منشآت هيدروليكية

- الفصل الثاني -

المرحلة الرابعة
قسم الهندسة المدنية

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CHAPTER 2

THEORIES OF SEEPAGE

According to the hydraulic gradient theory (1902) , the hydraulic gradient in the structure should be less than the allowable value. Certain observations were established that the subsurface flow may cause the failure of the impervious floor either by piping or by uplift pressure.

2.1 Bligh's Theory (1910)

This is also called a creep theory, in which the length of the path thus traversed by the percolating water is called the length of creep or the creep length. As the water creeps from the upstream end to the downstream end, the head loss occurs. The head loss is proportional to the creep distance travelled. According to Bligh, in a previous foundation, the water percolates (seeps) along the base profile of the structure which is in contact with the subsoil. The length of the seepage path traversed by the water is called *creep length* (Lw). Also, the subsoil hydraulic gradient, which is the loss of head (H_L) per unit length of creep, is constant throughout the seepage path.

If H_L is the total head loss or the seepage head which is the difference of water levels between the upstream and downstream ends and L_w is the total creep length, so the loss of head per unit length is equal to H_L/L_w is called the *hydraulic gradient* (see Fig. 1).

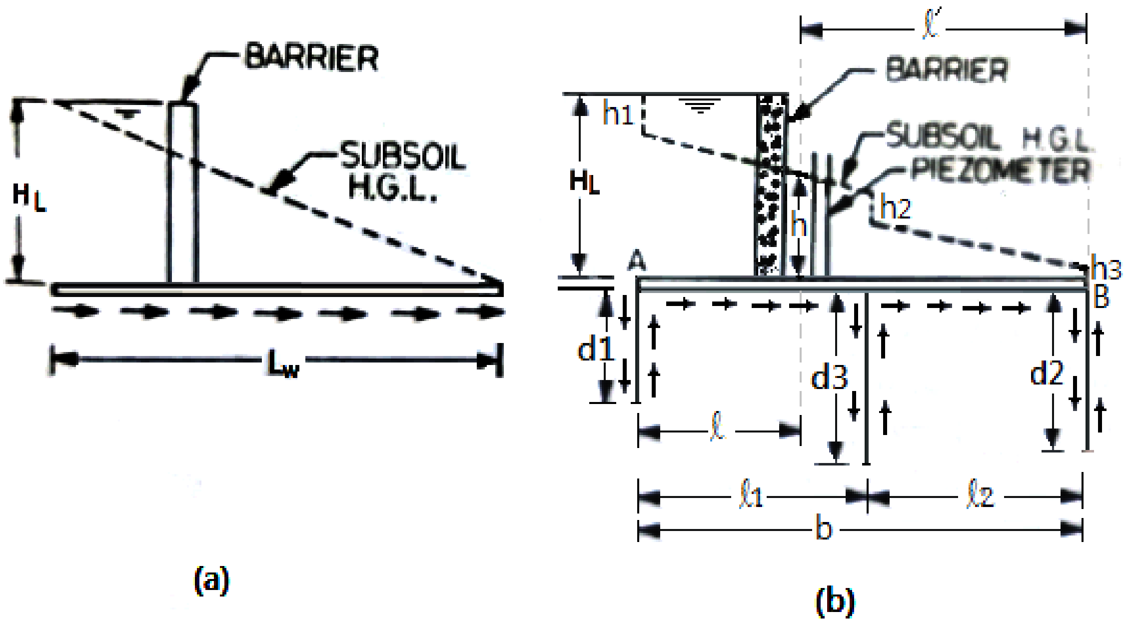


Figure (1): Examples of subsurface flows

In Figure 1, the water follows the path indicated by arrows. The total creep length (L_w) with cutoffs is given by:

$L_w = 2d_1 + l_1 + 2d_3 + l_2 + 2d_2$ …………………….…………………………………. (1)

Where d_1, d_2, and d_3 are the depths of the upstream, downstream and intermediate piles respectively. l_1 and l_2 are the lengths between the upstream and downstream piles.

The head loss per unit length or hydraulic gradient is given by (see Fig.1b):

$$i = \frac{H_L}{L_w} = \frac{H_L}{(l1+l2)+2.(d1+d2+d3)} = \frac{H_L}{b+2.(d1+d2+d3)}$$ ………………………..…….. (2)

where, $H_L = H_{U/S} - H_{D/S}$ = difference in water levels between u/s and d/s ends, $H_{U/S}$ = water depth at *U/S* end, and $H_{D/S}$ = water depth at *D/S* end.

The worst condition is that when no tail water exists at the *D/S* end, i.e. $H_{D/S} = 0$. In this case $H_L = H_{U/S}$.

Figure (1) shows the subsoil hydraulic gradient lines which presents the pressure heads at the point below the impervious floor due to subsurface (seepage) flow. Also, the Figure shows a sudden drop in the subsoil hydraulic gradient line at location of the piles (cutoffs).

The head loss at any point of apron which shown in Figure (2) can be written as follows:

Head loss occurs on upstream cutoff = $\frac{H_L}{L_w} 2d_1$

Head loss occurs on intermediate cutoff = $\frac{H_L}{L_w} 2d_2$

Head loss occurs on downstream cutoff = $\frac{H_L}{L_w} 2d_3$

Head at Point C = Total Head – Head loss occurs on U/S cutoff;

$H_C = H_{U/S} - \frac{H_L}{L_w} 2d_1 = H_L - \frac{H_L}{L_w} 2d_1 = \frac{H_L}{L_w} (L_w - 2d_1)$

