7. ONE-DIMENSIONAL FLUID FLOW IN SOILS

Seepage through soils is defined as the flow of a fluid (Water in the most cases) through the pores of the soil under a pressure gradient.

7-1 Types of Water in the Soils

1- Free Water (Gravitational Water): a water that is free to move through a soil mass under the influence of gravity.
2- Held Water: a water that is held by soil particles, like: i- Structural water ii- Adsorbed water iii- Capillary water

It is important to understand the principles of water flow through soil mass in order to solve many geotechnical problems in the field like:

1- Estimate the quantity of seepage under the various hydraulic structures
2- Determine the stability under hydraulic structures that are subjected to seepage force
3- Investigating problems involving the pumping of water for underground construction

In Fig.(1), water will flow from point (A) to point (B) in a winding (Zigzag) path from a pore to another with a variable velocities. However, in geotechnical problems, it is assumed that water will flow (from A to B) in a straight line with a constant effective velocity.

Fig.(1): Water Path Through Soil Voids

7-2 **Permeability:**

It is a property of a porous material which permits the passage or seepage of a fluid through its *interconnecting* voids.

According to the above definition, coarse gravel is highly permeable soil, while stiff clay is the least permeable soil.

7-3 **Bernoulli's Equation**

According to Bernoulli's equation, the total head at a point in motion water is the sum of the pressure, velocity, and elevation heads, as explained below:

$$h = \frac{u}{\gamma_w} + \frac{v^2}{2g} + Z$$

where:

h: total head u:

pressure *v*: velocity g:

gravity acceleration

For water flow in a soil media, the term $\frac{v^2}{2g}$ can be neglected because the seepage velocity is small, so

$$h = \frac{u}{\gamma_w} + Z$$

Fig.(2) below shows the relationship between pressure, elevation and total heads for water flow in the soil, piezometer tubes installed in points **A** and **B** to measure the pressure head, the elevation head for a point is the vertical distance from this point to any assumed horizontal datum.

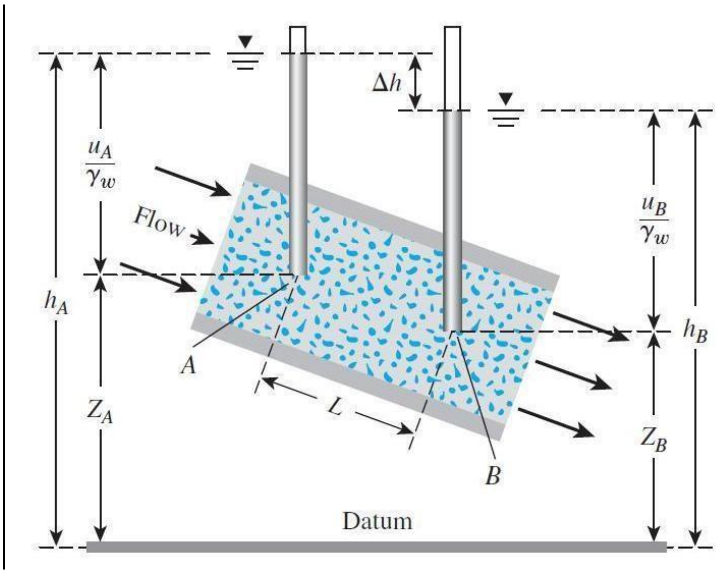

Fig.(2): Relationship Between (i) Pressure, (ii) Elevation, (iii) total head. For Water Flow in Soil

The head loss between points A and B can be expressed as:

$$\Delta h = h_A - h_B = \left(\frac{u_A}{\gamma_w} + Z_A\right) - \left(\frac{u_B}{\gamma_w} + Z_B\right)$$

The head loss (Δh) will be expressed in a non-dimensional form as:

$$i = \frac{\Delta h}{L}$$

where:

i= hydraulic gradient

L: the length of flow (distance from A to B)

The variation of the velocity (v) with the hydraulic gradient (i) is shown in Fig.(3). This relationship is divided into three zones:

