Manometry

A standard technique for measuring pressure involves the use of liquid columns in vertical or inclined tubes. Pressure-measuring devices based on this technique are called manometers. The mercury barometer is an example of one type of manometer, but there are many other configurations possible depending on the particular application. Three common types of manometers include the piezometer tube, the U-tube manometer, and the inclined-tube manometer.

2.6.1 Piezometer Tube

The simplest type of manometer, called a piezometer tube, consists of a vertical tube, open at the top, and attached to the container in which the pressure is desired, as illustrated in Fig. 2.6. The figure in the margin shows an important device whose operation is based on this principle. It is a sphygmomanometer, the traditional instrument used to measure blood pressure. Because manometers involve columns of fluids at rest, the fundamental equation describing their use is Eq. 2.8

\[ p = \gamma h + p_0 \]

which gives the pressure at any elevation within a homogeneous fluid in terms of a reference pressure \( p_0 \) and the vertical distance \( h \) between \( p \) and \( p_0 \). Remember that in a fluid at rest pressure will increase as we move downward and will decrease as we move upward. Application of this equation to the piezometer tube of Fig. 2.6 indicates that the pressure \( p_A \) can be determined by a measurement of \( h_1 \) through the relationship

\[ p_A = \gamma_1 h_1 \]

where \( \gamma_1 \) is the specific weight of the liquid in the container. Note that since the tube is open at the top, the pressure \( p_0 \) can be set equal to zero (we are now using gage pressure), with the height \( h_1 \) measured from the meniscus at the upper surface to point (1). Because point (1) and point A within the container are at the same elevation, \( p_A = p_1 \).
Although the piezometer tube is a very simple and accurate pressure-measuring device, it has several disadvantages. It is only suitable if the pressure in the container is greater than atmospheric pressure (otherwise air would be sucked into the system), and the pressure to be measured must be relatively small so the required height of the column is reasonable. Also the fluid in the container in which the pressure is to be measured must be a liquid rather than a gas.

2.6.2 U-Tube Manometer

To overcome the difficulties noted previously, another type of manometer that is widely used consists of a tube formed into the shape of a U as is shown in Fig. 2.7. The fluid in the manometer is called the gage fluid. To find the pressure $p_A$ in terms of the various column heights, we start at one end of the system and work our way around to the other end, simply utilizing Eq. 2.8. Thus, for the U-tube manometer shown in Fig. 2.7, we will start at point A and work around to the open end. The pressure at points A and (1) are the same, and as we move from point (1) to (2) the pressure will increase by $\gamma_1 h_1$. The pressure at point (2) is equal to the pressure at point (3), since the pressures at equal elevations in a continuous mass of fluid at rest must be the same. Note that we could not simply "jump across" from point (1) to a point at the same elevation in the right-hand tube since these would not be points within the same continuous mass of fluid. With the pressure at point (3) specified we now move to the open end where the pressure is zero. As we move vertically upward the pressure decreases by an amount $\gamma_2 h_2$. In equation form these various steps can be expressed as

$$p_A + \gamma_1 h_1 - \gamma_2 h_2 = 0$$

and, therefore, the pressure $p_A$ can be written in terms of the column heights as

$$p_A = \gamma_2 h_2 - \gamma_1 h_1$$

(2.12)

A major advantage of the U-tube manometer lies in the fact that the gage fluid can be different from the fluid in the container in which the pressure is to be determined. For example, the fluid in A in Fig. 2.7 can be either a liquid or a gas. If A does contain a gas, the contribution of the gas column, $\gamma_1 h_1$, is almost always negligible so that $p_A \approx p$ and, in this instance, Eq. 2.12 becomes

$$p_A = \gamma_2 h_2$$
Example 2.2 Simple U-Tube Manometer

Given: A closed tank contains compressed air and oil (SG_air = 0.90) as shown in Fig. E2.2. A U-tube manometer using mercury (SG_Hg = 13.6) is connected to the tank as shown. The column heights are $h_1 = 36$ in., $h_2 = 6$ in., and $h_3 = 9$ in.

Find: Determine the pressure reading (in psi) of the gage.