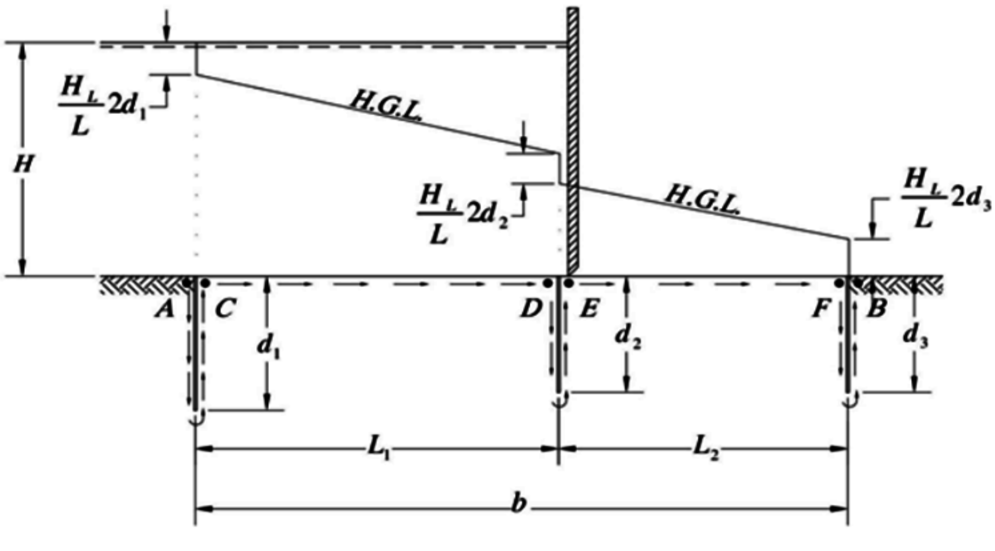


Figure (2): Head loss at cutoffs.

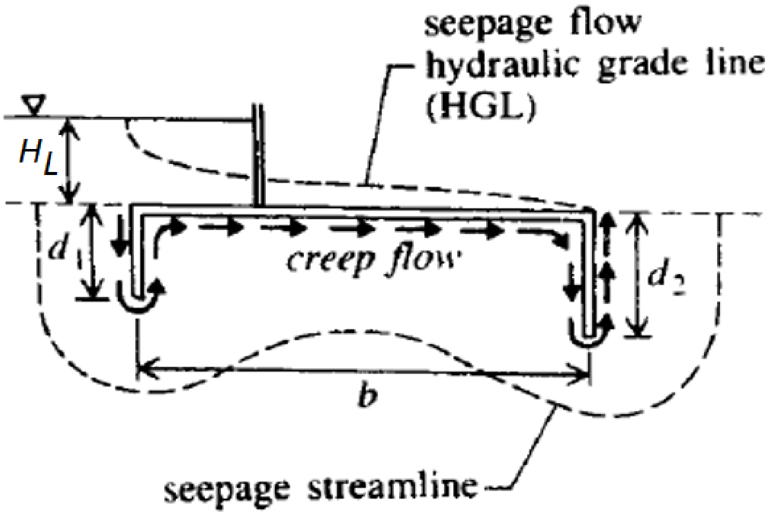


Figure (3): Bligh's Creep

عودة للشكل(1b) الذي يوضح خط الانحدار الهيدروليكي لماء التربة والذي يبين فيه ضغط الماء في نقاط تحت سطح المنشأ غير النفاذ نتيجة رشح الماء تحت المنشأ. ولو غرزنا انابيب بيزومترية في نقاط مختلفة لأرتفع الماء في هذه الانابيب الى مستويات موضحة بالانحدار الهيدروليكي لماء التربة.

2.1.1 Safety against piping حماية ارضية المنشأ الهيدروليكي بسبب نحر التربة بفعل تسرب الماء تحت أرضية المنشأ

The exit gradient is the hydraulic gradient of the seepage flow under the base of the weir floor. The rate of seepage increases with the increase in exit gradient, and such an increase would cause 'boiling' of surface soil, the soil being washed away by the percolating water. The flow concentrates into the resulting depression thus removing more soil and creating progressive scour backwards (i.e. upstream). This phenomenon is called 'piping', and eventually undermines the weir foundations.

For the safety of the hydraulic structure on pervious foundation, the subsoil hydraulic gradient i, should be less than the permissible value to prevent piping failure.

Piping failure will not occur if the hydraulic gradient is equal to or less than a safe value. Thus for a safe design,

$$i = \frac{H_L}{L_w} \leq \frac{1}{C_1} \quad \text{or;} \qquad L_w = C_1 H_L \quad \ldots\ldots\ldots (3)$$

where H_L, is the difference of water levels between upstream and downstream ends (no water is shown at the downstream end), L_w is the creep length, and C_1 is Bligh's creep coefficient, which depends upon the type of soil (see Table 1).

Table 1: Bligh's Creep Coefficient

S. No.	Type of soil	Creep coefficient, C_1	Safe hydraulic gradient, $1/C_1$
1.	Light sand and mud	18	1/18
2.	Fine micaceous sand	15	1/15
3.	Coarse grained sand	12	1/12
4.	Boulders and gravel mixed with sand	5 to 9	1/5 to 1/9

The piping phenomenon can be minimized by reducing the exit gradient, i.e. by increasing the creep length. The creep length can be increased by increasing the impervious floor length and by providing upstream and downstream cut-off piles.

2.1.2 Safety against Uplift Pressures حماية أرضية المنشأ الهيدروليكي بتأثير ضغط ماء التربة تحت أرضية المنشأ الهيدروليكي

The base of the impervious floor is subjected to uplift pressures as the water seeps through below it. The uplift upstream of the weir is balanced by the weight of water standing above the floor in the pond (Fig. 3 & 4), whereas on the downstream side there may not be any such balancing water weight. The design consideration must assume the worst possible loading conditions, i.e. when the gates are closed and the downstream side is practically dry. The floor should be sufficiently thick to prevent its rupture due to uplift pressure, i.e. the weight of the gravity floor must be sufficient to counterbalance the uplift pressure.

The impervious base floor may crack or rupture if its weight is not sufficient to resist the uplift pressure. Any rupture thus developed in turn reduces the effective length of the impervious floor (i.e. reduction in creep length), which increases the exit gradient. The provision of increased creep lengths and sufficient floor thickness prevents this kind of failure. Excessively thick foundations are costly to construct below the river bed under water. Hence, piers can sometimes be extended up to the end of the downstream apron and thin reinforced concrete floors provided between the piers to resist failure by bending.