1. Laminar flow zone (Zone I)
2. Transition zone (Zone II)
3. Turbulent flow zone (Zone III)

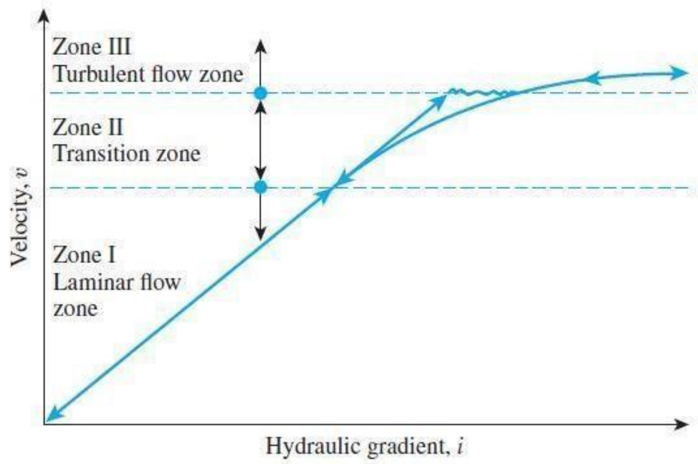

Fig.(3): Relationship Between (v) and (i)

In most soils, the flow of water through the void spaces can be considered as laminar;

Thus; $v \propto i$

In fractured rocks, stones, gravels, and very coarse sands, turbulent flow conditions may exist, and the above equation may not be valid.

7-4 <u>Darcy's Law</u>

In 1856, a French scientist called **Henry Darcy** published a simple equation to describe the relationship between the hydraulic gradient and the discharge velocity as below:

$$v = ki$$

where:

v= discharge velocity k= coefficient of permeability, has a velocity units like

(cm/sec) (ft/min)… etc and

$$Q = kiA$$

where: A=cross-sectional area of the soil

Darcy's law is applicable for <u>laminar flow condition</u> and for <u>wide range of soil</u> <u>types</u> in <u>saturation state</u>

7-5 <u>Discharge Velocity</u> (*v*) <u>and Seepage Velocity</u> *(v_s)*

The actual velocity of water (the seepage velocity) (v_s) is higher than the discharge velocity (*v*), according to Fig.(4) the quantity of flowing water through the soil in a unit of time is (**q**), then

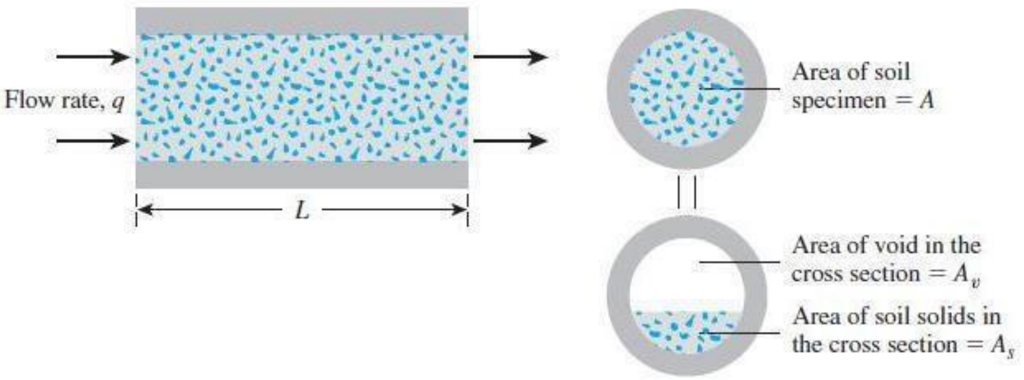

Fig.(4): Difference Between Discharge and Seepage Velocities

$$q = vA = A_v v_s$$

where: A_v= area of voids

But $A = A_v + A_s$

where A_s= area of solids

$$q = v(A_v + A_s) = v_s A_v$$

$$v_s = \frac{v(A_v + A_s)}{A_v} \times \frac{L}{L}$$

$$v_s = \frac{v(V_v + V_s)}{V_v} \times \frac{V_s}{V_s}$$

$$v_s = v\frac{\left(1 + \frac{V_v}{V_s}\right)}{\frac{V_v}{V_s}} = v\frac{(1+e)}{e}$$

$$v_s = \frac{v}{n}$$

where: e=void ratio

n=porosity

7-6 <u>Factors Affecting the Coefficient of Permeability</u>

1- Fluid viscosity
2- Pore-size distribution
3- Grain-size distribution
4- Void ratio
5- Roughness of mineral particles
6- Degree of saturation
   And for clay soils there are three more factors adding to the above
7- Structures
8- Ionic concentration
9- Thickness of the soil layer

