 * 23 = \underline{27.6\,kV}$$

Line to Line



① find perunit impedance

② find Em of the motor

Take $V_B = 22$ kV at bus bar 1

$S_B = 100$ MVA

Sol)

$$V_{G_B} = 22 * \frac{15}{35} = 9.4 \text{ kV} \quad (\text{base})$$

$$V_{m_B} = 22 * \frac{15}{40} = 8.25 \text{ kV} \quad (\text{base})$$

VLoad

$$X_{G1} = 0.5 \left(\frac{100}{50}\right)\left(\frac{23}{9.4}\right)^2 = j5.98 \text{ pu}$$

$$X_{T1} = 0.6 \left(\frac{100}{30}\right)\left(\frac{35}{22}\right)^2 = j5.0619 \text{ pu}$$

secondary

$$X_{T2} = 0.3 \left(\frac{100}{40}\right)\left(\frac{40}{22}\right)^2 = j2.47 \text{ pu}$$

primary

$$X_m = 0.3 \left(\frac{100}{30}\right)\left(\frac{18}{8.25}\right)^2 = j4.76 \text{ pu}$$

(1-30)

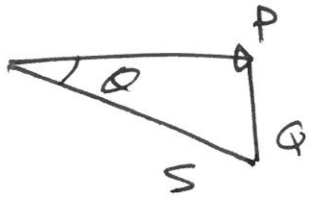
$$Z_{B\,Line} = \frac{V_B^2}{S_B} = \frac{22^2}{100} = 4.84\ \Omega$$

$$Z_{Line}\ pu = \frac{\text{actual}}{\text{Base}} = \frac{3+j4}{4.84} = 0.62 + j0.826\ pu$$

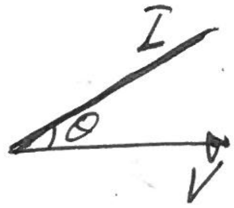
$$Z_{Load} = \frac{V_{LL}^2}{S^*_L} \Longrightarrow S_L = 40\angle{-\cos^{-1}0.65}$$