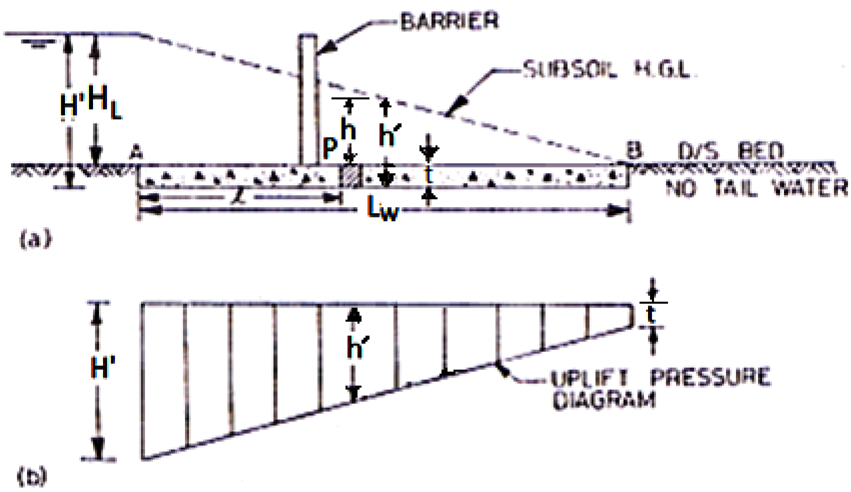


Figure (4): Flow under a weir

Figure (4a) shows a simple horizontal floor of length L_w, subjected to seepage head of H_L. The residual head (h) at any point p is given by:

$$h = H_L - \frac{H_L}{L_w}\, l \qquad \text{(without cut off)} \qquad \ldots\ldots (4)$$

where l is the horizontal length between point A and p.

ملاحظة: عندما يكون لدينا ركيزة pile في مقدم ارضية المنشأ الهيدروليكي فأن l تكون مساوية للمسافة بين نقطتين مضافا اليها عمق الـ pile مضروباً x 2 (انظر المثال رقم 1).

The residual head (h) can also be obtained from the subsoil hydraulic gradient line (H.G.L.).

$h' = h + t$

where t is the thickness of floor.

Figure *(4-b)* shows the uplift pressure diagram on the bottom surface.

The upward force, F acting on the unit area (i.e. A = 1) of the floor due to uplift pressure is given by:

$P = \frac{F}{A}$ or; F = PA i.e. $F = \gamma_w h A$

In this case the pressure head is equal h', so

$F = \gamma_w h' \times 1 = \gamma_w (h + t)$ ………………………………………..……………..…. (5)

where, γ_w is the specific weight of water.

The downward force W due to the weight of the floor material is given by:

$W = \gamma_f V = G_f \gamma_w V = (G_f \gamma_w)\, t \times 1$ …...………..…………………………………. (6)

In which G_f is the specific gravity of the floor material .

Equating the last two equations (5) and (6) results:

F = W

$\gamma_w h' = G_f \gamma_w t$

$\gamma_w (h + t) = G_f \gamma_w t$

$h = G_f t - t$

$h = t\,(G_f - 1)$

$t = \frac{h}{(G_f - 1)}$ ………………………………………..……………….…..…… (7)

For concrete material $\gamma_f = 25\ ton/m^3$,and $G_f - 1 = 1.5 \Rightarrow t = \frac{2}{3} h$

In general, a factor of safety 4/3 is adopted. Thus

$t = \frac{4}{3} \frac{h}{(G_f - 1)}$ ………………………………………………………………..…… (8)

2.1.3 Limitations of Bligh's Theory

1. The Bligh theory does not differentiate between the vertical creep and the horizontal creep and gives the same weightage to both, which is not correct. Actually, the vertical creep is more effective than the horizontal creep.

2. The theory assumes a linear variation of the head loss, which is not correct. The actual head loss variation is non-linear (see Fig. 2).

3. No distinction is made between the head loss on the outer faces and that on the inner faces of the sheet piles. Actually, the outer faces are more effective than the inner faces.

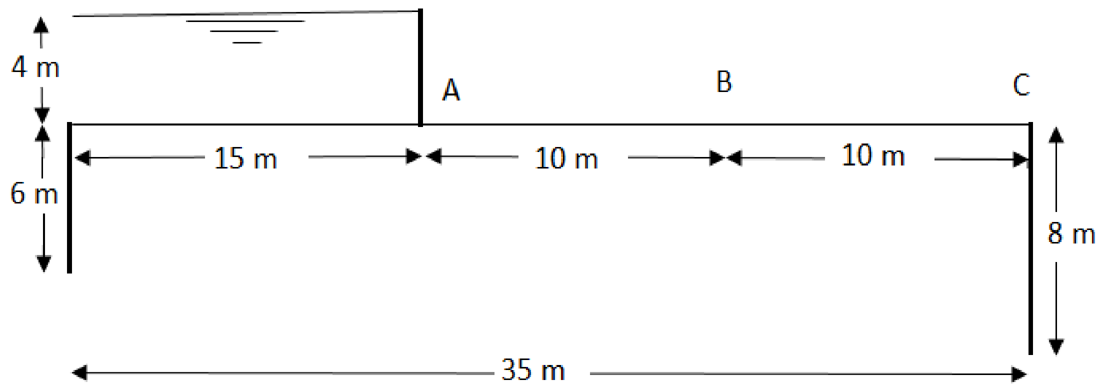
4. The theory does not emphasize the importance of the downstream pile without which piping failure occurs. It considers the downstream pile as a component of the total creep length and not as a controlling factor for the exit gradient and the piping.

5. The theory does not give any theoretical or practical method for the determination of the creep coefficient C_1.

6. Bligh did not consider the effect of the intermediate pile.

7. The theory does not give the approximate results if the horizontal distance between the piles is less than twice their depths.

