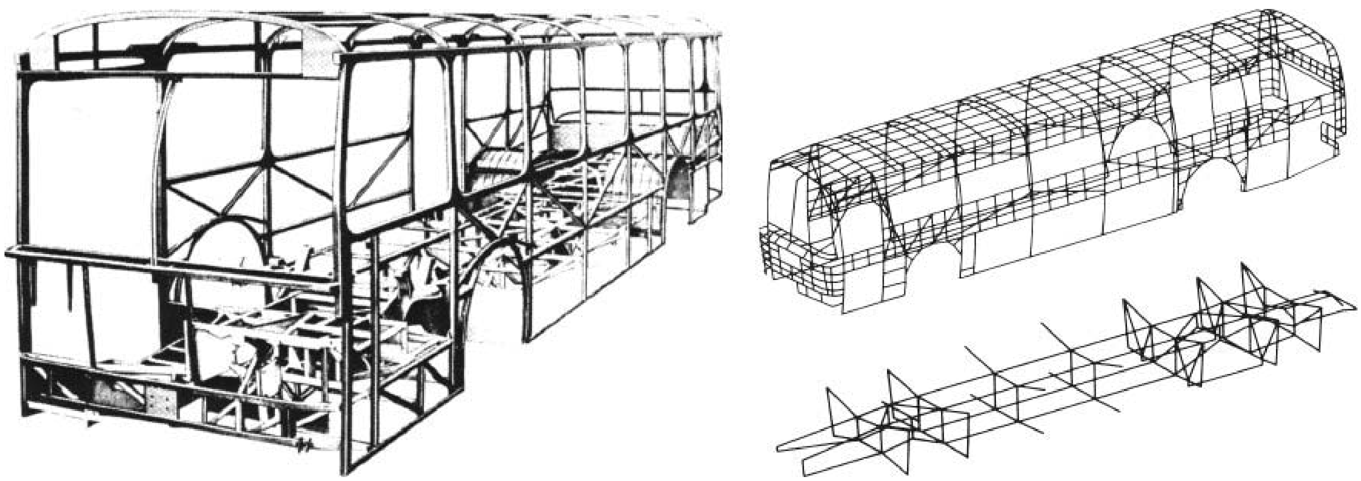

FIGURE 1.6 Finite element idealization of the body of a bus [1.16]. (Reprinted with permission © 1974 Society of Automotive Engineers, Inc.)

the way for the application of random vibrations to practical engineering problems. The monographs of Crandall and Mark and of Robson systematized the existing knowledge in the theory of random vibrations [1.12, 1.13].

Until about 40 years ago, vibration studies, even those dealing with complex engineering systems, were done by using gross models, with only a few degrees of freedom. However, the advent of high-speed digital computers in the 1950s made it possible to treat moderately complex systems and to generate approximate solutions in semidefinite form, relying on classical solution methods but using numerical evaluation of certain terms that cannot be expressed in closed form. The simultaneous development of the finite element method enabled engineers to use digital computers to conduct numerically detailed vibration analysis of complex mechanical, vehicular, and structural systems displaying thousands of degrees of freedom [1.14]. Although the finite element method was not so named until recently, the concept was used centuries ago. For example, ancient mathematicians found the circumference of a circle by approximating it as a polygon, where each side of the polygon, in present-day notation, can be called a finite element. The finite element method as known today was presented by Turner, Clough, Martin, and Topp in connection with the analysis of aircraft structures [1.15]. Figure 1.6 shows the finite element idealization of the body of a bus [1.16].

1.3 Importance of the Study of Vibration

Most human activities involve vibration in one form or other. For example, we hear because our eardrums vibrate and see because light waves undergo vibration. Breathing is associated with the vibration of lungs and walking involves (periodic) oscillatory motion of legs and hands. Human speech requires the oscillatory motion of larynges (and tongues) [1.17]. Early scholars in the field of vibration concentrated their efforts on understanding the natural phenomena and developing mathematical theories to describe the vibration of physical systems. In recent times, many investigations have been motivated by the engineering applications of vibration, such as the design of machines, foundations, structures, engines, turbines, and control systems.

Most prime movers have vibrational problems due to the inherent unbalance in the engines. The unbalance may be due to faulty design or poor manufacture. Imbalance in diesel engines, for example, can cause ground waves sufficiently powerful to create a nuisance in urban areas. The wheels of some locomotives can rise more than a centimeter off the track at high speeds due to imbalance. In turbines, vibrations cause spectacular mechanical failures. Engineers have not yet been able to prevent the failures that result from blade and disk vibrations in turbines. Naturally, the structures designed to support heavy centrifugal machines, like motors and turbines, or reciprocating machines, like steam and gas engines and reciprocating pumps, are also subjected to vibration. In all these situations, the structure or machine component subjected to vibration can fail because of material fatigue resulting from the cyclic variation of the induced stress. Furthermore, the vibration causes more rapid wear of machine parts such as bearings and gears and also creates excessive noise. In machines, vibration can loosen fasteners such as nuts. In metal cutting processes, vibration can cause chatter, which leads to a poor surface finish.