7-7 <u>Determination of the Coefficient of Permeability</u>

A- Laboratory Tests
   i- Constant Head Test
   ii- Falling Head Test
B- Field Methods
   i- Wells Pumping Method
   ii- Auger Holes Method

<u>A-i) Constant Head Test</u>

As shown in Fig. (5), the water is supplying from at the inlet in such a way to keep the difference of head between the inlet and outlet still constant. Water is allocated in graduated flask for a known duration of time

then

$$Q = Avt = A(ki)t$$

where:

Q: Volume of water in the flask

A: cross-sectional area of the soil sample

t: duration of collection water but

$$i = \frac{h}{L}$$

where:

h: the difference between the inlet and outlet

L=length of the soil sample

Then

$$k = \frac{QL}{Aht}$$

This test is suitable for course soils that have high values of (*k*).

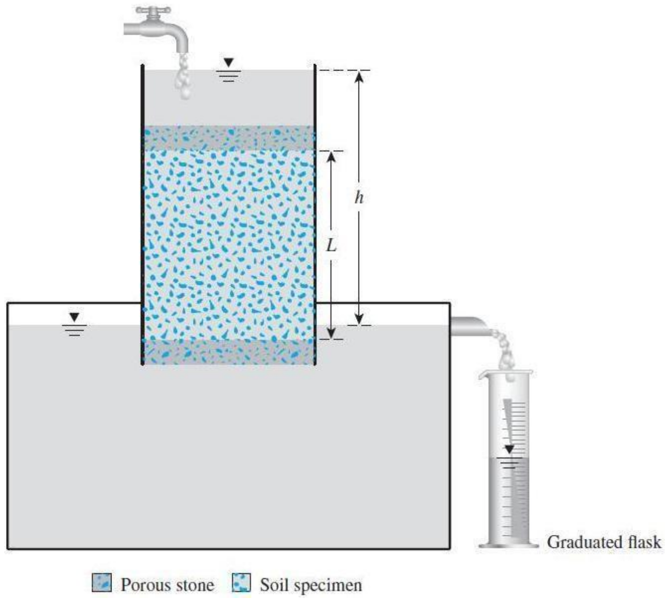

Fig.(5): Constant Head Test

<u>**A-ii) Falling Head Test**</u> As shown in Fig.(6)

h_1 = is the initial head difference at time(t=0)

h_2 = is the final head difference at time ($t=t_F$)

A: cross-sectional area of the soil sample

a: cross-sectional area of the stand pipe

The coefficient of permeability can be calculated from the equation below

$$k = \frac{a}{A}\frac{L}{(t_2 - t_1)} ln\frac{h_1}{h_2}$$

This test is suitable for fine soils that have low values of (*k*)