Example 1: In the Figure below, a hydraulic structure built on fine sand (C_1 = 15). Determine (a) whether the percolation gradient is safe. (b) Uplift pressure at points A, B, and C at distances 15, 25, and 35 m from the upstream end. (c) Thickness of floor at these points. Use Bligh's theory. Take G_f = 2.24.



Solution:

(a) Creep length, L_w = 6x2 + 15 + 10 + 10 + 8 x 2 = 63 m,

H_L= U/S water level – D/S water level = 4.0 m – 0.0 = 4.0 m

Hydraulic gradient, $i = H_L/L_w$

$$i = 4/63 = 1/15.75 < 1/15 \text{ (safe)}$$

(b) Uplift pressure head h, at point A = $H_L - \frac{H_L}{L_W} \times (2d + l)$

$$= 4 - [\frac{1}{15.75} \times (2 \times 6 + 15)] = 2.29 \text{ m}$$

$$\text{at point B} = 4 - [\frac{1}{15.75} \times (2 \times 6 + 25)] = 1.65 \text{ m}$$

$$\text{at point C} = 4 - [\frac{1}{15.75} \times (2 \times 6 + 35)] = 1.02 \text{ m}$$

Check uplift pressure head at point C = $\frac{1}{15.75} \times (2 \times 8) = 1.02$ m (OK)

(c) Thickness of floor,

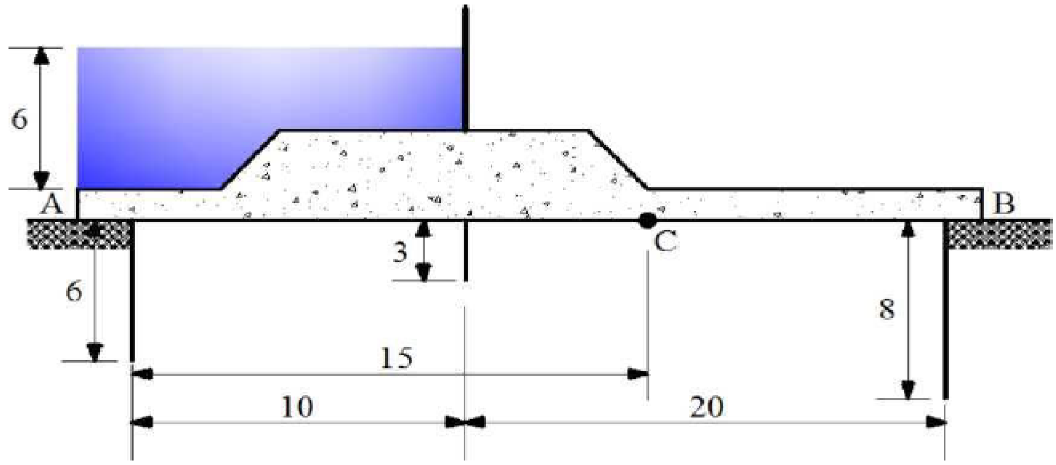
$$t = \frac{4}{3} (\frac{h}{G_f - 1})$$

At point A, $t = \frac{4}{3} (\frac{2.29}{2.24-1}) = 2.46$ m

At point B, $t = \frac{4}{3} (\frac{1.65}{2.24-1}) = 1.77$ m

At point C, $t = \frac{4}{3} (\frac{1.02}{2.24-1}) = 1.10$ m

Example 2: Find the hydraulic gradient and uplift pressure and the thickness of floor at a point C, 15 m from the upstream end of the floor in the Figure below. All dimensions in meter.



Solution:

Water percolates at point ***A*** and emerges at point ***B***

Total creep length (L_w) = 2x6+10+ 2x3+ 20+ 2x8 = 64m

Head of water on structure ($H_{U/S}$) = 6 m = H_L

Hydraulic gradient, i = H_L/L_w = 6 /64 = $1/C_1$ = 1/10.67

According to Bligh's theory, the structure would be safe on sand mixed with boulders & Gravel

Creep length up to point *C* = 2x6 + 2x3 + 15 = 33m

The residual uplift pressure (h) at the point C is:

$h = H_L - (\frac{H_L}{L_w})[\, l + 2d_1 + 2d_3]$, where d_1 = 6 m and d_2 = 3 m

$= 6 - \frac{6}{64}(2x6 + 15 + 2x3) = 2.91$ m

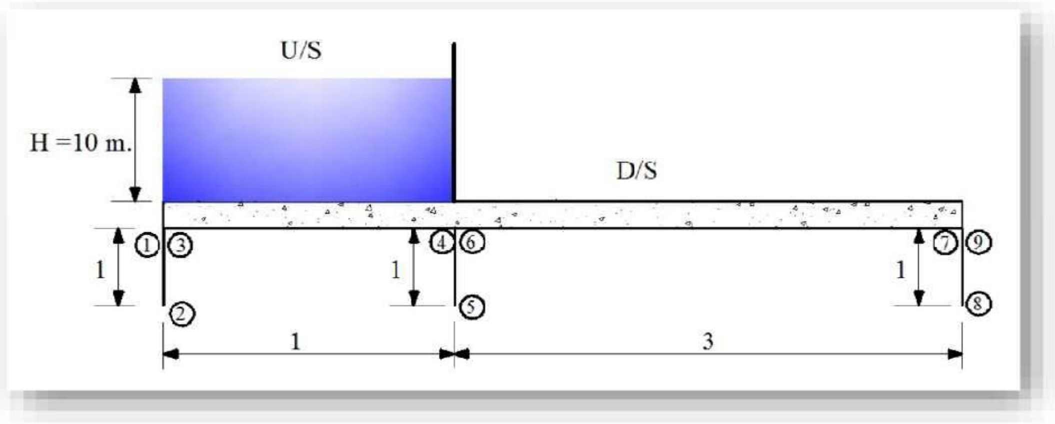
The thickness of floor at C is:

$t = \frac{4}{3}(\frac{h}{G_f - 1}) = \frac{4}{3}(\frac{2.91}{2.4-1}) = 2.77$ m

Homework No. 1: For the hydraulic structure shown below:

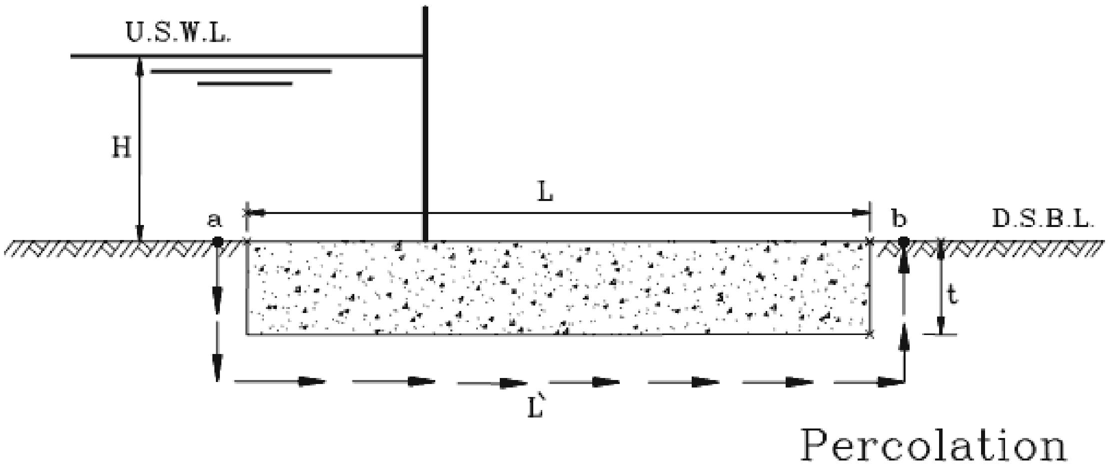
1. Find the uplift pressure at key points 4, and 7.

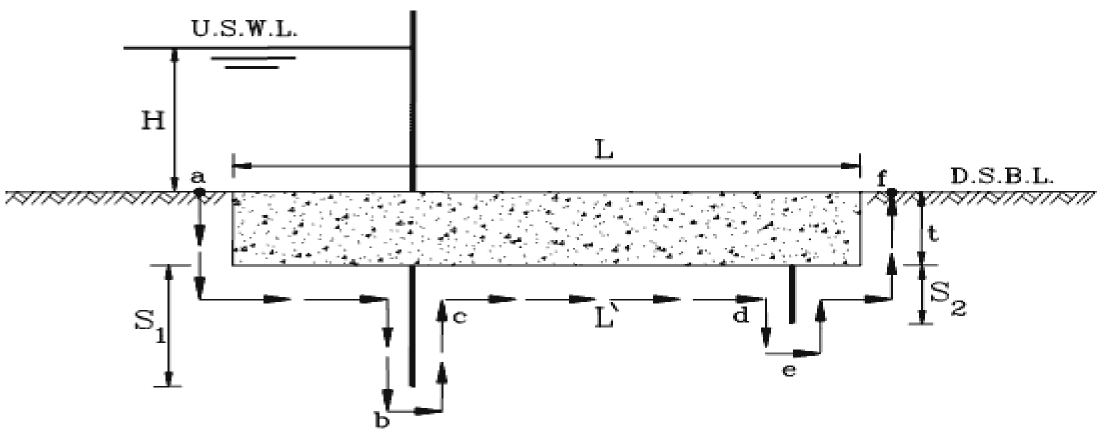
2. Find the thickness of floor at key point 6.



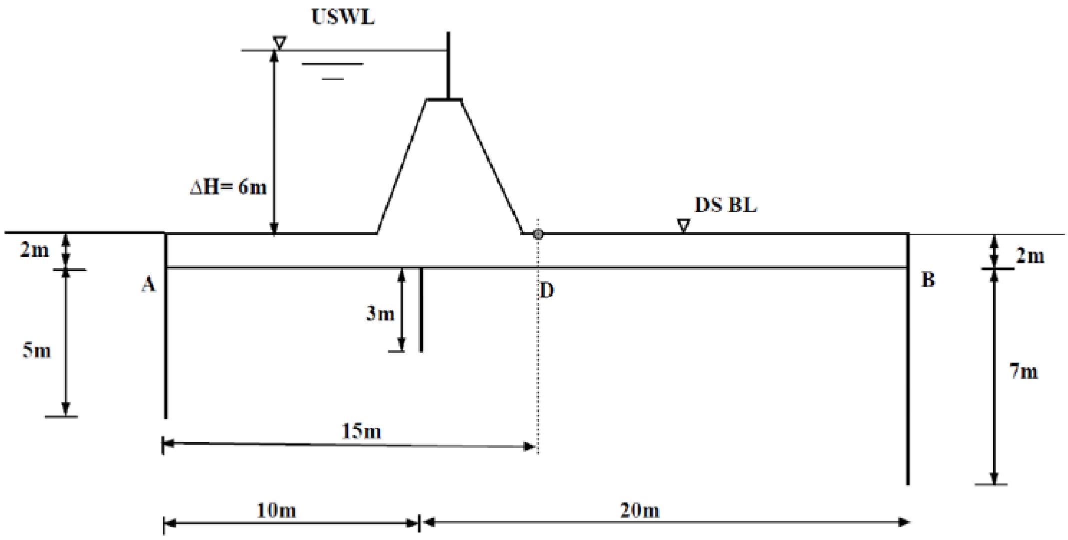
Homework No. 2: For the hydraulic structures shown below:

Determine the creep length L_W





Example 3: Find the hydraulic gradient and the head at point D of the following structure for static condition



Solution:

The total creep length, L_w= 2 + 5*2 +10 +2*3 + 20 + 2*7 + 2 = 64 m

Hydraulic gradient, i = H_L/L_w= $\Delta H/L_w$= 6/64 = 1/10.66

According to Bligh's creep coefficient (see Table 1), the structure should be safe if $H_L/L_W \leq 1/C_1$. From Table 1 the structure is safe on sand mixed with boulders & gravel, i.e.