Whenever the natural frequency of vibration of a machine or structure coincides with the frequency of the external excitation, there occurs a phenomenon known as *resonance*, which leads to excessive deflections and failure. The literature is full of accounts of system failures brought about by resonance and excessive vibration of components and systems (see Fig. 1.7). Because of the devastating effects that vibrations can have on machines

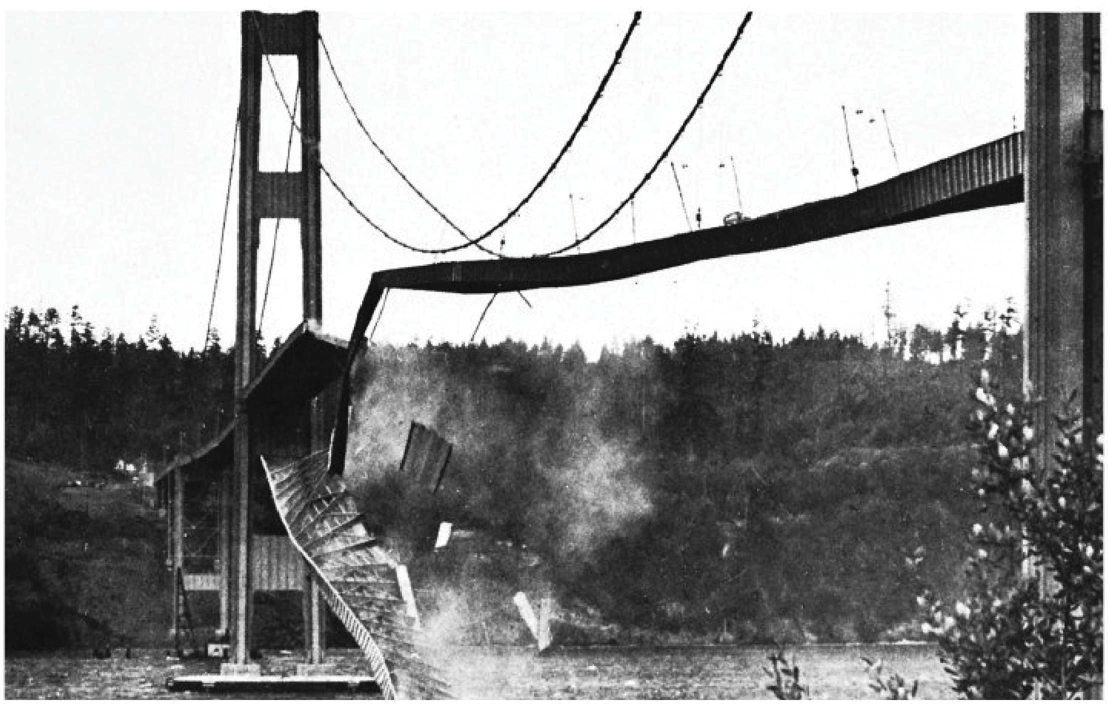

FIGURE 1.7 Tacoma Narrows bridge during wind-induced vibration. The bridge opened on July 1, 1940, and collapsed on November 7, 1940. (Farquharson photo, Historical Photography Collection, University of Washington Libraries.)

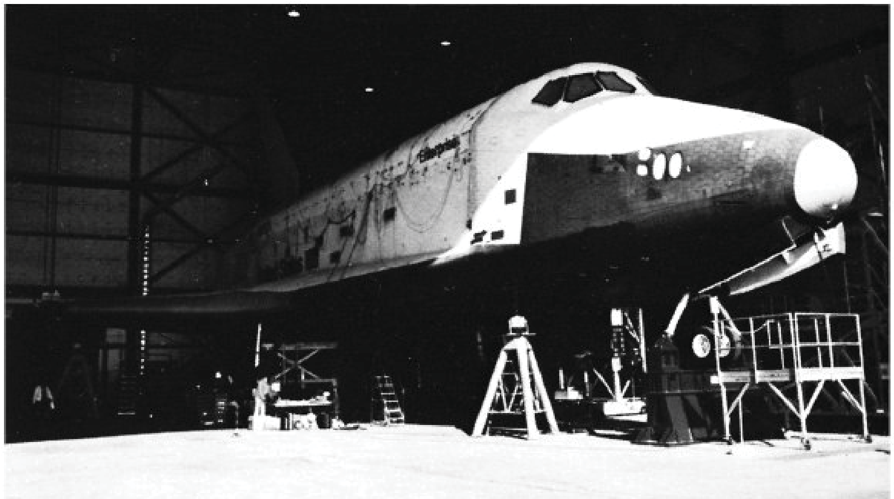

FIGURE 1.8 Vibration testing of the space shuttle *Enterprise*. (Courtesy of NASA.)

and structures, vibration testing [1.18] has become a standard procedure in the design and development of most engineering systems (see Fig. 1.8).

