Principles of Baseband Digital Data Transmission

Line Codes and their Power Spectra

The digital data can be transmitted by various transmission or line codes that represent baseband transmission such as unipolar, polar, bipolar and so on. This is called *line coding* that is used for short or medium distance communication. These signals are transmitted without carrier modulation. Several considerations should be taken into account in choosing an appropriate data format for a given application. Among these are: *efficient transmission bandwidth*, *power efficiency*, *error detection and correction capability*, *Self-synchronization*, *Transparency, good bit error probability performance*.

Let be defined $s_0(t)$, $s_1(t)$ as waveform transmitted signals for binary '0' and '1' respectively. Then the following line coding techniques are studied.

1. Unipolar Trans

Unipolar Non Return to Zero (NRZ)

$$s_0(t) = 0 \quad 0 \le t < T_b$$
$$s_1(t) = +A \quad 0 \le t < T_b$$

where T_b is the duration of the information bits (sec/bit).

Let $a_k, k = 0, \pm1, \pm2, \ldots.$, represents the binary digits sequence ($a_k = 0 \text{ or } 1 \text{ for all } k$). The binary sequence waveform

$$a(t) = \sum_{k=-\infty}^{\infty} a_k \, \delta(t - kT_b)$$

The function $h(t)$ is a deterministic pulse-type waveform

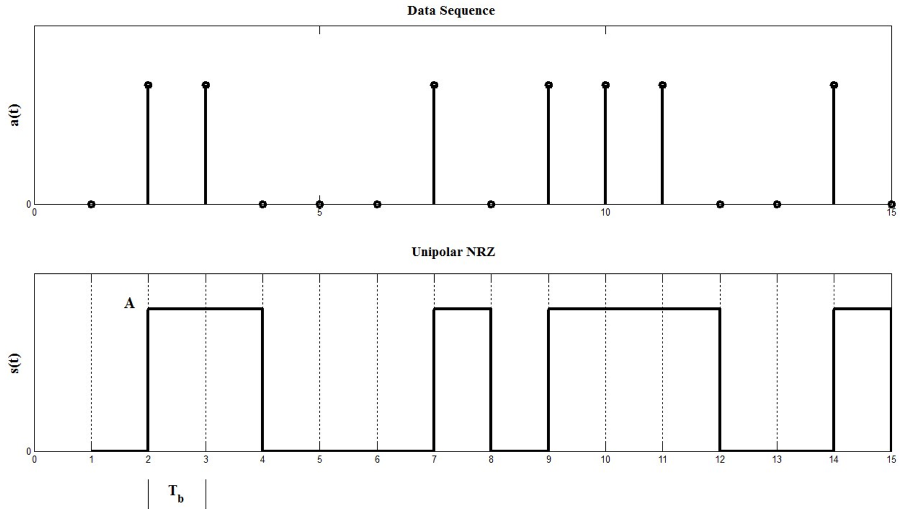
$$h(t) = rect\left[\frac{t - (T_b/2)}{T_b}\right]$$

And the corresponding the frequency response P(w)

$$H(w) = T_b \, \mathrm{Sa}\left(\frac{wT_b}{2}\right) e^{-jwT_b/2}$$

Then the transmitted unipolar NRZ waveform

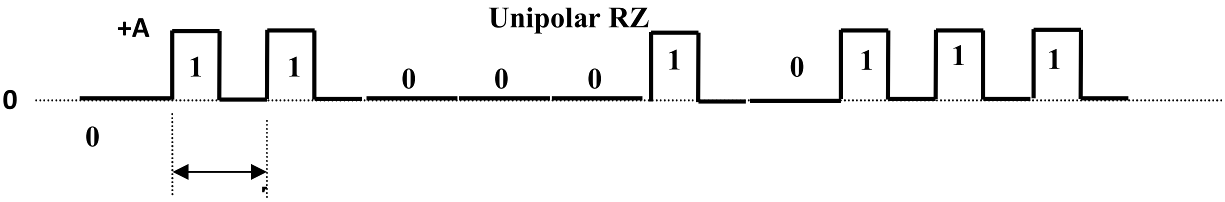
$$s(t) = h(t) \circledast a(t) = \sum_{k=-\infty}^{\infty} a_k \, rect\left[\frac{t - (T_b/2) - kT_b}{T_b}\right]$$



Unipolar Return to Zero (RZ)

$s_0(t) = 0 \quad 0 \le t < T_b$

$$s_1(t) = \begin{cases} +A & 0 \le t < T_b/2 \\ 0 & T_b/2 \le t < T_b \end{cases}$$