$i = \frac{H_L}{L_w} \leq \frac{1}{C_1}$ or $L_w = C_1 H_L$

$1/10.66 \leq 1/5$ to $1/9$

Creep length up to point D is L_D = 2 + 5*2 + 15 + 3*2 = 33 m

The residual uplift pressure head (h) at D = $H_L - (H_L/L_w)\, l$ = 6 - (6/64)* [2 +5*2 +15 +3*2] = 2.9 m

The thickness of floor at any point should be sufficient to resist the residual uplift pressure.

$t = \frac{4}{3}\left(\frac{h}{G_f - 1}\right)$ = 1.33*[2.9/(2.24-1)] = 3.1 m

At the end of this theory, it should be noted that Bligh's theory is quite simple and convenient.

عدد كبير من المنشأت الهيدروليكية قد صممت سابقا باستخدام هذه النظرية ولازال بعض هذه المنشات موجودة ليومنا هذا والبعض الاخر قد فشلت بمرور الزمن وهذا يعود الى أن هذه النظرية قد بنيت على عدد من المحددات والتي ذكرت سابقا. ومن النادر أستخدام هذه النظرية في يومنا هذا ، ولكن في بعض الاحيان يمكن أستخدامها لبعض المنشأت الصغيرة كحل بدائي لمنشآت كبيرة يراد تصميمها.

2.2 Lane's Weighted Creep Theory (1932)

This theory gives different weightage to the vertical and horizontal creeps. Lane found that the vertical creep is 3 times more effective than the horizontal creep in reducing the uplift pressure. A weightage of unity was given to the vertical creep and 1/3 to the horizontal creep. Thus the weighted creep length (L_w) is given by:

$$L_w = \frac{1}{3} N + V \quad \text{...} (9)$$

where, N is the sum of all the horizontal contacts and the flat sloping contacts making an angle less than 45^o with the horizontal, V is the sum of all vertical contacts and the steep sloping contacts making an angle greater than 45^o with the horizontal.

According to Lane's weighted creep theory, an irrigation structure will be safe if (H_L/L_w) is less than the safe hydraulic gradient ($1/C_1$) for that soil, where H_L is the seepage head, and C_1 is Lane's creep coefficient as it was given in Table 2.

Thus, $\frac{H_L}{L_w} \leq \frac{1}{C_1}$

or; $L_w = C_1 H_L$

The thickness of the floor at any point can be determined by computing the residual uplift pressure head (h) and using equation (8). Thus

$$t = \frac{4}{3} \times \left(\frac{h}{G_f - 1}\right)$$

While computing the residual head (h), proper weightage should be given to creep length. For example, the residual head (h) at point p at a distance l from the upstream end (see Fig. 1) is given by:

$$h = H_L - \frac{H_L}{L_w}\left(\frac{1}{3} l + 2d_1\right) \quad \text{(with u/s cutoff)} \quad \text{.......................................} (10)$$

وتعتبر هذه الطريقة اكثر مقبولية من الطريقة الاولى بسبب انها تأخذ بنظر الأعتبار وزنا أكبر للنضح العمودي, أما محدداتها فهي نفس محددات معادلة Bligh ماعدا ماذكر أعلاه حول اهمية النضح العمودي كونه أكثر فاعلية من الوزن الافقي. مثال ذلك الشكل التالي يراد حساب قيمة L_w وحسب المعادلة رقم (9) وكما يلي:

Example 4: Calculate the creep length for the weir below.

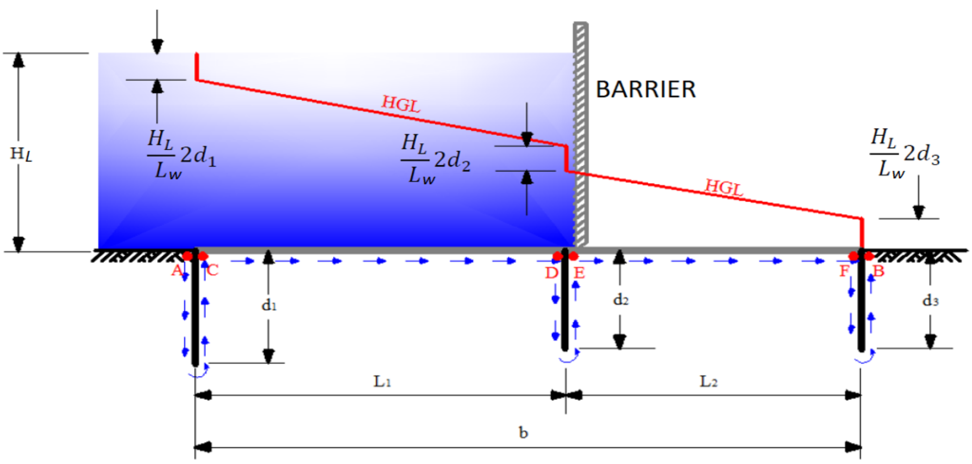


Figure (5): Flow under a weir

The total Lane's creep length (L_w) is given by:

$L_w = (d_1 + d_1) + (1/3)\ L_1 + (d_2 + d_2) + (1/3)\ L_2 + (d_3 + d_3)$

$= (1/3)\ (L_1 + L_2) + 2(d_1 + d_2 + d_3) = (1/3)\ b + 2(d1 + d2 + d3)$

لقد بين Lane أن قيم C1 كما وردت في الجدول المرفق في أدناه وحسب نوع التربة وكما في الجدول رقم 2.