In many engineering systems, a human being acts as an integral part of the system. The transmission of vibration to human beings results in discomfort and loss of efficiency. The vibration and noise generated by engines causes annoyance to people and, sometimes, damage to property. Vibration of instrument panels can cause their malfunction or difficulty in reading the meters [1.19]. Thus one of the important purposes of vibration study is to reduce vibration through proper design of machines and their mountings. In this

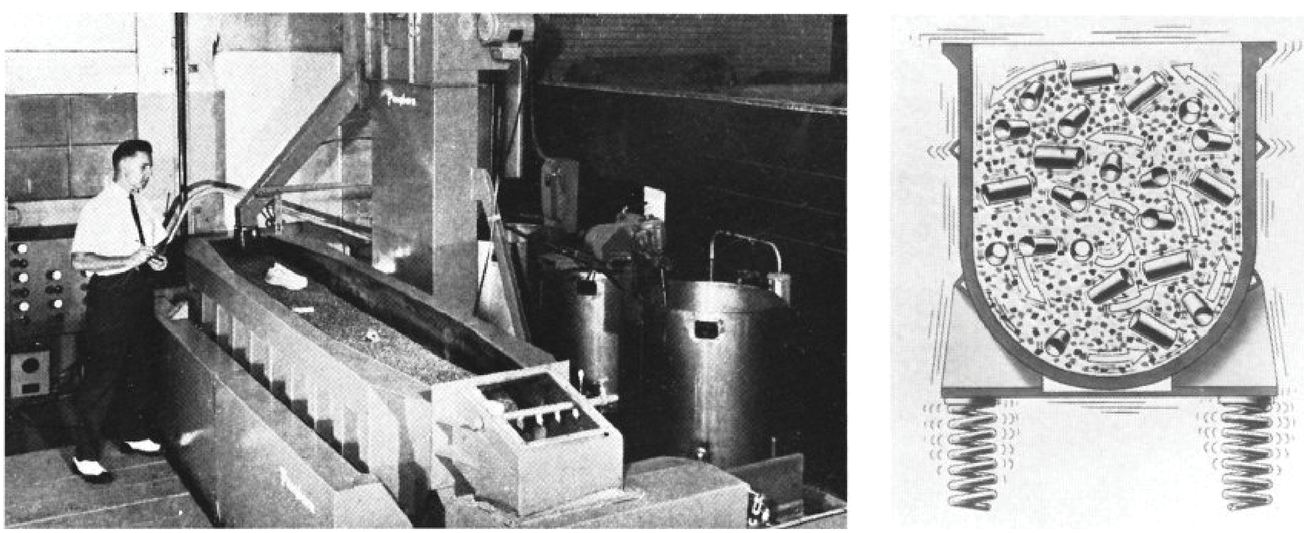

FIGURE 1.9 Vibratory finishing process. (Reprinted courtesy of the Society of Manufacturing Engineers, © 1964 The Tool and Manufacturing Engineer.)

connection, the mechanical engineer tries to design the engine or machine so as to minimize imbalance, while the structural engineer tries to design the supporting structure so as to ensure that the effect of the imbalance will not be harmful [1.20].

In spite of its detrimental effects, vibration can be utilized profitably in several consumer and industrial applications. In fact, the applications of vibratory equipment have increased considerably in recent years [1.21]. For example, vibration is put to work in vibratory conveyors, hoppers, sieves, compactors, washing machines, electric toothbrushes, dentist's drills, clocks, and electric massaging units. Vibration is also used in pile driving, vibratory testing of materials, vibratory finishing processes, and electronic circuits to filter out the unwanted frequencies (see Fig. 1.9). Vibration has been found to improve the efficiency of certain machining, casting, forging, and welding processes. It is employed to simulate earthquakes for geological research and also to conduct studies in the design of nuclear reactors.

1.4 Basic Concepts of Vibration

1.4.1 Vibration

Any motion that repeats itself after an interval of time is called *vibration* or *oscillation*. The swinging of a pendulum and the motion of a plucked string are typical examples of vibration. The theory of vibration deals with the study of oscillatory motions of bodies and the forces associated with them.

1.4.2 Elementary Parts of Vibrating Systems

A vibratory system, in general, includes a means for storing potential energy (spring or elasticity), a means for storing kinetic energy (mass or inertia), and a means by which energy is gradually lost (damper).

The vibration of a system involves the transfer of its potential energy to kinetic energy and of kinetic energy to potential energy, alternately. If the system is damped, some energy is dissipated in each cycle of vibration and must be replaced by an external source if a state of steady vibration is to be maintained.

As an example, consider the vibration of the simple pendulum shown in Fig. 1.10. Let the bob of mass m be released after being given an angular displacement θ. At position 1 the velocity of the bob and hence its kinetic energy is zero. But it has a potential energy of magnitude $mgl(1 - \cos\theta)$ with respect to the datum position 2. Since the gravitational force mg induces a torque $mgl \sin\theta$ about the point O, the bob starts swinging to the left from position 1. This gives the bob certain angular acceleration in the clockwise direction, and by the time it reaches position 2, all of its potential energy will be converted into kinetic energy. Hence the bob will not stop in position 2 but will continue to swing to position 3. However, as it passes the mean position 2, a counterclockwise torque due to gravity starts acting on the bob and causes the bob to decelerate. The velocity of the bob reduces to zero at the left extreme position. By this time, all the kinetic energy of the bob will be converted to potential energy. Again due to the gravity torque, the bob continues to attain a counterclockwise velocity. Hence the bob starts swinging back with progressively increasing velocity and passes the mean position again. This process keeps repeating, and the pendulum will have oscillatory motion. However, in practice, the magnitude of oscillation (θ) gradually decreases and the pendulum ultimately stops due to the resistance (damping) offered by the surrounding medium (air). This means that some energy is dissipated in each cycle of vibration due to damping by the air.