$$= 40\angle{-49.45}$$

$$Z_{Load} = \frac{18^2}{40\angle{49.45}} = 8.1\angle{-49.45}$$



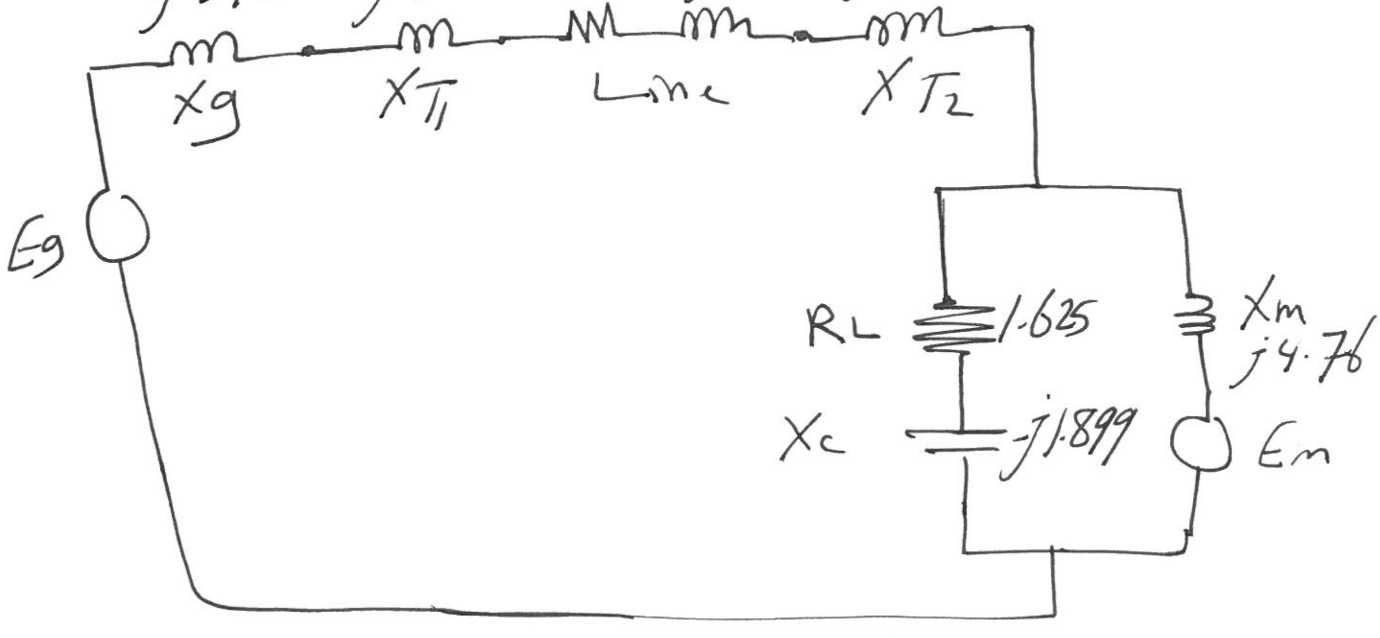
$$Z_{Load} = (5.265 - j6.154)\ \Omega$$



$$Z_{B\,Load} = \frac{V_B^2}{S_B} = \frac{18^2}{100} = 3.24\ \Omega$$

leading p.f

$$Z_{Load\,pu} = \frac{\text{actual}}{\text{Base}} = \frac{5.265 - j6.154}{3.24} = 1.625 - j1.899\ pu$$



$E_m = V_m - \hat{I}_m X_m$ (1-31)

$$V_m = \frac{\text{actual}}{\text{Base}} = \frac{18}{8.25} = 2.18 \text{ pu} = V_3$$

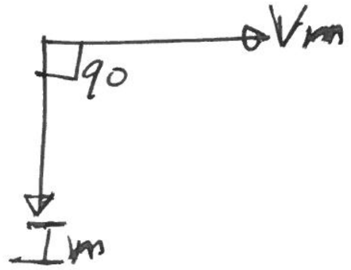
$$S_m = V_m \hat{I}_m^* \Longrightarrow \hat{I}_m = \frac{S_m^*}{V_m^*}$$

$$\hat{I}_m = \frac{S_m^*}{V_m^*}$$

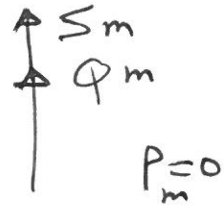
$S_m = 30 \angle{-90}$
because it has only reactance $X_m = j0.476$
(leading p.f)

$$S_m = \frac{30 \angle 90}{100} = 0.3 \angle 90 = j0.3 \text{ pu}$$

$$\hat{I}_m = \frac{S_m^*}{V_m^*} = \frac{-j0.3}{2.18} = -j0.1376 \text{ pu}$$



$$E_m = V_m - \hat{I}_m X_m$$
$$= 2.18\angle 0 - (-j0.1376)^* (j4.76)$$



$$= 2.18 - 0.654$$
$$= 1.526 \text{ pu}$$

OR $E_m = V_2 - (\hat{I}_m + \hat{I}_L) X_{T_2} - \hat{I}_m X_m$

$$V_2 = V_m + (\hat{I}_m + \hat{I}_L) X_{T_2}$$

(1-32)

$$E_m = V_2 - (\hat{I}_m + \hat{I}_L) X_T - \hat{I}_m X_m$$

$$V_2 = (\hat{I}_m + \hat{I}_L) X_T + V_m$$

$$E_m = \cancel{(\hat{I}_m + \hat{I}_L) X_T} + V_m - \cancel{(\hat{I}_m + \hat{I}_L) X_T} - \hat{I}_m X_m$$