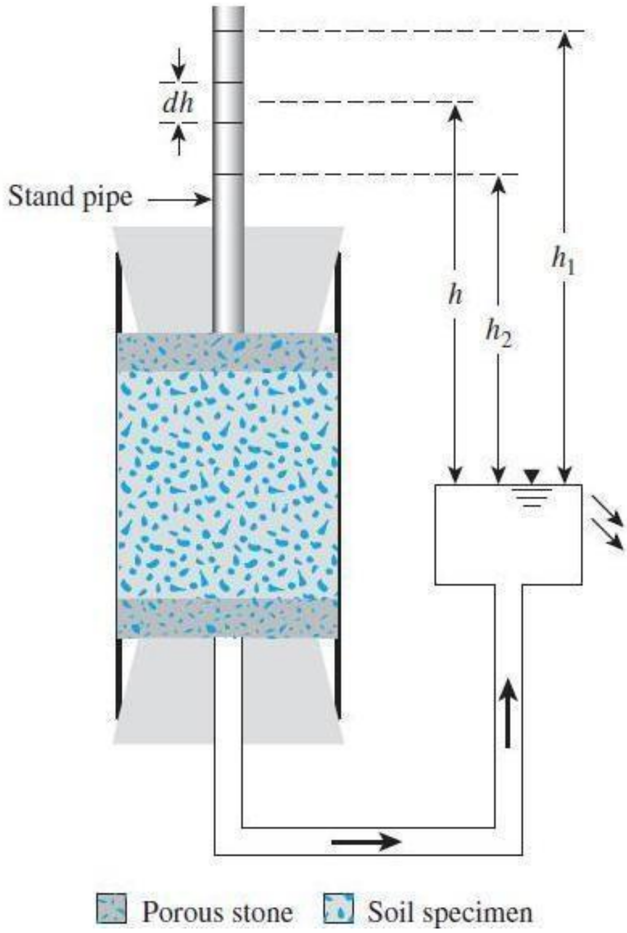

Fig.(6): Falling Head Test

B-i) Well-Pumping Method

This method is used in order to determine the (k) for an unconfined flow in soil layer underlined by impermeable layer. During the test, water is pumped out at a constant rate from a test well that has a perforated casing. Several observation wells at various radial distances are made around the test well. Continuous observations of the water level in the test well and in the observation wells are made after the start of pumping, until a steady state is reached. The steady state is established when the water level in the test and observation wells becomes constant. As shown in the Fig.(7), the below formula may be applied to find (k)

$$k = \frac{2.303q \; log\left(\frac{r_1}{r_2}\right)}{\pi(h_1^2 - h_2^2)}$$

And in order to determine (k) of an confined aquifer (i.e.) (located between two impermeable layers) as shown in Fig.(8) the below formula may be used

$$k = \frac{q \log\left(\frac{r_1}{r_2}\right)}{2.727(h_1 - h_2)}$$

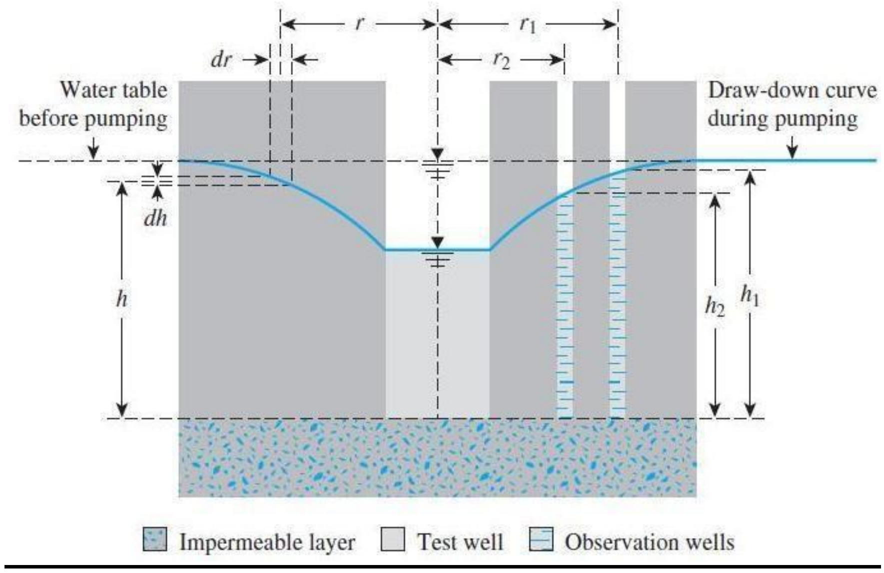

Fig.(7): Pumping Wells Method

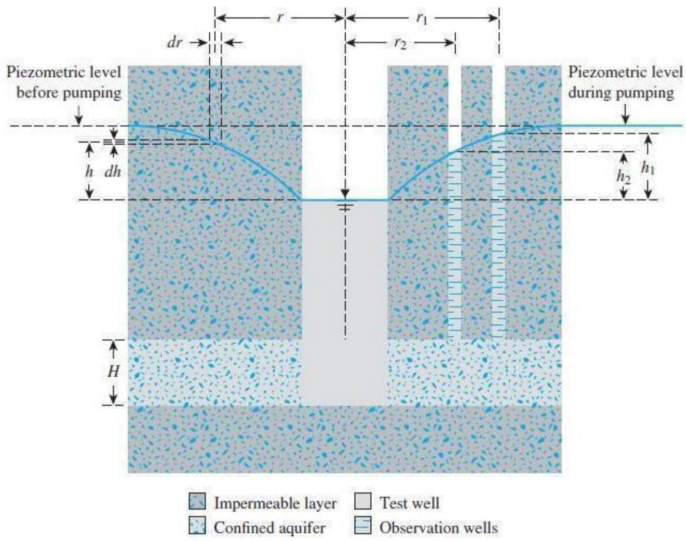

Fig.(8): Pumping Wells Method (confined aquifer)

B-ii) Auger Holes Method

As shown in Fig.(9), after making the hole in the field, water is also pumped out from the hole, after a time , water will flow to the hole making the water level rising again.

$$k = \frac{40}{\left(20 + \frac{L}{r}\right)\left(2 - \frac{y}{L}\right)} \frac{r}{y} \frac{\Delta y}{\Delta t}$$

where:

r= radius of the auger hole y= average value of distance of the water level in the auger hole measuring from the G.W.T. during a time interval Δt.

