Vectors and Two Dimensional Motion

1) Vector and Scalar Quantities

a) Scalar quantity possesses magnitude only with specific number.
Examples:
Speed, mass and temperature ------etc.
e.g. (50 km/hr)

b) Vector quantities contains both magnitude and direction.
Examples:
Displacement, velocity, acceleration, and force.
Magnitude and direction means, for example, 50 km/hr, south

c) Two vectors are equal if they have the same magnitude and direction. Moving a vector to a different location does not change the vector as long as its direction and magnitude remain constant.

d) Usefulness of Vectors

1. Principle of superposition
If an object is subjected to two separate influences, each producing a type of motion, it responds to each without modifying its response to the other.

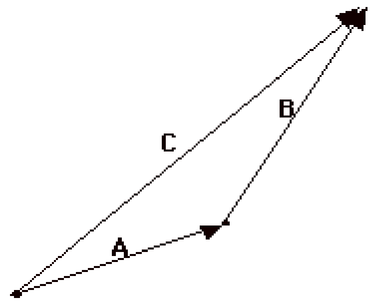
2. A vector analysis allows you to separate two-dimensional motion into Two one-dimensional motions and then combine them at the end of the problem. Example (projectile motion)

2) Rules for vector addition

a) For vectors along a straight line use a positive sign for a vector to the right and a negative sign for a vector to the left.

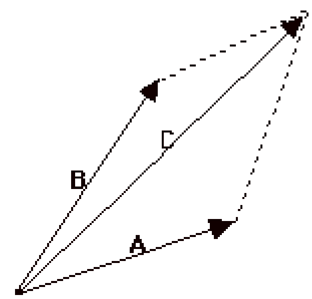
b) For vectors at an angle to each other use:

1. The polygon method. In Fig. 1a below, the vectors are placed head to tail. The resultant vector goes from the tail of the first vector A to the head of the second vector B to give C = A + B



(a) pologon method

2. The parallelogram method. Draw the vectors with their tails at one point. Complete the parallelogram and draw the diagonal to find the resultant. See Fig. 1b below.



(b) parallelgram method

Fig. 1

c) Cartesian coordinates

1. Unit vectors. The unit vector i, j, and k are unit vectors along the X, Y, and Z-axes, respectively. When multiplied by a number or a symbol that represents a quantity it becomes a vector with the magnitude of the quantity (or symbol). For example, 5m i is a vector of length 5 m along the X-axis.

2. Components of vectors

a) If a vector lies in the X-Y plane, it can be written as a component in the X-direction added vectorally to a component in the Y-direction. $A = A_x i + A_y j$, where A_x and A_y are the X and Y components of the vector A (Fig. 2a below).

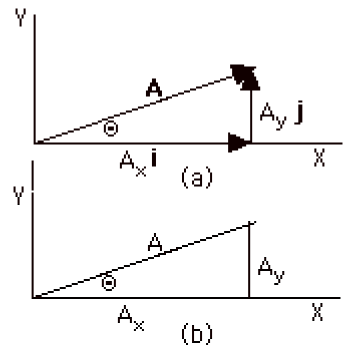


Fig. 2

b) From Fig. 2b above, we see that

A_x=Acosθ

A_y=Asinθ

and

$A = (A_x^2 + A_y^2)^{1/2}$

c) Utility of components.

If two vectors are equal, their components along any chosen axis are equal. If C=D, then $C_x = D_x$ and $C_y = D_y$

Example (Fig. 3 below)

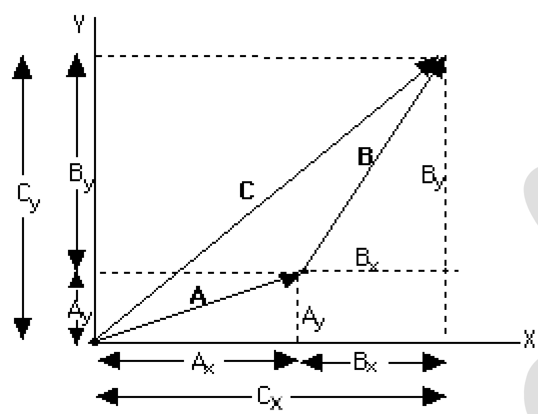


Fig. 3

$A = A_x i + A_x j$ $B = B_x i + B_x j$

$A + B = (A_x i + A_y j) + (B_x i + B_y j)$

$= (A_x + B_{x)})i + (A_y + B_y)j$

$C = C_x i + C_x j.$

If $C = A + B$, $C_x = (A_x + B_{x)})$, and

$C_y = (A_y + B_y)$ [Fig. 3 above]