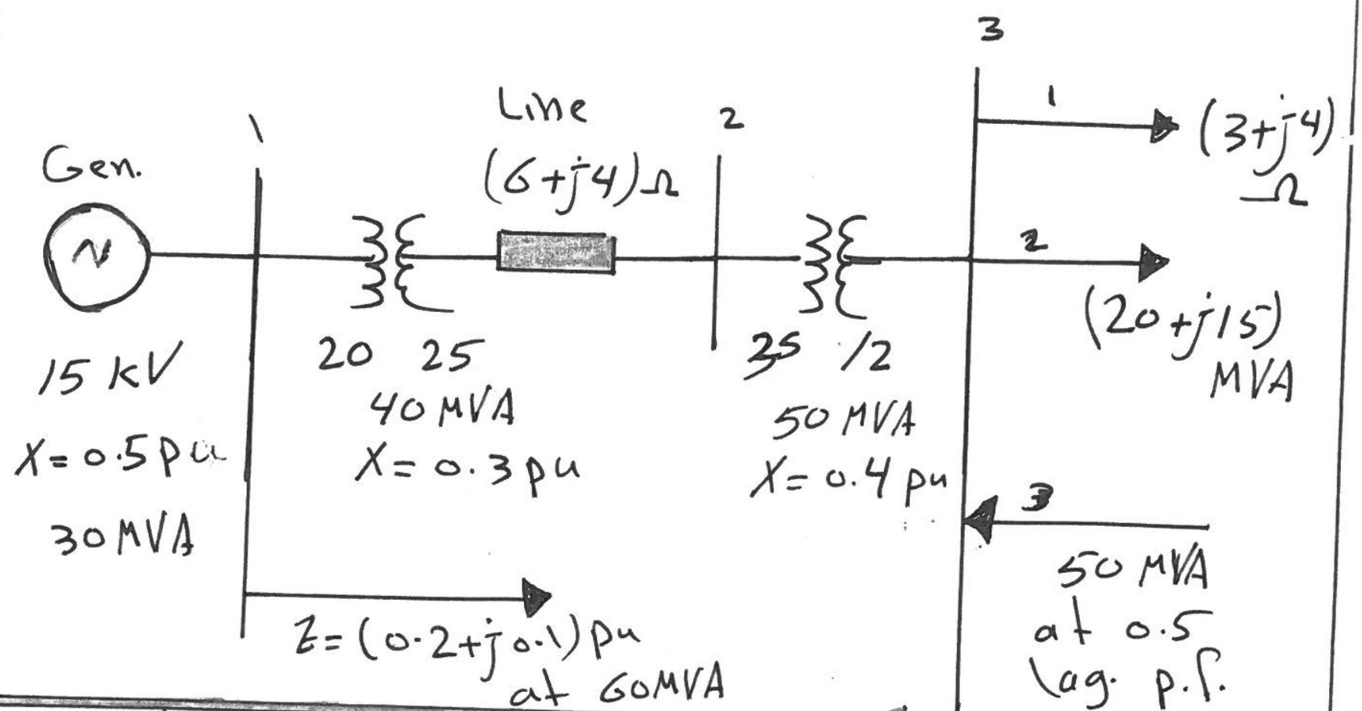
$$\boxed{E_m = V_m - \hat{I}_m X_m}$$

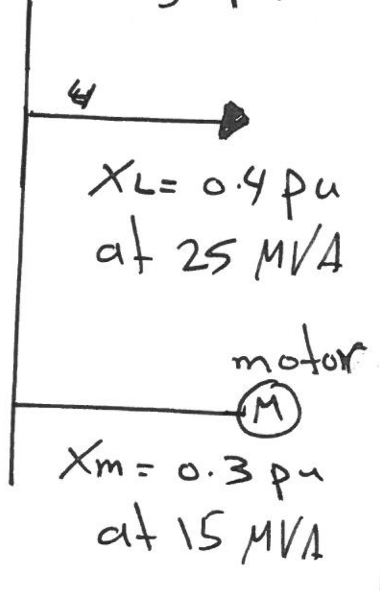
$$E_G = V_m + (\hat{I}_m + \hat{I}_L) (Z + X_G)$$

$$Z = X_{T_2} + X_{Line} + X_{T_1}$$

Q)

In the fig. below





The base MVA is 100 MVA and the base voltage is at the Line circuit is 40 kV

The terminal voltage at bus bar 3 is 15 kV

Find

1- The perunit impedance
2- The line current (perunit and actual)
3- The actual generator current
4- The emf of the generator and motor
5- Draw the perunit cct.