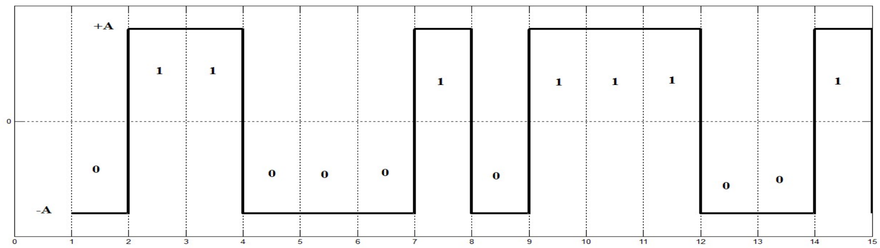
2. **Polar Trans**
   Polar Non Return to Zero (NRZ)

$s_0(t) = -A \quad 0 \le t < T_b$

$s_1(t) = +A \quad 0 \le t < T_b$

The transmitted polar NRZ waveform,

$$s(t) = \sum_{k=-\infty}^{\infty} a_k \, rect\left[\frac{t - (T_b/2) - kT_b}{T_b}\right]$$



Polar Return to Zero (RZ)

$$s_0(t) = \begin{cases} -A & 0 \le t < T_b/2 \\ 0 & T_b/2 \le t < T_b \end{cases}$$

$$s_1(t) = \begin{cases} +A & 0 \le t < T_b/2 \\ 0 & T_b/2 \le t < T_b \end{cases}$$

The function $h(t)$ for Polar RZ is

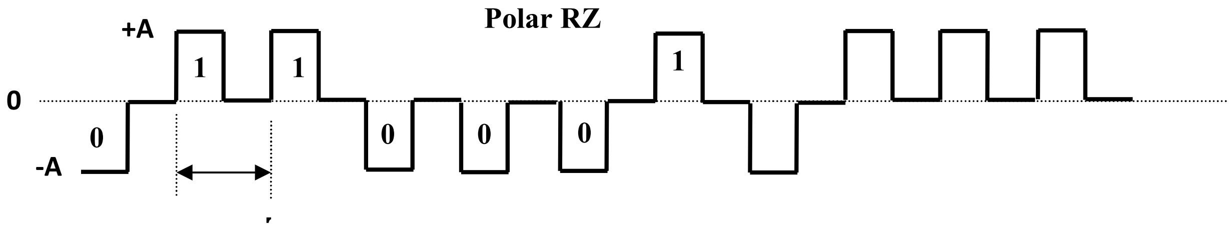
$$h(t) = rect\left[\frac{t-(T_b/4)}{T_b/2}\right]$$

And the corresponding the frequency response P(w)

$$H(w) = \frac{T_b}{2}\text{Sa}\left(\frac{wT_b}{4}\right)e^{-jwT_b/4}$$

Then the transmitted polar RZ waveform

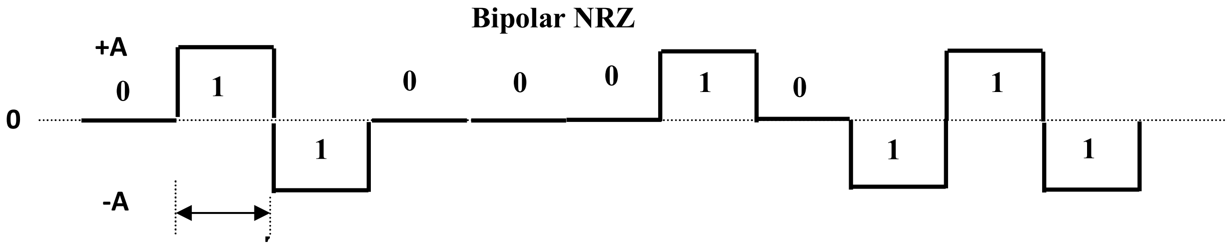
$$s(t) = h(t) \circledast a(t) = \sum_{k=-\infty}^{\infty} a_k\, rect\left[\frac{t-(T_b/4)-kT_b}{T_b/2}\right]$$