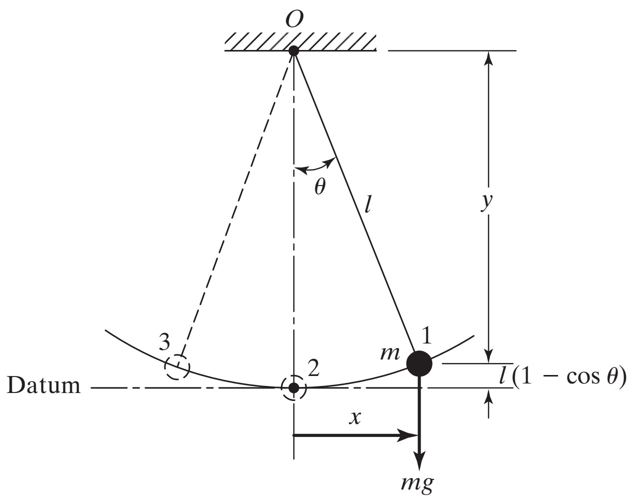

FIGURE 1.10 A simple pendulum.

1.4.3 Number of Degrees of Freedom

The minimum number of independent coordinates required to determine completely the positions of all parts of a system at any instant of time defines the number of degrees of freedom of the system. The simple pendulum shown in Fig. 1.10, as well as each of the systems shown in Fig. 1.11, represents a single-degree-of-freedom system. For example, the motion of the simple pendulum (Fig. 1.10) can be stated either in terms of the angle θ or in terms of the Cartesian coordinates x and y. If the coordinates x and y are used to describe the motion, it must be recognized that these coordinates are not independent. They are related to each other through the relation $x^2 + y^2 = l^2$, where l is the constant length of the pendulum. Thus any one coordinate can describe the motion of the pendulum. In this example, we find that the choice of θ as the independent coordinate will be more convenient than the choice of x or y. For the slider shown in Fig. 1.11(a), either the angular coordinate θ or the coordinate x can be used to describe the motion. In Fig. 1.11(b), the linear coordinate x can

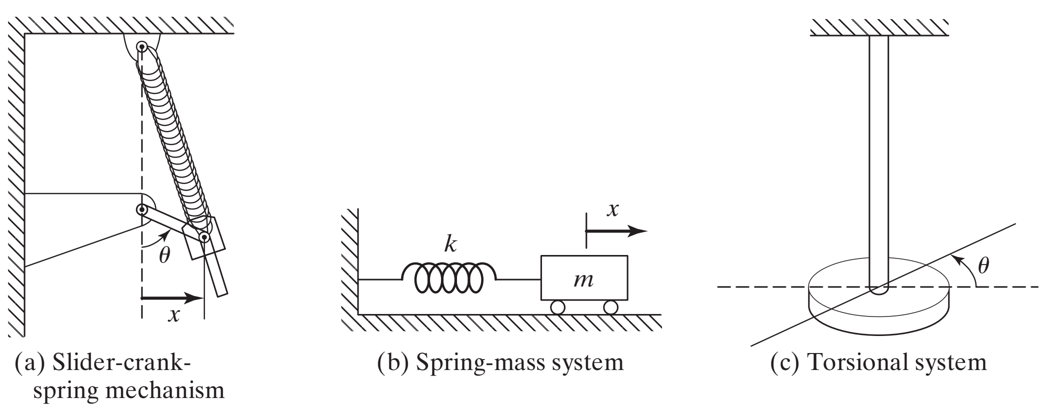

FIGURE 1.11 Single-degree-of-freedom systems.

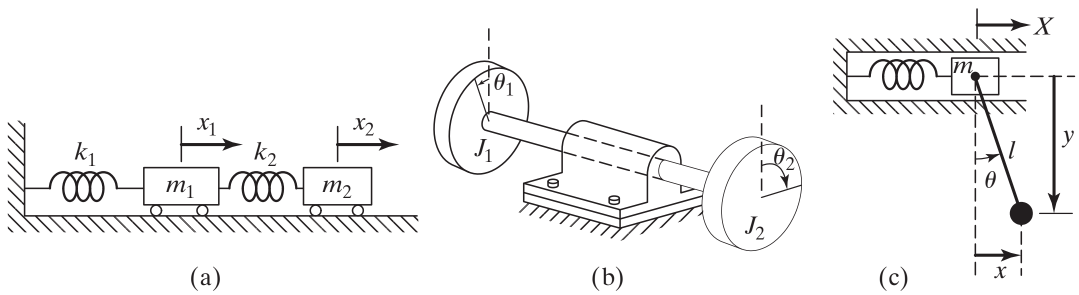

FIGURE 1.12 Two-degree-of-freedom systems.