Sol)

$$V_{B1} = V_{BG} = 40 * \frac{20}{25} = 32\ kV$$

$$V_{B3} = 40 * \frac{12}{35} = 13.71\ kV$$

$$X_{G_{new}} = X_{old}\left(\frac{S_{B\,new}}{S_{B\,old}}\right)\left(\frac{V_{B\,old}}{V_{B\,new}}\right)^2$$

$$X_G = 0.5\left(\frac{100}{30}\right)\left(\frac{15}{32}\right)^2 =$$

$$= j\,0.3662\ pu$$

$$X_{T_1\,pri.} = 0.3\left(\frac{100}{40}\right)\left(\frac{20}{32}\right)^2 =$$

$$= j\,0.293\ pu$$

$$X_{T_2\,pri} = 0.4\left(\frac{100}{50}\right)\left(\frac{35}{40}\right)^2 =$$

$$= j\,0.6125\ pu$$

$$Z_{B_{Line}} = \frac{V_B^2}{S_B} = \frac{40^2}{100} = 16\ \Omega$$

$$Z_{\substack{Line\\pu}} = \frac{actual}{Base} = \frac{(6+j4)}{16} = 0.375+j0.25 \; pu$$

$$Z_{\substack{Load\\3-1}} = (3+j4)\,\Omega$$

$$Z_B = \frac{V_B^2}{S_B} = \frac{13.71^2}{100} = 1.879\,\Omega$$

$$Z_{\substack{Load\,3-1\\pu}} = \frac{actual}{Base} = \frac{3+j4}{1.879} = 1.596+j2.128 \; pu$$

$$Z_{\substack{Load\,3-2\\actual}} = \frac{V_L^2}{S^*_{L\,3-\phi}} = \frac{15^2}{(20+j15)^*} = \frac{15^2}{20-j15}$$

$$= 7.2+j5.4 \quad \Omega$$

$$Z_{\substack{Load\,3-2\\Base}} = \frac{V_B^2}{S_B} = \frac{13.71^2}{100} = 1.879\,\Omega$$

$$Z_{\substack{Load\,3-2\\pu}} = \frac{actual}{Base} = \frac{7.2+j5.4}{1.879}$$

$$= 3.83+j2.87 \; pu$$

$$Z_{Load\,3\text{-}3\,actual} = \frac{V_L^2}{S_{L\,3\phi}^*} = \frac{15^2}{(50\angle \cos^{-1}0.5)^*}$$

$$= \frac{15^2}{(50\angle 60^\circ)^*} = \frac{15^2}{50\angle -60^\circ}$$

$$= 4.5\angle 60^\circ\ \Omega$$

$$Z_{Load\,3\text{-}3\,pu} = \frac{\text{actual}}{\text{Base}} = \frac{4.5\angle 60^\circ}{1.879}$$

$$= 2.395\angle 60^\circ$$

$$= 1.197 + j2.074\ \text{pu}$$

$$Z_{Load\,3\text{-}4\,pu} = j0.4\left(\frac{100}{25}\right)\left(\frac{15}{13.71}\right)^2$$

$$= j1.915\ \text{pu}$$

$$X_m = 0.3\left(\frac{100}{15}\right)\left(\frac{15}{13.71}\right)^2 =$$

$$= j2.3941\ \text{pu}$$

$$I_G = I_{Line_{pu}} = I_{L3-1} + I_{L3-2} + I_{L3-3} + I_{L3-4} + I_m$$

$$I_{L3-1_{pu}} = \frac{V_{L\,pu}}{Z_{L_{3-1_{pu}}}}$$

$$V_{L_{pu}} = \frac{V_{L\,actual}}{V_{L\,Base}} = \frac{15}{13.71} = 1.094\angle 0$$