3. Bipolar Trans

Bipolar NRZ

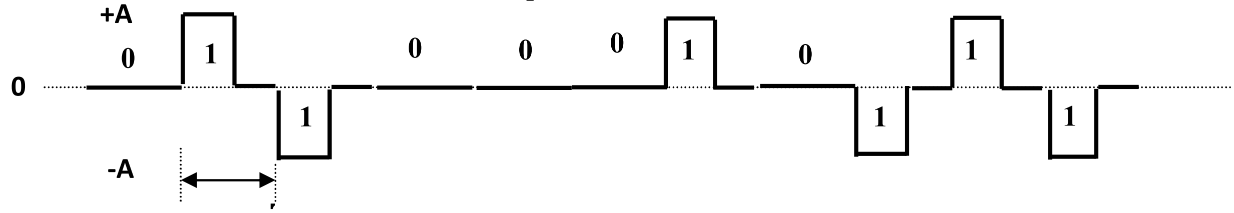
In the bipolar NRZ format, a code 0s is transmitted by no pulse and 1s are sent as pulses of alternating polarity. NRZ can also be implemented, usually with pulse duration T_b. Bipolar modulation also called ***pseudoternary*** since it has three values +A,0, and –A. Another name is (***alternate mark inversion*** (**AMI**)).



Bipolar RZ

The code '0' is represented by a 0 level; '1's are represented by RZ pulses that alternate in sign. RZ can also be implemented, usually with pulse duration Tb/2.

Bipolar RZ



4. Split Phase Manchester Code

$$s_1(t) = \begin{cases} +A & 0 \le t < T_b/2 \\ -A & T_b/2 \le t < T_b \end{cases}$$

$$s_0(t) = \begin{cases} -A & 0 \le t < T_b/2 \\ +A & T_b/2 \le t < T_b \end{cases}$$

Also called ***bi-phase level*** and ***twinned binary code***.

The function $p(t)$ for Manchester is

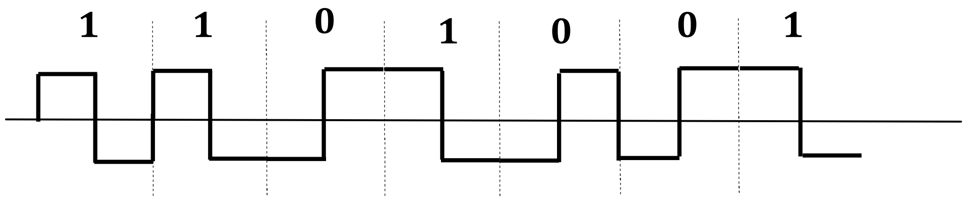
$$h(t) = rect\left[\frac{t + (T_b/4)}{T_b/2}\right] - rect\left[\frac{t - (T_b/4)}{T_b/2}\right]$$

And the corresponding the frequency response P(w)

$$H(w) = jT_b \mathrm{Sa}\left(\frac{wT_b}{4}\right) sin\left(\frac{wT_b}{4}\right)$$

Then the transmitted Manchester waveform

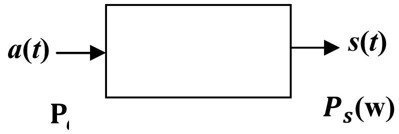
$$s(t) = \sum_{k=-\infty}^{\infty} a_k \left(rect\left[\frac{t - (T_b/4) - kT_b}{T_b/2}\right] - rect\left[\frac{t - (3T_b/4) - kT_b}{T_b/2}\right]\right)$$



Power Spectral Density for various Line Codes

Let us assume the 0s and 1s are equal probability with p(0)=p(1)=1/2.

If the power spectrum of a(t) and s(t) are denoted as $P_a(w)$ and $P_s(w)$ respectively. Then



$$\boldsymbol{P_s(w) = P_a(w)|H(w)|^2}$$

$$\boldsymbol{P_s(f) = \frac{1}{T_b}|H(f)|^2 \sum_{k=-\infty}^{\infty} R(k)e^{j2\pi kf\;\;_b}}$$

$(\ \textit{see Leon Page } 416\ \textit{Eq.}\,6-70)$

where R(k) is the autocorrelation of the data that is given by

$$\boldsymbol{R(k) = \sum_{i=1}^{I}(a_n a_{n+k})_i\, p_i = E[a_n a_{n+k}]},$$

where a_n and a_{n+k} are the voltage levels of the data pulse at the nth and (n+k)th symbol position, and p_i is the probability of having the ith $a_n a_{n+k}$ product.