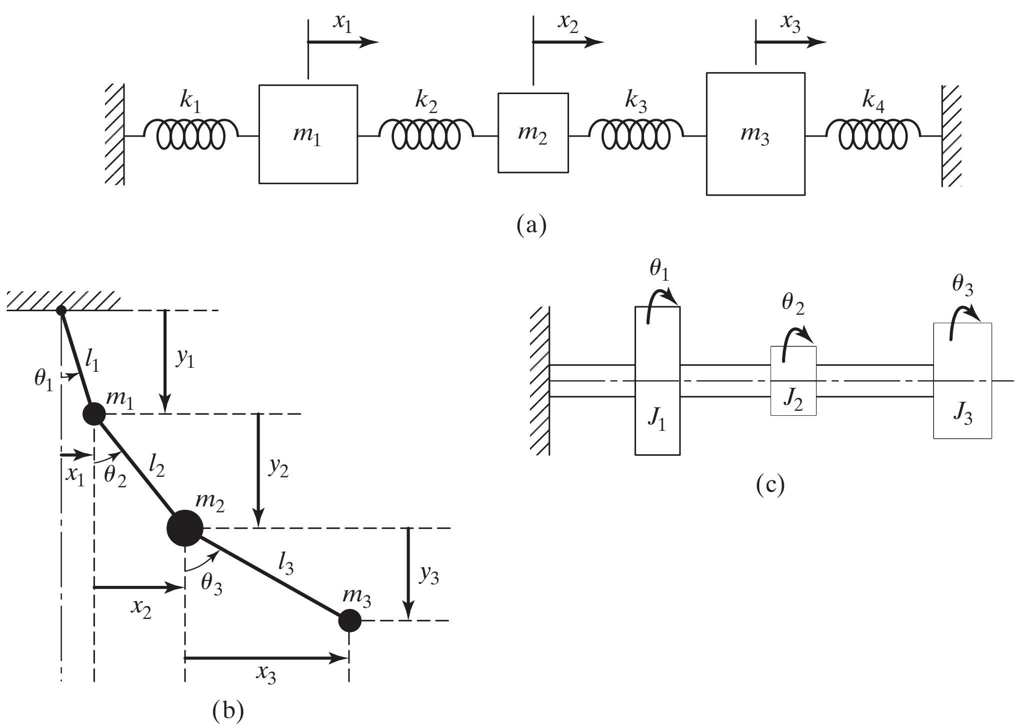

FIGURE 1.13 Three–degree-of-freedom systems.

be used to specify the motion. For the torsional system (long bar with a heavy disk at the end) shown in Fig. 1.11(c), the angular coordinate θ can be used to describe the motion.

Some examples of two- and three-degree-of-freedom systems are shown in Figs. 1.12 and 1.13, respectively. Figure 1.12(a) shows a two-mass, two-spring system that is described by the two linear coordinates x_1 and x_2. Figure 1.12(b) denotes a two-rotor system whose motion can be specified in terms of θ_1 and θ_2. The motion of the system shown in Fig. 1.12(c) can be described completely either by X and θ or by x, y, and X. In the latter case, x and y are constrained as $x^2 + y^2 = l^2$ where l is a constant.

For the systems shown in Figs. 1.13(a) and 1.13(c), the coordinates $x_i \, (i = 1, 2, 3)$ and $\theta_i \, (i = 1, 2, 3)$ can be used, respectively, to describe the motion. In the case of the

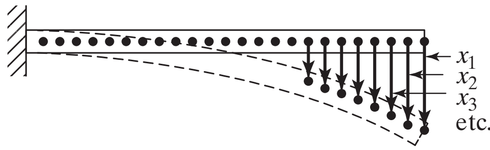

FIGURE 1.14 A cantilever beam (an infinite-number-of-degrees-of-freedom system).

system shown in Fig. 1.13(b), $\theta_i (i = 1, 2, 3)$ specifies the positions of the masses $m_i (i = 1, 2, 3)$. An alternate method of describing this system is in terms of x_i and $y_i (i = 1, 2, 3)$; but in this case the constraints $x_i^2 + y_i^2 = l_i^2 (i = 1, 2, 3)$ have to be considered.

The coordinates necessary to describe the motion of a system constitute a set of *generalized coordinates*. These are usually denoted as $q_1, q_2, \ldots$ and may represent Cartesian and/or non-Cartesian coordinates.

1.4.4 Discrete and Continuous Systems

A large number of practical systems can be described using a finite number of degrees of freedom, such as the simple systems shown in Figs. 1.10 to 1.13. Some systems, especially those involving continuous elastic members, have an infinite number of degrees of freedom. As a simple example, consider the cantilever beam shown in Fig. 1.14. Since the beam has an infinite number of mass points, we need an infinite number of coordinates to specify its deflected configuration. The infinite number of coordinates defines its elastic deflection curve. Thus the cantilever beam has an infinite number of degrees of freedom. Most structural and machine systems have deformable (elastic) members and therefore have an infinite number of degrees of freedom.

Systems with a finite number of degrees of freedom are called *discrete* or *lumped parameter* systems, and those with an infinite number of degrees of freedom are called *continuous* or *distributed* systems.

Most of the time, continuous systems are approximated as discrete systems, and solutions are obtained in a simpler manner. Although treatment of a system as continuous gives exact results, the analytical methods available for dealing with continuous systems are limited to a narrow selection of problems, such as uniform beams, slender rods, and thin plates. Hence most of the practical systems are studied by treating them as finite lumped masses, springs, and dampers. In general, more accurate results are obtained by increasing the number of masses, springs, and dampers—that is, by increasing the number of degrees of freedom.

1.5 Classification of Vibration

Vibration can be classified in several ways. Some of the important classifications are as follows.