$$I_{L3-1_{pu}} = \frac{1.094}{1.596 + j2.128}$$

$$I_{L3-1} = 0.246 - j0.329 \quad \text{(lagging}$$

$$= 0.4113\angle -53.13^\circ \text{ pu}$$

$$I_{L3-2_{pu}} = \frac{V_L}{Z_{L_{3-2}}} = \frac{1.094\angle 0}{7.2 + j5.4} =$$

$$= 0.0972 - j0.0729 \text{ pu}$$

$$I_{L3-3_{pu}} = \frac{V_L}{Z_{L3-3}} = \frac{1.094\angle 0}{1.197 + j2.074}$$

$$= 0.2284 - j0.3957 \text{ pu}$$

$$I_{L_{3-4}\,pu} = \frac{V_L}{Z_{3-4}} = \frac{1.094\angle 0}{j1.915} =$$

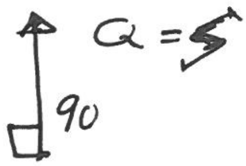
$$= -j0.5713 \text{ pu}$$

$$I_m = \frac{S_m^*}{V_m^*} \quad \text{where} \quad S_m = V_m I_m^*$$

$$S_{m\,pu} = \frac{S_m \text{ actual}}{S_m \text{ Base}} = \frac{15}{100} = 0.15 \text{ pu}$$

$$I_m = \frac{(0.15\angle +90^\circ)^*}{1.094\angle 0} = 0.1371\angle -90^\circ$$

where $X_m = j2.394$
means that this motor have
an inductive effect (lagging p.f)
pure inductive in this case.



$$I_G = I_{Line} =$$

$$(0.246 - j0.329) + (0.0972 - j0.0729)$$

$$-(0.2284 - j0.3957) + (-j0.5713)$$

$$+ (-j0.1371) =$$

$$I_G = I_{Line} = 0.1148 - j0.7146 \text{ pu}$$

$$V_1 = V_3 + I_{Line} * (\underbrace{X_{T_2} + Z_{Line} + X_{T_1}}_{Z})$$

$$V_3 = 1.094\angle 0$$

$$I_{Line} = 0.1148 - j0.7146$$

$$Z = (j0.6125) + (0.375 + j0.25) + (j0.293)$$

$$Z = 0.375 + j1.155 \text{ pu}$$

$$V_1 = 1.094 + (0.1148 - j0.7146) * (0.375 + j1.155)$$

$$= \underline{1.9628 + j0.135} \text{ pu} = 1.967 \angle -3.93^\circ \text{ pu}$$

$$= 1.967 * 32_{kV} = \underline{62.95_{kV}}$$

$$ZL_{1\text{-}1\,pu} = (0.2 + j0.1) * \left(\frac{100}{60}\right)\left(\frac{62.95}{32}\right)^2$$

$$= \underline{1.289 + j0.645} \text{ pu}$$

$$E_{g\,pu} = V_1 + I_G X_G$$

$$= (1.9628 - j0.135) + (0.1148 - j0.7146) * j0.3662$$

$$E_g = 2.2245 - j0.093 \text{ p.u}$$

$$2.2264 \angle -2.393^\circ \text{ pu}$$

$$E_{g\,actual} = 2.264 * 32$$

$$= 71.244 \text{ kV}$$

$$I_{G\,actual} = (1.9628 - j0.135) * I_B$$

$$I_B = \frac{S_B}{\sqrt{3}\, V_{B\,gen.}} = \frac{100*10^6}{\sqrt{3} * 32*10^3}$$

at the generator cct.

$$I_B = 1.80427*10^3 \text{ A}$$

$$= 1804.27 \text{ A}$$

$$I_G = (1.9628 - j0.135) * 1804.27$$

$$= 3541.4 - j243.57 \text{ A}$$

$$I_{\text{Line actual}} = (1.9628 - j0.135) * I_B$$

$$I_{\text{actual}} = I_{pu} * I_B$$

$$I_{B\,\text{Line}} = \frac{S_B}{\sqrt{3}\, V_{B\,\text{Line}}} = \frac{100*10^6}{\sqrt{3} * 40*10^3}$$