Unipolar NRZ signaling:

The possible levels for the a's are **+A** and **0** V. Now to find R(k), for k=0 the possible products of $a_n a_n$ are $A \times A$=A^2 and 0×0=0, and consequently, I=2. For random data, the probability of having A^2 is 1/2 and the probability of having 0 is 1/2, so that

$$R(0) = \sum\nolimits_{i=1}^{2}(a_n a_n)_i\, p_i = A^2.\frac{1}{2} + 0.\frac{1}{2} = \frac{A^2}{2}$$

For k≠0, there are I=4 possibilities for the product values: $A \times A$, $A \times 0$, and $0 \times A$, 0×0. They occur with a probability of 1/4 . Thus

$$R(k) = \sum\nolimits_{i=1}^{4}(a_n a_{n+k})p_i = A^2.\frac{1}{4} + 0.\frac{1}{4} + 0.\frac{1}{4} + 0.\frac{1}{4} = \frac{A^2}{4}$$

Hence

$$R_{unipolar}(k) = \begin{cases} \frac{1}{2}A^2 & k = 0 \\ \frac{1}{4}A^2 & k \neq 0 \end{cases}$$

For rectangular NRZ pulse shapes, the Frequency response is

$$|H(f)| = T_b \frac{sin(nfT_b)}{nfT_b}$$

Then, the PSD for the unipolar NRZ line code is

$$\mathcal{P}_{unipolar\ NRZ}(f) = \frac{1}{T_b}\left|T_b \frac{sin(\pi f T_b)}{\pi f T_b}\right|^2 \left[\sum_{k=0} \frac{A^2}{2} e^{j2\pi kfT_b} + \sum_{\substack{k=-\infty \\ k\neq 0}}^{\infty} \frac{A^2}{4} e^{j2\pi kfT_b}\right]$$

$$= \frac{A^2T_b}{2}\left(\frac{sin(\pi fT_b)}{\pi f\;\;_b}\right)^2 + \frac{A^2T_b}{4}\left(\frac{sin(\pi f\;\;_b)}{\pi f\;\;_b}\right)^2 \sum_{\substack{k=-\infty \\ k\neq 0}}^{\infty} e^{j2\pi kf\;\;_b}$$

But

$$\sum_{\substack{k=-\infty \\ k\neq 0}}^{\infty} e^{j2\pi kfT_b} = \frac{1}{T_b}\sum\nolimits_{m=-\infty}^{\infty} \delta\left(f - \frac{m}{T_b}\right) \qquad ,(Poisson's\ expression)$$

Thus,

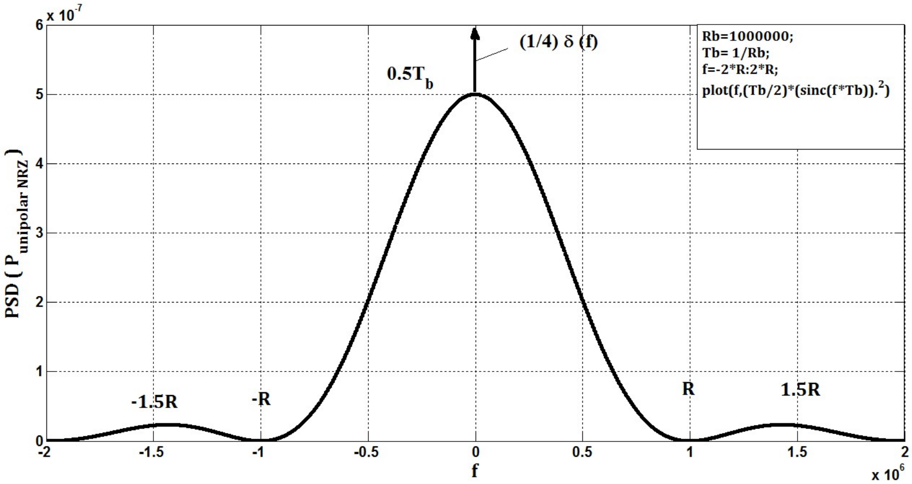
$$\mathcal{P}_{unipolar\ NRZ}(f) = \frac{A^2T_b}{2}\left(\frac{sin(\pi f T_b)}{\pi f T_b}\right)^2 + \frac{A^2}{4}\left(\frac{sin(\pi f T_b)}{\pi f T_b}\right)^2 \sum_{m=-\infty}^{\infty} \delta\left(f - \frac{m}{T_b}\right)$$

Because $\frac{sin(nfT_b)}{nf\ {}_b} = 0\ at\ f = \frac{m}{T_b}$ for m≠ 0, then the term

$$\left(\frac{sin(\pi f\ {}_b)}{\pi f T_b}\right)^2 \sum_{m=-\infty}^{\infty} \delta\left(f - \frac{m}{T_b}\right) = \delta(f)$$

and,

$$\boldsymbol{\mathcal{P}_{unipolar\ NRZ}(f) = \frac{A^2T_b}{2}\left(\frac{sin(\pi f T_b)}{\pi f\ {}_b}\right)^2 + \frac{A^2}{4}\delta(f)}$$