1.5.1 Free and Forced Vibration

Free Vibration. If a system, after an initial disturbance, is left to vibrate on its own, the ensuing vibration is known as *free vibration.* No external force acts on the system. The oscillation of a simple pendulum is an example of free vibration.

Forced Vibration. If a system is subjected to an external force (often, a repeating type of force), the resulting vibration is known as *forced vibration.* The oscillation that arises in machines such as diesel engines is an example of forced vibration.

If the frequency of the external force coincides with one of the natural frequencies of the system, a condition known as *resonance* occurs, and the system undergoes dangerously large oscillations. Failures of such structures as buildings, bridges, turbines, and airplane wings have been associated with the occurrence of resonance.

1.5.2 Undamped and Damped Vibration

If no energy is lost or dissipated in friction or other resistance during oscillation, the vibration is known as *undamped vibration.* If any energy is lost in this way, however, it is called *damped vibration.* In many physical systems, the amount of damping is so small that it can be disregarded for most engineering purposes. However, consideration of damping becomes extremely important in analyzing vibratory systems near resonance.

1.5.3 Linear and Nonlinear Vibration

If all the basic components of a vibratory system—the spring, the mass, and the damper—behave linearly, the resulting vibration is known as *linear vibration.* If, however, any of the basic components behave nonlinearly, the vibration is called *nonlinear vibration.* The differential equations that govern the behavior of linear and nonlinear vibratory systems are linear and nonlinear, respectively. If the vibration is linear, the principle of superposition holds, and the mathematical techniques of analysis are well developed. For nonlinear vibration, the superposition principle is not valid, and techniques of analysis are less well known. Since all vibratory systems tend to behave nonlinearly with increasing amplitude of oscillation, a knowledge of nonlinear vibration is desirable in dealing with practical vibratory systems.

1.5.4 Deterministic and Random Vibration

If the value or magnitude of the excitation (force or motion) acting on a vibratory system is known at any given time, the excitation is called *deterministic.* The resulting vibration is known as *deterministic vibration.*

In some cases, the excitation is *nondeterministic* or *random;* the value of the excitation at a given time cannot be predicted. In these cases, a large collection of records of the excitation may exhibit some statistical regularity. It is possible to estimate averages such as the mean and mean square values of the excitation. Examples of random excitations are wind velocity, road roughness, and ground motion during earthquakes. If the excitation is random, the resulting vibration is called *random vibration.* In this case the vibratory response of the system is also random; it can be described only in terms of statistical quantities. Figure 1.15 shows examples of deterministic and random excitations.

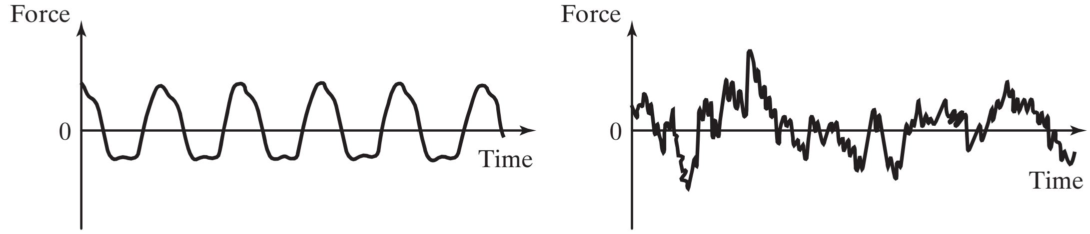

FIGURE 1.15 Deterministic and random excitations.

1.6 Vibration Analysis Procedure

A vibratory system is a dynamic one for which the variables such as the excitations (inputs) and responses (outputs) are time dependent. The response of a vibrating system generally depends on the initial conditions as well as the external excitations. Most practical vibrating systems are very complex, and it is impossible to consider all the details for a mathematical analysis. Only the most important features are considered in the analysis to predict the behavior of the system under specified input conditions. Often the overall behavior of the system can be determined by considering even a simple model of the complex physical system. Thus the analysis of a vibrating system usually involves mathematical modeling, derivation of the governing equations, solution of the equations, and interpretation of the results.

Step 1: Mathematical Modeling. The purpose of mathematical modeling is to represent all the important features of the system for the purpose of deriving the mathematical (or analytical) equations governing the system's behavior. The mathematical model should include enough details to allow describing the system in terms of equations without making it too complex. The mathematical model may be linear or nonlinear, depending on the behavior of the system's components. Linear models permit quick solutions and are simple to handle; however, nonlinear models sometimes reveal certain characteristics of the system that cannot be predicted using linear models. Thus a great deal of engineering judgment is needed to come up with a suitable mathematical model of a vibrating system.