Noting that (L≡ meters)

7-8 Equivalent Permeability in Stratified Soil

In a stratified soil, the hydraulic conductivity changes from layer to layer, so in order to simplify the calculation, an equivalent hydraulic conductivity (k_{eq}) can be computed.

A- Horizontal Direction of Flow

Figure (10) shows (n) layers of soil with flow in the *horizontal direction*, for a unit length of cross-sectional area perpendicular to the direction of flow

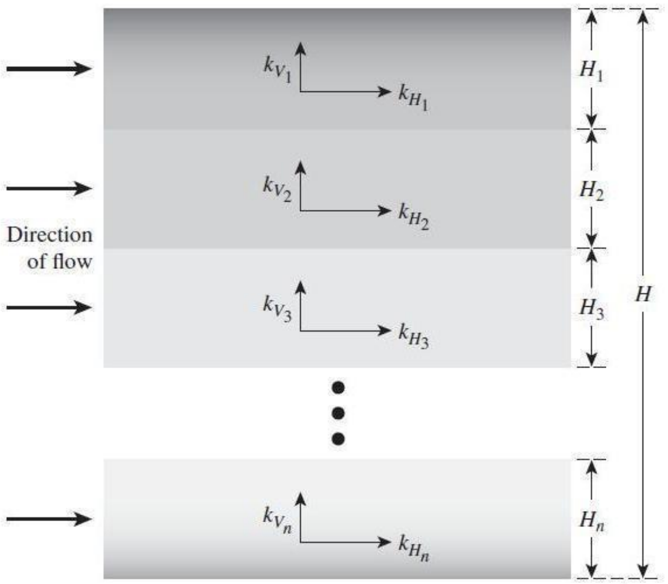

Fig.(10): Determination ($k_{H(eq)}$) in Case of Horizontal Flow in Stratified Soil

$$q = v.1.H$$

Where: v=

$$q = v_1.1.H_1 + v_2.1.H_2 + v_3.1.H_3 + \cdots + v_n.1.H_n$$

equivalent velocity $v_1, v_2, v_3 \ldots . v_n$= discharge velocities for layers 1, 2, 3….. n, respectively if $k_{H1}, k_{H2}, k_{H3}, \ldots . k_{Hn}$, are the horizontal hydraulic conductivity for layers 1, 2, 3…n and if $k_{H(eq)}$ is the equivalent horizontal hydraulic conductivity, so according to

Darcy's Law

$$v = k_{H(eq)} i_{(eq)};\quad v_1 = k_{H1} i_1;\quad v_2 = k_{H2} i_2;\quad v_3 = k_{H3} i_3;\quad v_n = k_{Hn} i_n$$

Noting that $i_{(eq)}=i_1=i_2=i_3 \ldots \ldots =i_n$

$$k_{H(eq)} = \frac{1}{H}(k_{H1}H_1 + k_{H2}H_2 + k_{H3}H_3 + \cdots + k_{Hn}H_n) = \frac{\sum_1^n k_H H}{\sum_1^n H}$$

B- <u>Vertical Direction of Flow</u>

Figure (11) shows (n) layers of soil with flow in the *vertical direction*, for a unit length of cross-sectional area perpendicular to the direction of flow

In this case, the velocity of flow through all the layers is the same. However, the total head loss, h, is equal to the sum of the head losses in all layers. Thus,