Unipolar RZ signaling:

The unipolar RZ has the same autocorrelation function of unipolar NRZ. For RZ signaling, the pulse duration is $T_b/2$ instead of T_b, as used in NRZ signaling. That is

$$|H(f)| = \frac{T_b}{2}\frac{sin(nfT_b/2)}{nfT_b/2}$$

Then, the PSD for the unipolar NRZ line code is

$$\mathcal{P}_{unipolar\ RZ}(f) = \frac{1}{T_b}\left|\frac{T_b}{2}\frac{sin(\pi f\ {}_b/2)}{\pi f\ {}_b/2}\right|^2 \left[\sum_{k=0} \frac{A^2}{2} e^{j2\pi k f\ {}_b} + \sum_{\substack{k=-\infty \\ k\neq 0}}^{\infty} \frac{A^2}{4} e^{j2\pi k f\ {}_b}\right]$$

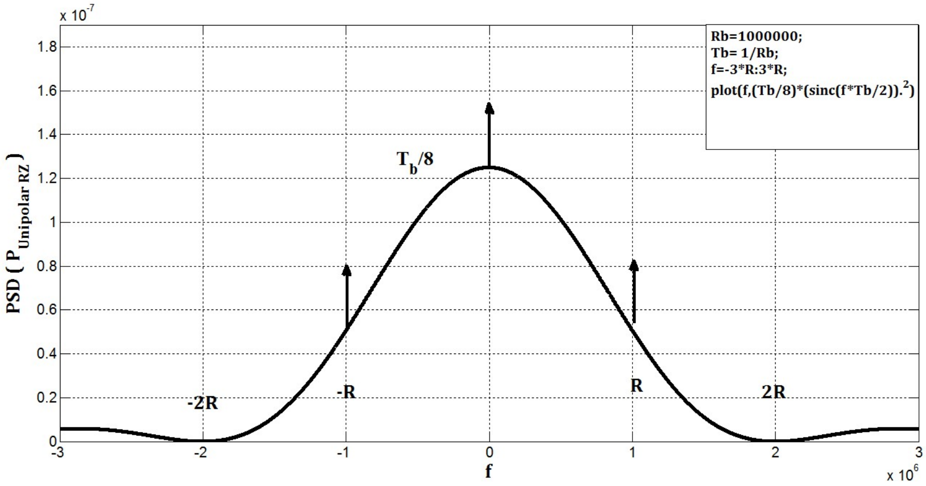
$$= \frac{A^2T_b}{8}\left(\frac{sin(\pi f\ {}_b/2)}{\pi f\ {}_b/2}\right)^2 + \frac{A^2T_b}{16}\left(\frac{sin(\pi f T_b/2)}{\pi f T_b/2}\right)^2 \sum_{\substack{k=-\infty \\ k\neq 0}}^{\infty} e^{j2\pi k\ \ {}_b}$$

But

$$\sum_{\substack{k=-\infty \\ k\neq 0}}^{\infty} e^{j2\pi k f\ {}_b} = \frac{1}{T_b}\sum_{m=-\infty}^{\infty} \delta\left(f - \frac{m}{T_b}\right) \qquad ,(Poisson's\ expression)$$

Thus,

$$\mathcal{P}_{unipolar\ RZ}(f) = \frac{A^2 T_b}{8}\left(\frac{\sin(\pi f T_b/2)}{\pi f T_b/2}\right)^2 + \frac{A^2}{16}\left(\frac{\sin(\pi f T_b/2)}{\pi f T_b/2}\right)^2 \sum_{m=-\infty}^{\infty} \delta\left(f - \frac{m}{T_b}\right)$$



Polar NRZ signaling:

The possible levels for the a's are **+A** and **–A**. Now to find R(k), for k=0 the possible products of $a_n a_n$ are $A \times A$=A^2 and $-A \times -A$=A^2, and consequently, I=2. For random data, the probability of having A^2 is 1/2 and the probability of having $(-A)^2$ is 1/2, so that

$$R(0) = \sum_{i=1}^{2} (a_n a_n)_i\, p_i = A^2.\frac{1}{2} + A^2.\frac{1}{2} = A^2$$

For k≠0, there are I=4 possibilities for the product values: $A \times A$, $A \times -A$, and $-A \times A$, $-A \times -A$. They occur with a probability of 1/4 . Thus

$$R(k) = \sum_{i=1}^{4} (a_n a_{n+k}) p_i = A^2.\frac{1}{4} + (-A)(A).\frac{1}{4} + (A)(-A).\frac{1}{4} + (-A)^2.\frac{1}{4} = 0$$