For one dimensional motion

In the X-direction $V_x = V_{ox} + a_x t$ and $x = xo + V_{ox}\,t + 1/2\,a_x t^2$
In the Y-direction
$V_y = V_{oy} + a_y t$ and $y = y_o + V_{oy}\,t + 1/2\,a_y t^2$

For two dimensional motion

General Kinematic Equations for Constant Acceleration in Two Dimensions

x component (horizontal)	*y* component (vertical)
$v_x = v_{x0} + a_x t$	$v_y = v_{y0} + a_y t$
$x = x_0 + v_{x0} t + \frac{1}{2} a_x t^2$	$y = y_0 + v_{y0} t + \frac{1}{2} a_y t^2$
$v_x^2 = v_{x0}^2 + 2a_x(x - x_0)$	$v_y^2 = v_{y0}^2 + 2a_y(y - y_0)$

We can simplify Eqs. to use for projectile motion because we can set $a_x = 0$, which assumes y is positive upward, so $a_y = -g = -9.80\ \text{m/s}^2$.

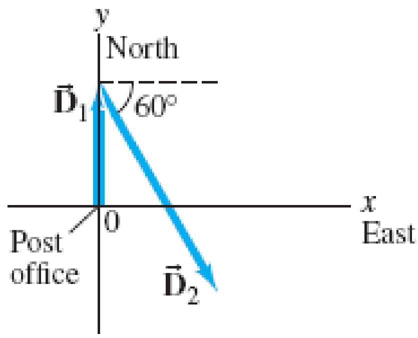
Kinematic Equations for Projectile Motion
(y positive upward; $a_x = 0$, $a_y = -g = -9.80\ \text{m/s}^2$)

Horizontal Motion ($a_x = 0$, v_x = constant)	Vertical Motion† ($a_y = -g$ = constant)
$v_x = v_{x0}$	$v_y = v_{y0} - gt$
$x = x_0 + v_{x0} t$	$y = y_0 + v_{y0} t - \frac{1}{2} g t^2$
	$v_y^2 = v_{y0}^2 - 2g(y - y_0)$

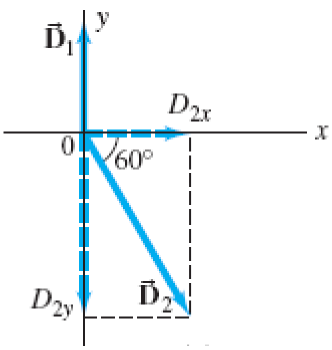
† If y is taken positive downward, the minus ($-$) signs in front of g become $+$ signs.

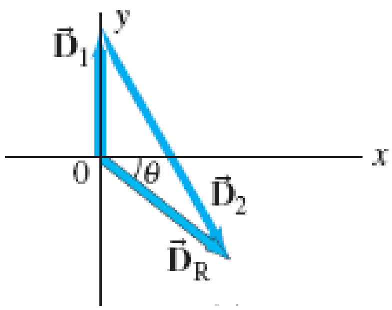
Examples and exercises

1- A mail carrier leaves the post office and drives 22.0 km in a northerly direction. He then drives in a direction 60.0° south of east for 47.0 km what is her displacement from the post office?



Solution





$$D_{1x} = 0, \qquad D_{1y} = 22.0\ \text{km}.$$

$\vec{D}_2$ has both x and y components:

$$D_{2x} = +(47.0\ \text{km})(\cos 60°) = +(47.0\ \text{km})(0.500) = +23.5\ \text{km}$$
$$D_{2y} = -(47.0\ \text{km})(\sin 60°) = -(47.0\ \text{km})(0.866) = -40.7\ \text{km}.$$

Notice that D_{2y} is negative because this vector component points along the negative y axis. The resultant vector, $\vec{D}_R$, has components:

$$D_{Rx} = D_{1x} + D_{2x} = 0\ \text{km} + 23.5\ \text{km} = +23.5\ \text{km}$$
$$D_{Ry} = D_{1y} + D_{2y} = 22.0\ \text{km} + (-40.7\ \text{km}) = -18.7\ \text{km}.$$

This specifies the resultant vector completely:

$$D_{Rx} = 23.5\ \text{km}, \qquad D_{Ry} = -18.7\ \text{km}.$$

$$D_R = \sqrt{D_{Rx}^2 + D_{Ry}^2} = \sqrt{(23.5\ \text{km})^2 + (-18.7\ \text{km})^2} = 30.0\ \text{km}$$

$$\tan\theta = \frac{D_{Ry}}{D_{Rx}} = \frac{-18.7\ \text{km}}{23.5\ \text{km}} = -0.796.$$

$$\theta = \tan^{-1}(-0.796) = -38.5°$$