Sometimes the mathematical model is gradually improved to obtain more accurate results. In this approach, first a very crude or elementary model is used to get a quick insight into the overall behavior of the system. Subsequently, the model is refined by including more components and/or details so that the behavior of the system can be observed more closely. To illustrate the procedure of refinement used in mathematical modeling, consider the forging hammer shown in Fig. 1.16(a). It consists of a frame, a falling weight known as the tup, an anvil, and a foundation block. The anvil is a massive steel block on which material is forged into desired shape by the repeated blows of the tup. The anvil is usually mounted on an elastic pad to reduce the transmission of vibration to the foundation block and the frame [1.22]. For a first approximation, the frame, anvil, elastic pad, foundation block, and soil are modeled as a single–degree of freedom system as shown in Fig. 1.16(b). For a refined approximation, the weights of the frame and anvil and

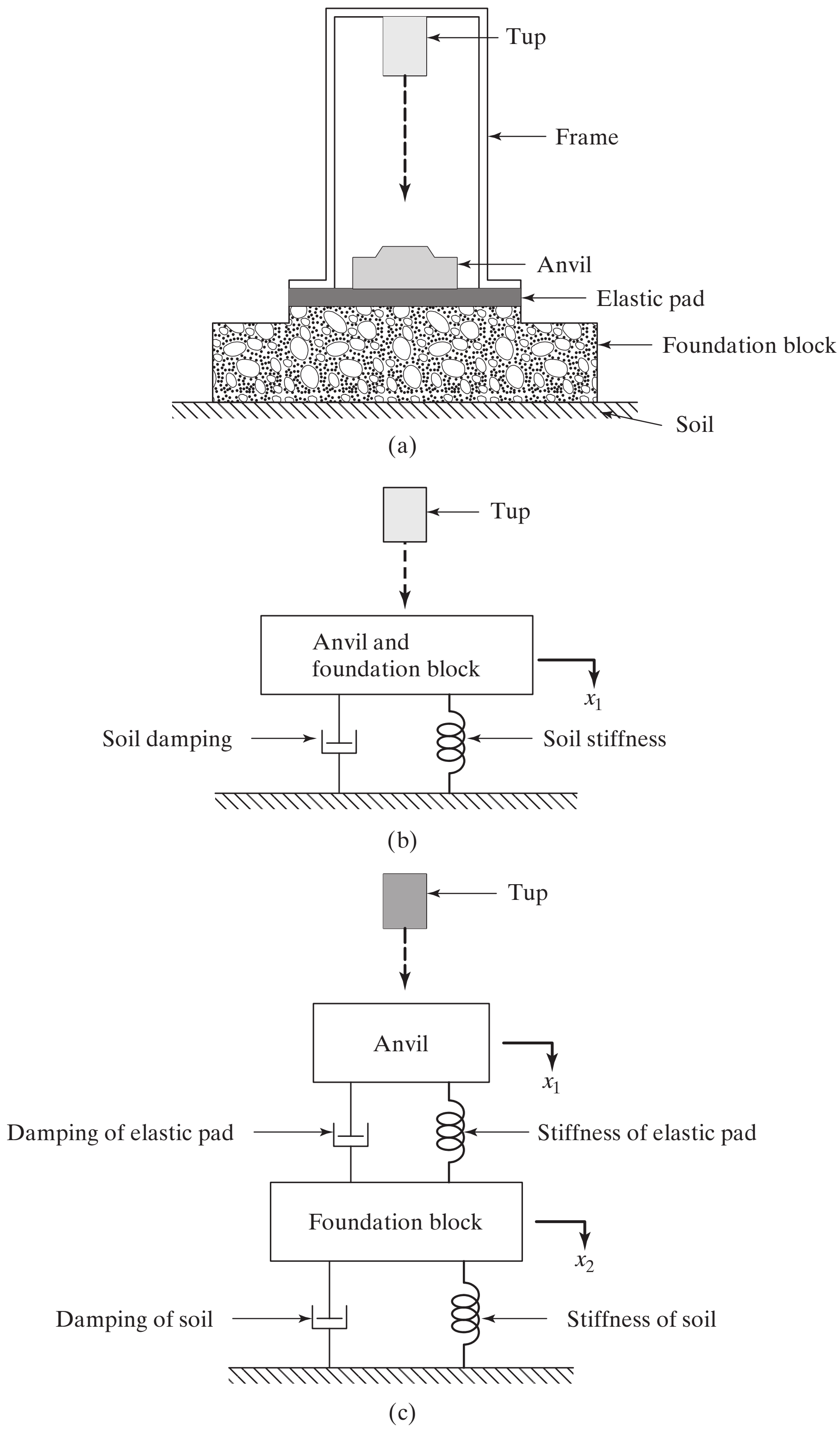

FIGURE 1.16 Modeling of a forging hammer.

the foundation block are represented separately with a two-degree-of-freedom model as shown in Fig. 1.16(c). Further refinement of the model can be made by considering eccentric impacts of the tup, which cause each of the masses shown in Fig. 1.16(c) to have both vertical and rocking (rotation) motions in the plane of the paper.

Step 2: Derivation of Governing Equations. Once the mathematical model is available, we use the principles of dynamics and derive the equations that describe the vibration of the system. The equations of motion can be derived conveniently by drawing the free-body diagrams of all the masses involved. The free-body diagram of a mass can be obtained by isolating the mass and indicating all externally applied forces, the reactive forces, and the inertia forces. The equations of motion of a vibrating system are usually in the form of a set of ordinary differential equations for a discrete system and partial differential equations for a continuous system. The equations may be linear or nonlinear, depending on the behavior of the components of the system. Several approaches are commonly used to derive the governing equations. Among them are Newton's second law of motion, D'Alembert's principle, and the principle of conservation of energy.