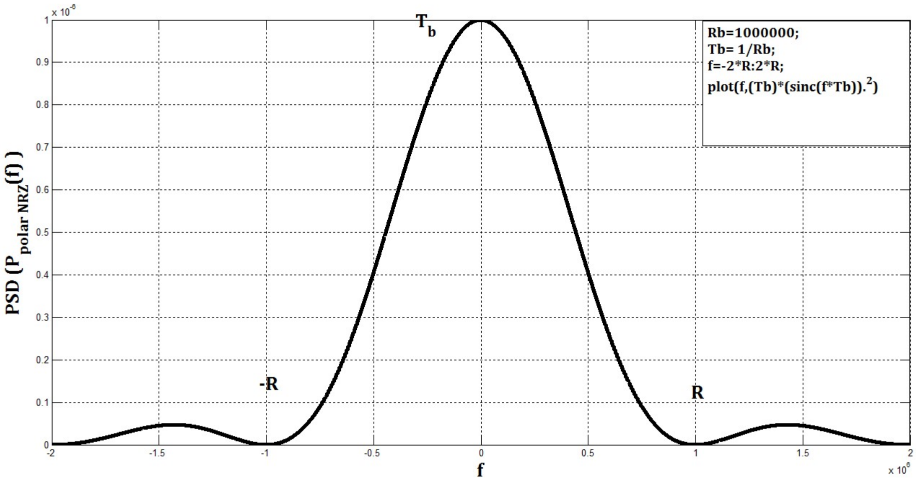
Hence

$$R_{polar}\ (k) = \begin{cases} A^2 & k = 0 \\ 0 & k \neq 0 \end{cases}$$

Then, the PSD for the polar NRZ line code is

$$\mathcal{P}_{polar\ NRZ}(f) = \frac{1}{T_b}\left|T_b \frac{\sin(\pi f\ \ _b)}{\pi f T_b}\right|^2 \left[\sum_{k=0} A^2\, e^{j2\pi k f\ \ _b} + \sum_{\substack{k=-\infty \\ k\neq 0}}^{\infty} (0) e^{j2\pi k f\ \ _b}\right]$$

$$\mathcal{P}_{polar\ NRZ}(f) = A^2 T_b \left(\frac{\sin(\pi f\ \ _b)}{\pi f T_b}\right)^2$$



Bipolar RZ and NRZ signaling:

The permitted values for a_n are +A,-A and 0, where binary 1's are represented by alternating +A and –A values a binary 0 is presented by $a_n=0$; these occur with probabilities 1/4, 1/4, and 1/2 respectively. Thus when k=0,

$$R(0) = 0^2\left(\frac{1}{2}\right) + A^2\frac{1}{4} + (-A)^2\frac{1}{4} = \frac{A^2}{2}$$

For k=1 (the adjacent bit case) and the data sequences (1,1),(1,0),(0,1) and (0,0), the possible $a_n\ a_{n+1}$ products are $-A^2$,0,0,0. Each of these sequences occurs with a probability of 1/4. Thus

$$R(1) = \sum_{i=1}^{4} (a_n a_{n+1})_i\, p_i = -\frac{A^2}{4}$$

For k>1, the bits being considered are not adjacent, and the $a_n a_{n+k}$ products are $\mp A$, 0,0,0, these occur with a probability of 1/4. then

$$R(k > 1) = \sum_{i=1}^{4} (a_n a_{n+k})_i\, p_i = -\frac{A^2}{8} + \frac{A^2}{8} = 0$$

$$R_{bipolar}\,(k) = \begin{cases} \dfrac{A^2}{2} & k = 0 \\ \dfrac{-A^2}{4} & |k| = 1 \\ 0 & |k| > 1 \end{cases}$$

The PSD of bipolar NRZ format is

$$\mathcal{P}_{bipolar\ NRZ}(f) = \frac{1}{T_b}\left|T_b\frac{sin(\pi f T_b)}{\pi f T_b}\right|^2 \left[\sum_{k=0}\frac{A^2}{2}\,e^{j2\pi k\;_b} + \sum_{k=\pm 1} -\frac{A^2}{4}\,e^{j2\pi kf\;_b} + \sum_{\substack{k=-\infty\\k>1}}^{\infty}(0)e^{j2\pi kfT_b}\right]$$

$$= T_b\left(\frac{sin(\pi f T_b)}{\pi f T_b}\right)^2 \left[\frac{A^2}{2} - \frac{A^2}{4}(e^{j2\pi fT_b} + e^{-j2\pi f\;_b})\right]$$