Solution:

Following the general procedure of starting at one end of the manometer system and working around to the other, we will start at the air-oil interface in the tank and proceed to the open end where the pressure is zero. The pressure at level (1) is

$$ p_1 = p_{atm} + \gamma_{ol}(h_1 + h_2) $$

This pressure is equal to the pressure at level (2), as these two points are at the same elevation in a homogeneous fluid at rest. As we move from level (2) to the open end, the pressure must decrease by $\gamma_{ol}h_3$, and at the open end the pressure is zero. Thus, the manometer equation can be expressed as

$$ p_{atm} + \gamma_{ol}(h_1 + h_2) - \gamma_{ol}h_3 = 0 $$

or

$$ p_{atm} + (SG_{air})(\gamma_{ol})(h_1 + h_2) - (SG_{Hg})(\gamma_{ol})(h_3) = 0 $$

For the values given

$$ p_{atm} = -(0.9)(62.4 \text{ lb/in}^2) \left( \frac{36 + 6}{12} \text{ ft} \right) $$

$$ + (13.6)(62.4 \text{ lb/in}^2) \left( \frac{9}{12} \text{ ft} \right) $$

so that

$$ p_{atm} = 440 \text{ lb/ft}^2 $$

Comment: Assume that the gage pressure remains at 3.06 psi, but the manometer is altered so that it contains only oil. That is, the mercury is replaced by oil. A simple calculation shows that in this case the vertical oil-filled tube would need to be $h_2 = 11.3$ ft tall, rather than the original $h_2 = 9$ in. There is an obvious advantage of using a heavy fluid such as mercury in manometers.

The U-tube manometer is also widely used to measure the difference in pressure between two containers or two points in a given system. Consider a manometer connected between containers $A$ and $B$ as is shown in Fig. 2.8. The difference in pressure between $A$ and $B$ can be found by again starting at one end of the system and working around to the other end. For example, at $A$ the pressure is $p_A$, which is equal to $p_1$, and as we move to point (2) the pressure increases by $\gamma_1h_1$. The pressure at $p_2$ is equal to $p_3$, and as we move upward to point (4) the pressure decreases by $\gamma_2h_2$. Similarly, as we continue to move upward from point (4) to (5) the pressure decreases by $\gamma_3h_3$. Finally, $p_S = p_B$, as they are at equal elevations. Thus,

$$ p_A + \gamma_1h_1 - \gamma_2h_2 - \gamma_3h_3 = p_B $$

Or, as indicated in the figure in the margin on the previous page, we could start at $B$ and work our way around to $A$ to obtain the same result. In either case, the pressure difference is

$$ p_A - p_B = \gamma_2h_2 + \gamma_3h_3 - \gamma_1h_1 $$


Example 2.3 U-Tube Manometer

Given: As is discussed in Chapter 3, the volume rate of flow, $Q$, through a pipe can be determined by a means of a flow nozzle located in the pipe as illustrated in Fig. E2.3a. The nozzle creates a pressure drop, $p_a - p_b$, along the pipe, which is related to the flow through the equation, $Q = K(p_a - p_b)$, where $K$ is a constant depending on the pipe and nozzle size. The pressure drop is frequently measured with a differential U-tube manometer of the type illustrated.

Find:

(a) Determine an equation for $p_a - p_b$ in terms of the specific weight of the flowing fluid, $\gamma_1$, the specific weight of the gage fluid, $\gamma_2$, and the various heights indicated.

Solution:

(a) Although the fluid in the pipe is moving, fluids in the columns of the manometer are at rest so that the pressure variation in the manometer tubes is hydrostatic. If we start at point $A$ and move vertically upward to level (1), the pressure will decrease by $\gamma_1 h_1$ and will be equal to the pressure at (2) and at (3). We can now move from (3) to (4) where the pressure has been further reduced by $\gamma_2 h_2$. The pressures at levels (4) and (5) are equal, and as we move from (5) to $B$ the pressure will increase by $\gamma_1 (h_1 + h_2)$. Thus, in equation form

$$p_a = \gamma_1 h_1 = \gamma_2 h_2 + \gamma_1 (h_1 + h_2) - p_b$$

or

$$p_a - p_b = \gamma_1 (\gamma_2 - \gamma_1)$$

(Ans)

Comment: It is to be noted that the only column height of importance is the differential reading, $h_d$. The differential manometer could be placed 0.5 or 5.0 m above the pipe ($h_1 = 0.5$ m or $h_1 = 5.0$ m), and the value of $h_d$ would remain the same.