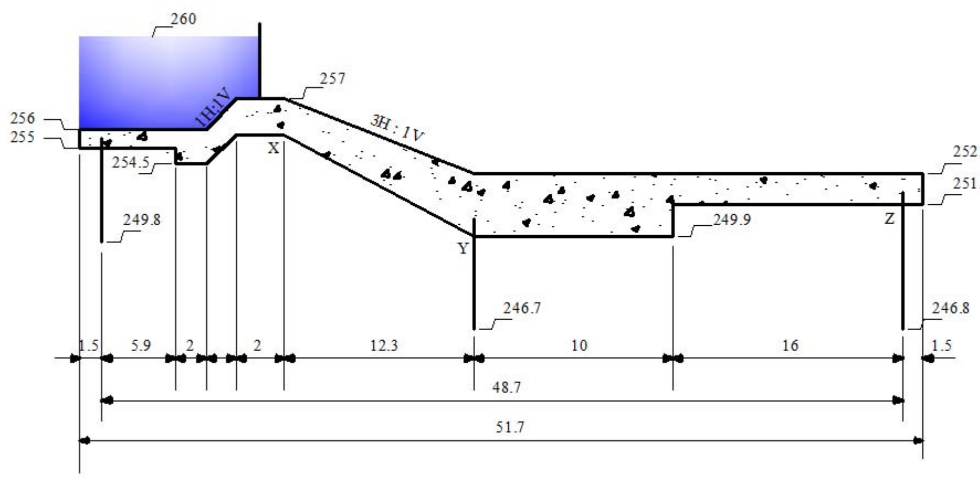
Table 2: Lane's creep coefficient for different types of soils.

No.	Type of soil	Lane's creep coefficient, C_1	Safe hydraulic gradient ($1/C_1$)
1.	Very fine sand or silt	8.5	1/8.5
2.	Fine sand	7.0	1/7.0
3.	Coarse sand	5.0	1/5.0
4.	Gravel and sand mixture	3.5-3.0	1/3.5-1/3.0
5.	Boulder, gravel and sand mixture	3.0-2.5	1/3.0-1/2.5
6.	Soft clay	3.0	1/3.0
7.	Hard clay	1.6	1/1.6

Example 5: A barrage structure on a river as shown in Figure below:

(a) It is required to check if the floor thickness at points X, Y and Z is sufficient to counteract the uplift pressure ($G_f = 2.4$).

(b) Check safety against piping if the soil type is coarse sand ($C_1 = 5$).



Solution:

$L_w = \frac{1}{3} N + V$

$N = 1.5 + 5.9 + 2 + 2 + \{(255 - 249.9)^2 + 12.3^{2})^{0.5}\} + 10 + 16 + 1.5$

$N = 1.5 + 5.9 + 2 + 2 + (5.1^2 + 12.3^2)^{0.5} + 10 + 16 + 1.5 = 52.2$ m

$V = 1 + 2 \times (255.0 - 249.8) + 0.5 + (0.5^2 + 0.5^2)^{0.5} + 2\ (249.9 - 246.7) + 1.5 + 2 \times (251.4 - 246.8) + 1.5$

$V = 1 + 2\times5.2 + 0.5 + (0.5^2 + 0.5^2)^{0.5} + 2\times3.2 + 1.5 + + 2\times4.6 + 1.5 = 31.2$ m

$\therefore L_w = \frac{1}{3} \times 52.2 + 31.2 = 48.6$ m

$H_L = 260 - 252.9 = 7.1$ m

$i = \frac{H_L}{L_w} = \frac{7.1}{48.6} = \frac{1}{6.84} < \frac{1}{5}$

Thus, the structure is safe against piping.

Lx = (256 -255) +1.5/3 + 2 x (255-249.8) + 5.9/3 + (255 – 254.5) + 2/3 + $(0.5^2+0.5^2)^{0.5}$ + 2/3 = 16.4 m,

L_Y = 20.83 m,

L_Z = 37.4 m

The head at points X, Y and Z is calculated from Figure below as follows:

H_X = (7.1/48.6) × (48.6 – 16.4) = 4.7 m of water

H_Y = (7.1/48.6) × (48.6 – 20.83) = 4.05 m of water

H_Z = (7.1/48.6) × (48.6 – 37.4) = 1.63 m of water

or, using the following equations:

$h = H_L - \frac{H_L}{L_W}(l)$

So, $h_x = H_L - \frac{H_L}{L_W}(l_x)$

$h_x = 7.1 - \frac{7.1}{48.6}(16.4) = 4.7$ m

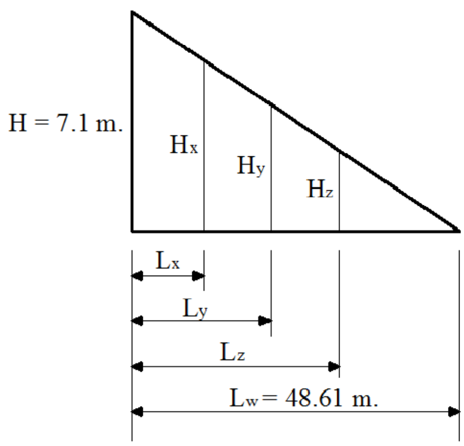
$h_Y = 7.1 - \frac{7.1}{48.6}(20.83) = 4.05$ m

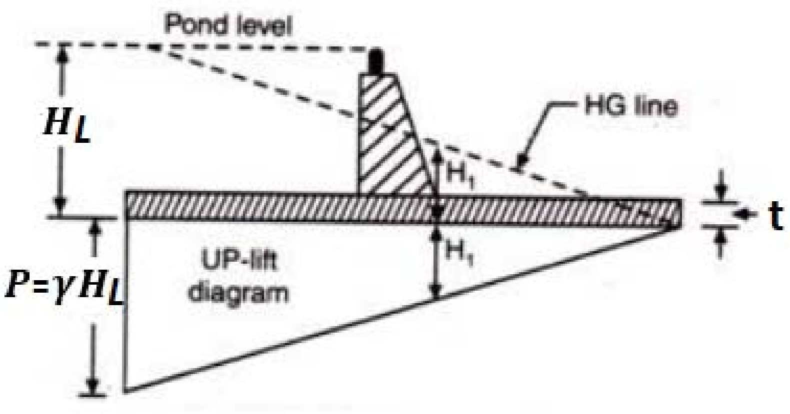
$h_Z = 7.1 - \frac{7.1}{48.6}(27.4) = 1.63$ m

t_X = 4.7 /(2.4-1) = 3.36 m of concrete > 2 not OK

t_Y = 4.05 /(2.4-1) = 2.89 m of concrete < 3 OK

t_Z = 1.63 /(2.4-1) = 1.16 m of concrete < 1.5 OK





2.3 Khosla's theory

بالرغم من الأستخدام الواسع لنظرية Bligh في تصميم منشأت الري على أرضية نفاذة فقد لوحظ أن أستخدام هذه النظرية أدت الى فشل عديد من المنشات الهيدروليكية.