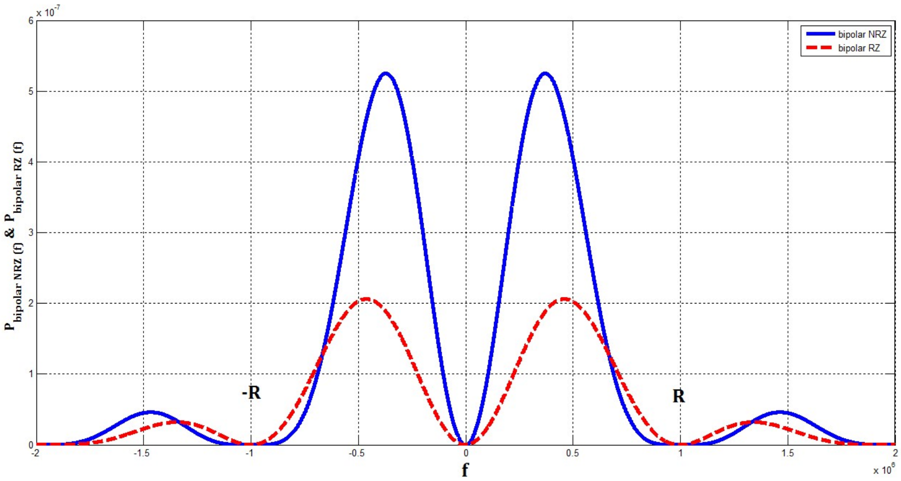
$$= \frac{T_b A^2}{2}\left(\frac{sin(\pi f T_b)}{\pi f T_b}\right)^2 \left[1 - \frac{1}{2}(e^{j2\pi fT_b} + e^{-j2\pi f\;_b})\right]$$

$$= \frac{T_b A^2}{2}\left(\frac{sin(\pi f T_b)}{\pi f\;_b}\right)^2 [1 - \cos(2\pi f T_b)]$$

$$\boldsymbol{\mathcal{P}_{bipolar\ NRZ}(f) = T_b A^2\left(\frac{sin(\pi f\;_b)}{\pi f T_b}\right)^2 sin^2(\pi f\;_b)}$$

Similarly for bipolar RZ, the PSD is given by

$$\boldsymbol{\mathcal{P}_{bipolar\ RZ}(f) = \frac{T_b A^2}{4}\left(\frac{sin(\pi f\;_b/2)}{\pi f\;_b/2}\right)^2 sin^2(\pi f T_b)}$$



Manchester NRZ Signaling:

The frequency response of the pulse shape for Manchester NRZ is,

$$H(f) = jT_b\left[\frac{sin(\pi f\;_b/2)}{\pi f\;_b/2}\right] sin\left(\frac{wT_b}{4}\right)$$

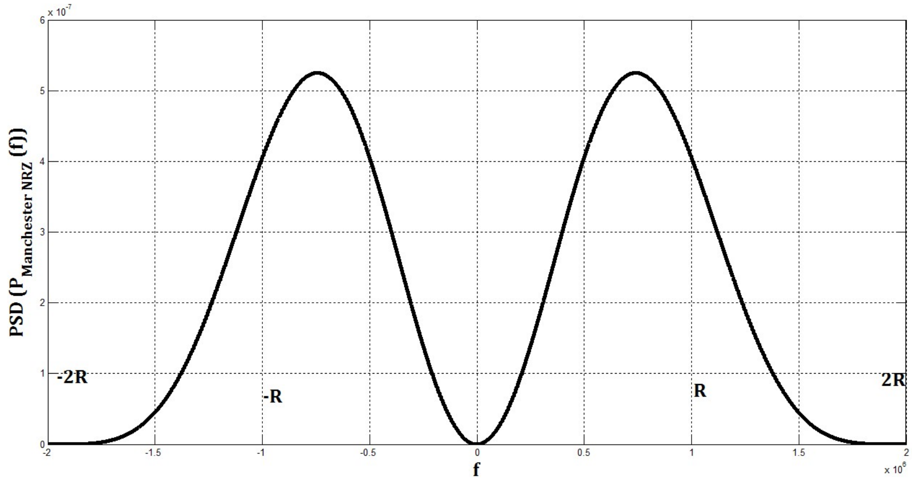
The autocorrelation for Manchester code is the same as the polar NRZ format,

$$R_{Mancheste}\ (k) = \begin{cases} A^2 & k = 0 \\ 0 & k \neq 0 \end{cases}$$

Hence, The PSD

$$\mathcal{P}_{Manchester\ NRZ}(f) = \frac{1}{T_b}\left| jT_b \frac{sin\left(\frac{\pi f T_b}{2}\right)}{\frac{\pi f T_b}{2}} \sin\left(\frac{wT_b}{4}\right) \right|^2 \left[\sum_{k=0} A^2\, e^{j2\pi kf\ \ b} + \sum_{\substack{k=-\infty \\ k\neq 0}}^{\infty} (0) e^{j2\pi kf\ \ b} \right]$$