$$= 1443.41 \quad A$$

$$I_{\text{Line actual}} = 1443.41 * (1.9628 - j0.135)$$

$$= 2833.1 - j194.86 \quad A$$

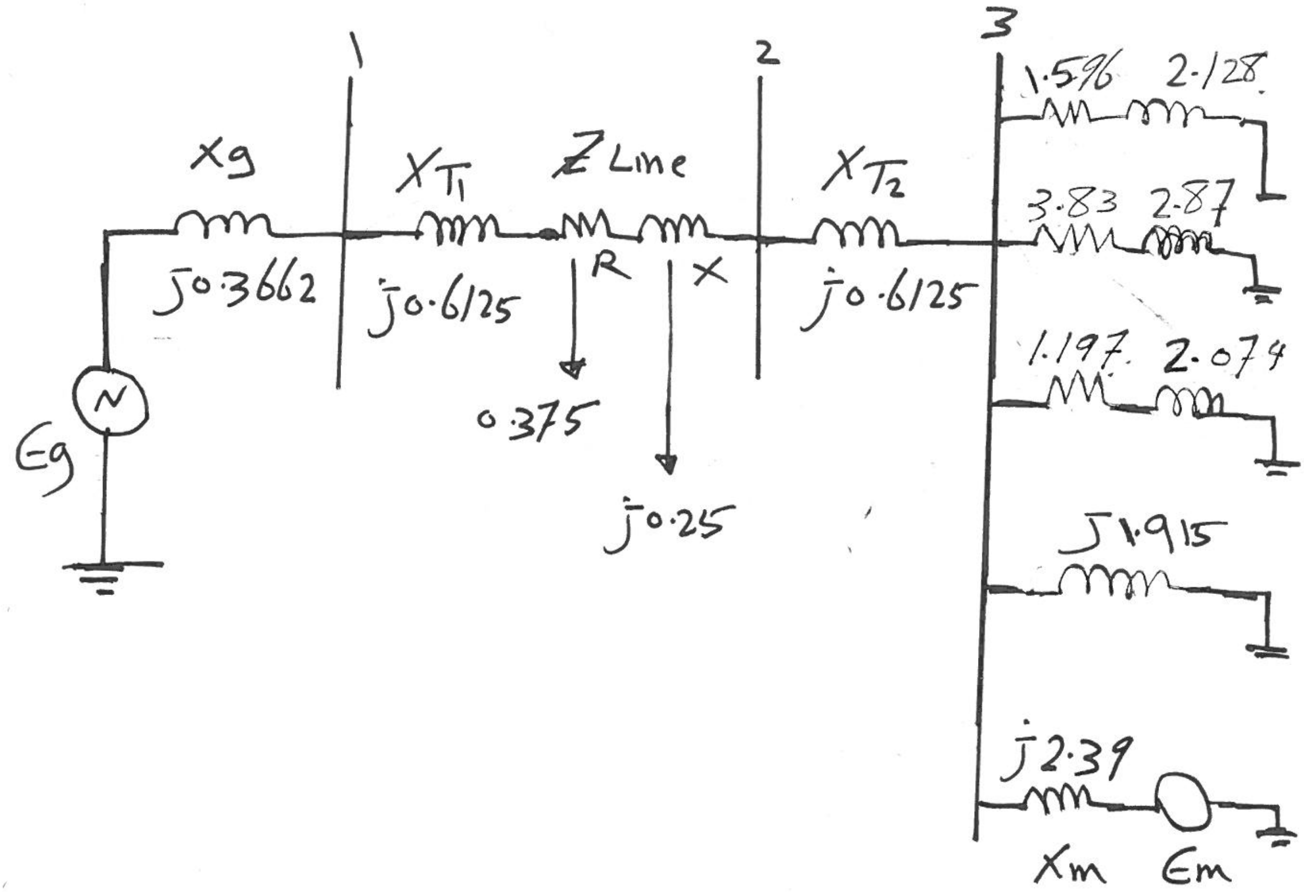
$$\text{emf}_{\text{motor pu}} = E_m = V_m - I_m X_m$$

$$= 1.094 \angle 0 - (-j0.1371) * (j2.394)$$

$$= 0.7657 \text{ pu}$$

$$\text{emf}_{m\,\text{actual}} = 0.7657 * 13.71 \text{ kV}$$

$$= 10.498 \text{ kV}$$


1
2
3
Xg
X_{T_1}
Z Line
X_{T_2}
j0.3662
j0.6125
R
X
j0.6125
0.375
j0.25
Eg
1.596
2.128
3.83
2.87
1.197
2.074
J1.915
j2.39
Xm
Em