Step 3: Solution of the Governing Equations. The equations of motion must be solved to find the response of the vibrating system. Depending on the nature of the problem, we can use one of the following techniques for finding the solution: standard methods of solving differential equations, Laplace transform methods, matrix methods,[1] and numerical methods. If the governing equations are nonlinear, they can seldom be solved in closed form. Furthermore, the solution of partial differential equations is far more involved than that of ordinary differential equations. Numerical methods involving computers can be used to solve the equations. However, it will be difficult to draw general conclusions about the behavior of the system using computer results.

Step 4: Interpretation of the Results. The solution of the governing equations gives the displacements, velocities, and accelerations of the various masses of the system. These results must be interpreted with a clear view of the purpose of the analysis and the possible design implications of the results.

EXAMPLE 1.1 Mathematical Model of a Motorcycle

Figure 1.17(a) shows a motorcycle with a rider. Develop a sequence of three mathematical models of the system for investigating vibration in the vertical direction. Consider the elasticity of the tires, elasticity and damping of the struts (in the vertical direction), masses of the wheels, and elasticity, damping, and mass of the rider.

Solution: We start with the simplest model and refine it gradually. When the equivalent values of the mass, stiffness, and damping of the system are used, we obtain a single-degree-of-freedom model

[1]The basic definitions and operations of matrix theory are given in Appendix A.

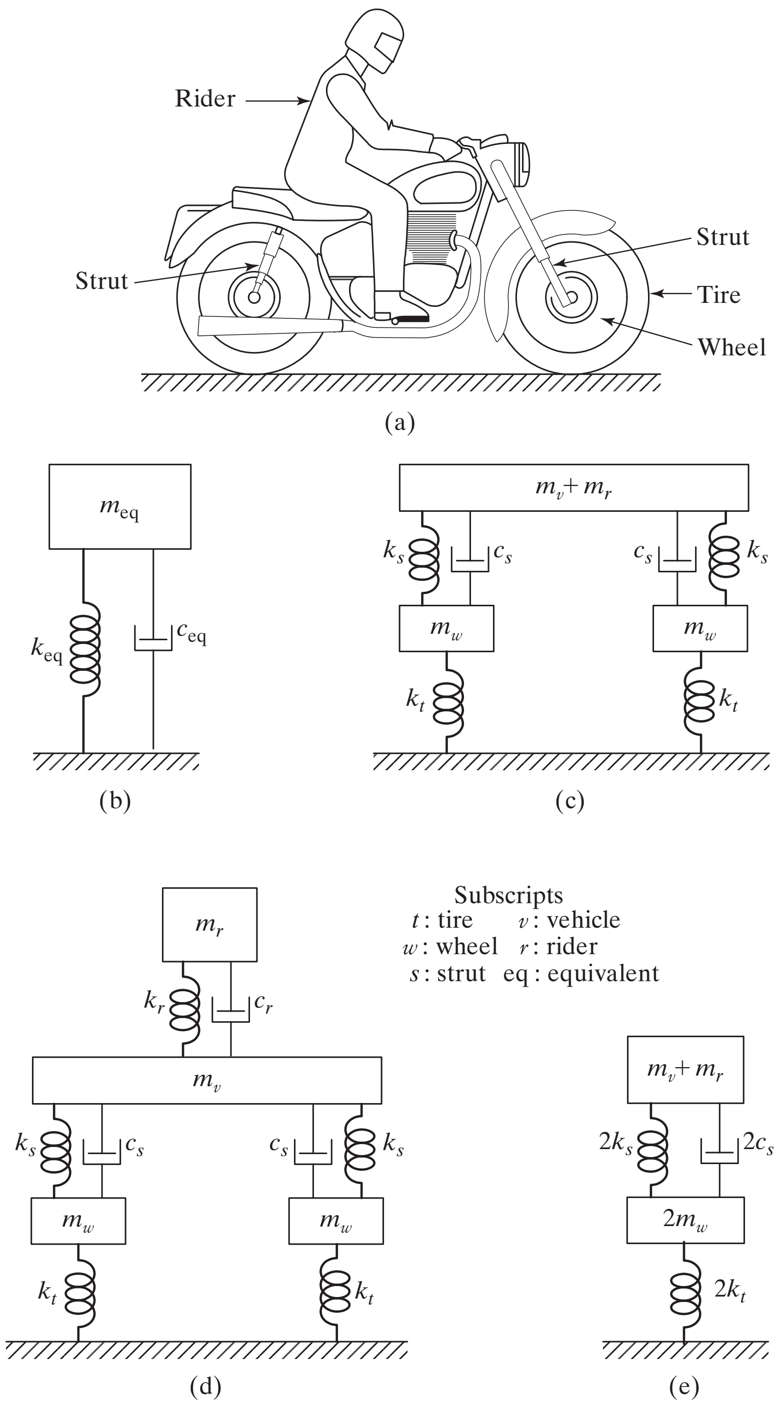

FIGURE 1.17 Motorcycle with a rider—a physical system and mathematical model.

of the motorcycle with a rider as indicated in Fig. 1.17(b). In this model, the equivalent stiffness (k_{eq}) includes the stiffnesses of the tires, struts, and rider. The equivalent damping constant (c_{eq}) includes the damping of the struts and the rider. The equivalent mass includes the masses of the wheels, vehicle body, and the rider. This model can be refined by representing the masses of wheels,