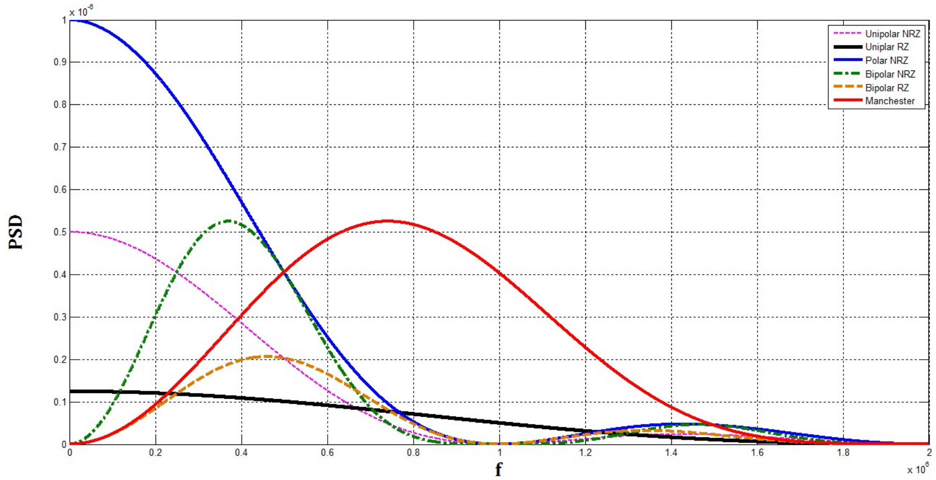
$$\mathcal{P}_{Manchester\ NRZ}(f) = A^2 T_b \left(\frac{sin(\pi f\ \ _b/2)}{\pi f T_b/2} \right)^2 \sin^2 \left(\frac{\pi f T_b}{2}\right)$$



Some important points:

1. Because there are no separation between the pulses in unipolar NRZ, therefore, the receiver needs synchronization to detect unipolar NRZ pulse.
2. The unipolar RZ contains line spectrum at $0, \pm R_b,\ \pm 2R_b, \ldots$ in addition to continuous spectrum. However, Unipolar format has some average DC value that does not carry any information.
3. The disadvantage of unipolar NRZis the waste power due to the Dc level and the fact that the spectrum is not approach zero near DC. Consequently, DC couple circuits are needed. The advantage of unipolar signaling are that it is easy generate using TTL and CMOS circuits and requires the use of only one power supply.
4. RZ type waveform requires more bandwidth than NRZ type.
5. The polar has the disadvantage of having a latge PSD near DC. On the other hand, polar signals are relatively easy to generate, although positive and negative power supplies are required.
6. Unipolar RZ requires 3 dB more signal power than polar signaling for the same probability of bit error.
7. Bipolar signaling has a single error detection capabilities built in, since a single error will cause a violation of the bipolar line code. The disadvantages of bipolar signals are that the receiver has to distinguish between three levels (+A,-A, and 0), instead of just two levels. Also, requires 3 dB more signal power that a polar signal for the same probability of error.

8. For Manchester code, irrespective of the probability of occurrence of symbol '1' and '0' the waveform has zero average value. Therefore by this mode, the power saving is quite more. But the drawback of this format is that it requires absolute sense of polarity at the receiver end.
9. The null bandwidth of Manchester format is twice that of the bipolar bandwidth.



Spectral Efficiency

It is given by the number of bits per second of data (R) that can be supported by each hertz of bandwidth (B).

$$\eta = \frac{R}{B}\ (bits/sec)/Hz$$

The maximum spectral efficiency is given by shannon's channel capacity formula,

$$\eta_{max} = \frac{C}{B} = log_2(1 + \frac{S}{N})$$

Code Type	First null bandwidth (Hz)	Spectral Efficiency η
Unipolar NRZ	R	1
Polar NRZ	R	1
Unipolar RZ	2R	1/2
Bipolar RZ	R	1
Manchester NRZ	2R	1/2