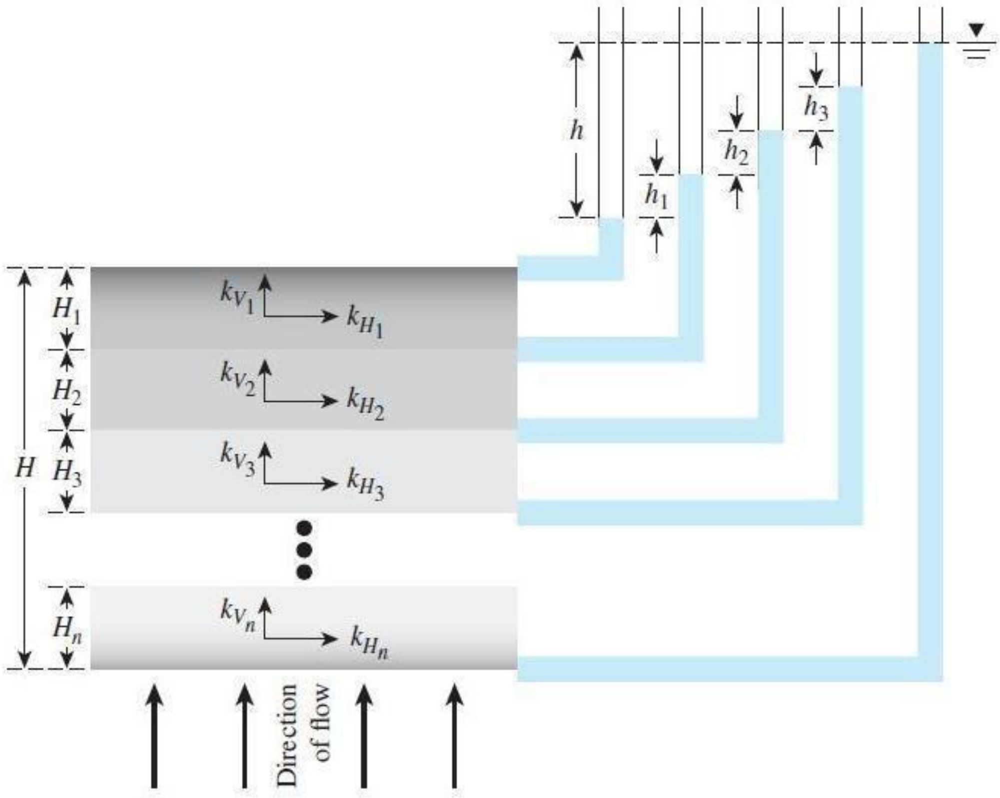

Fig.(10): Determination ($k_{V(eq)}$) in Case of Vertical Flow in Stratified Soil

$v=v_1=v_2=v_3 ... v_n$

$h=h_1+h_2+h_3+... h_n$

according to Darcy's law

$$k_{V(eq)}\left(\frac{h}{H}\right) = k_{V1}i_1 = k_{V2}i_2 = k_{V3}i_3 = \cdots = k_{Vn}i_n$$

where:

K_{V1}, k_{V2}, k_{H3},…. K_{Vn}, are the vertical hydraulic conductivity for layers 1, 2, 3…n

$K_{V(eq)}$ is the equivalent vertical hydraulic conductivity

But: $$h = H_1 i_1 + H_2 i_2 + H_3 i_3 + \cdots + H_n i_n$$

Then

$$k_{V(eq)} = \frac{H}{\left(\frac{H_1}{k_{V1}}\right) + \left(\frac{H_2}{k_{V2}}\right) + \left(\frac{H_3}{k_{V3}}\right) + \cdots + \left(\frac{H_n}{k_{Vn}}\right)} = \frac{\sum_1^n H}{\sum_1^n \frac{H}{k_V}}$$

7-9 Empirical Equations of Coefficient of Permeability

There are many empirical equations used to determine the value (*k*), as below:

$$k = cD_{10}^2$$

This equation valid for $C_u \leq 5$ where:

c = constant (1.0~5)

D_{10}= effective diameter≡ mm

Another equation for determination (*k*) as below

$$k = C_2 \; D_{10}^{2.32} \; C_u^{0.6} \; \frac{e^3}{1+e}$$

where: C_2=constant

Typical Values of Permeability Coefficient

Soil Type	*k* (cm/sec)
Clean Gravel	1~100
Coarse Sand	$1\sim10^{-2}$
Fine Sand	$10^{-2}\sim10^{-3}$
Silt	$10^{-3}\sim10^{-5}$
Clay	$<10^{-6}$

Table (1): Typical Ranges of Values of Hydraulic Conductivity for Soil Types