(b) The specific value of the pressure drop for the data given is

$$p_a - p_b = (0.5 \text{ m})(15.6 \text{ kN/m}^3 - 9.80 \text{ kN/m}^3) = 2.90 \text{ kPa}$$

(Ans)

Comment: By repeating the calculations for manometer fluids with different specific weights, $\gamma_2$, the results shown in Fig. E2.3b are obtained. Note that relatively small pressure differences can be measured if the manometer fluid has nearly the same specific weight as the flowing fluid. It is the difference in the specific weights, $\gamma_2 - \gamma_1$, that is important.
Inclined-Tube Manometer

To measure small pressure changes, a manometer of the type shown in Fig. 2.9 is frequently used. One leg of the manometer is inclined at an angle $\theta$, and the differential reading $\ell_2$ is measured along the inclined tube. The difference in pressure $p_A - p_B$ can be expressed as

$$p_A + \gamma_2 h_1 - \gamma_2\ell_2 \sin \theta - \gamma_1 h_1 = p_B$$

or

$$p_A - p_B = \gamma_2\ell_2 \sin \theta + \gamma_1 h_1 - \gamma_1 h_1$$

(2.13)

where it is to be noted the pressure difference between points (1) and (2) is due to the vertical distance between the points, which can be expressed as $\ell_2 \sin \theta$. Thus, for relatively small angles the differential reading along the inclined tube can be made large even for small pressure differences. The inclined-tube manometer is often used to measure small differences in gas pressures so that if pipes $A$ and $B$ contain a gas, then

$$p_A - p_B = \gamma_2\ell_2 \sin \theta$$

or

$$\ell_2 = \frac{p_A - p_B}{\gamma_2 \sin \theta}$$

(2.14)

where the contributions of the gas columns $h_1$ and $h_3$ have been neglected. As shown by Eq. 2.14 and the figure in the margin, the differential reading $\ell_2$ (for a given pressure difference) of the inclined-tube manometer can be increased over that obtained with a conventional U-tube manometer by the factor $1/\sin \theta$. Recall that $\sin \theta \to 0$ as $\theta \to 0$. 

**FIGURE 2.9** Inclined-tube manometer.
Mechanical and Electronic Pressure-Measuring Devices

Although manometers are widely used, they are not well suited for measuring very high pressures or pressures that are changing rapidly with time. In addition, they require the measurement of one or more column heights, which, although not particularly difficult, can be time consuming. To overcome some of these problems, numerous other types of pressure-measuring instruments have been developed. Most of these make use of the idea that when a pressure acts on an elastic structure, the structure will deform, and this deformation can be related to the magnitude of the pressure. Probably the most familiar device of this kind is the Bourdon pressure gage, which is shown in Fig. 2.10a. The essential mechanical element in this gage is the hollow, elastic curved tube (Bourdon tube), which is connected to the pressure source as shown in Fig. 2.10b. As the pressure within the tube increases, the tube tends to straighten, and although the deformation is small, it can be translated into the motion of a pointer on a dial as illustrated. Since it is the difference in pressure between the outside of the tube (atmospheric pressure) and the inside of the tube that causes the movement of the tube, the indicated pressure is gage pressure. The Bourdon gage must be calibrated so that the dial reading can directly indicate the pressure in suitable units such as psi, psf, or pascals. A zero reading on the gage indicates that the measured pressure is equal to the local atmospheric pressure. This type of gage can be used to measure a negative gage pressure (vacuum) as well as positive pressures.

For many applications in which pressure measurements are required, the pressure must be measured with a device that converts the pressure into an electrical output. For example, it may be desirable to continuously monitor a pressure that is changing with time. This type of pressure-measuring device is called a pressure transducer, and many different designs are used. A diaphragm-type electrical pressure transducer is shown in the figure in the margin.