After studying a lot of dam failures constructed based on Bligh's theory, Khosla came out with the following results;

1. From observation of Siphons designed on Bligh's theory, by actual measurement of uplift pressure at their bases with the help of pipes inserted in the floor of these siphons.

2. According to Khosla's theory, it was found that the actual uplift pressures were quite different from those computed by Bligh's theory. This led to the following provisional conclusions:-

a) The outer faces of the end sheet piles are much more effective than the inner ones and the horizontal length of the floor.

b) The intermediated piles of smaller length than the outer piles are ineffective except for local redistribution of pressure.

c) Undermining of floor started from tail end when the hydraulic gradient at the exit is greater than the critical gradient for a particular soil.

d) It is absolutely essential to have a reasonably deep vertical cut off at the downstream end to prevent piping.

لقد أوصى Khosla وفريق عمله بمزيد من البحوث وأجراء تجارب مختبرية وحقلية لحل المشاكل المتعلقة بفشل المنشأت الهيدروليكية المقامة على أرض نفاذة وعليه فأن:

e) Khosla and his associates took into account the flow pattern below the impermeable base of a hydraulic structure to calculate the uplift pressure and exit gradient (see Fig. 6).

وأعتمادا على الشكل أدناه والذي يوضح شبكة الجريان(flow net) تحت أرضية المنشأ النفاذة يلاحظ وجود خطوط الجريان (flow lines) المتعامدة عل خطوط تساوي الجهد (equipotential lines). أن خطوط الجريان تمثل المسارالذي يحدث فيه الجريان, في حين أن خطوط تساوي الجهد هي خطوط تتساوى فيها الشحنة *(h)*.

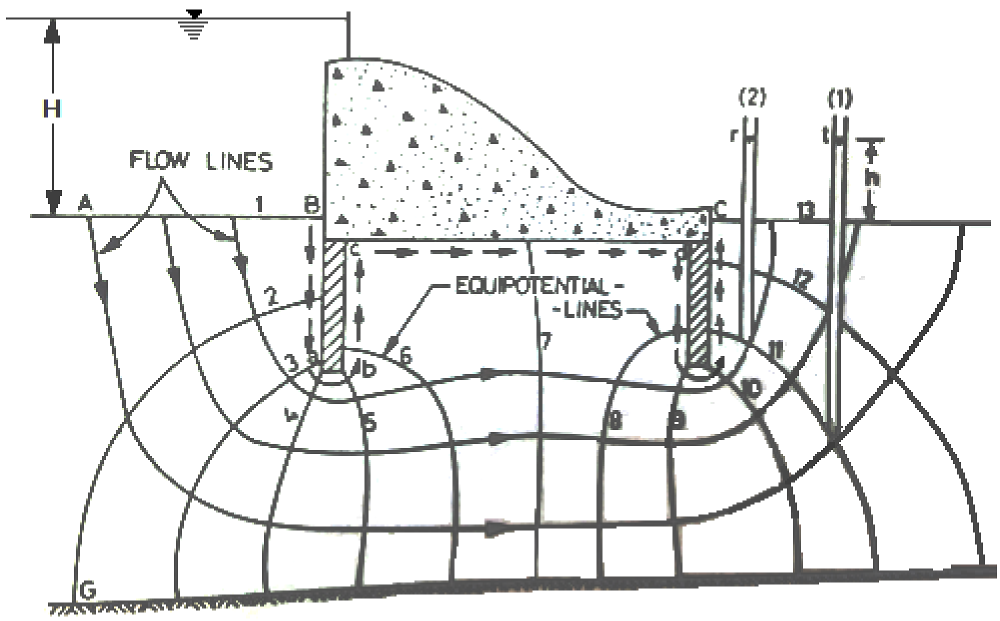


Figure (6): Flow net below the hydraulic structures.

The potential flow follows the Laplace equation to seepage Darcy flow:

$$\frac{\partial^2 \emptyset}{\partial_x{}^2} + \frac{\partial^2 \emptyset}{\partial y^2} = 0 \quad \ldots\ldots (11)$$

where, $\emptyset$ is the flow potential, or the velocity potential, given by:

$$\emptyset = - k\, h \quad \ldots\ldots (12)$$

In which *k* is the coefficient of permeability and *h* is the seepage head at any point in the soil.

Once the flow net has been drawn, it can be used for the determination of seepage discharge, the uplift pressure, the residual head, the hydraulic gradient and the exit gradient. The hydraulic gradient varies from point to point. The hydraulic gradient has a maximum value at the exit. The hydraulic gradient at the exit is known as the *exit gradient*. Piping will not occur if the exit gradient is equal or less than the critical gradient of the soil.

The exit gradient, G_E can be calculated from the following equation:

$$G_E = \frac{KH}{d} \quad \ldots\ldots (13)$$

where,

$$K = \frac{0.45}{\sqrt{1+m}} \quad \ldots\ldots (14)$$

$$m = \sqrt{1 + \left(\frac{b}{d}\right)^2} \quad \ldots\ldots (15)$$

b = horizontal length of the floor

d = depth of the downstream cut-off

H = U/S water level – D/S water level

A safety factor F should be considered as:

$$F = \frac{1}{G_E} \quad \ldots\ldots (16)$$

In Iraq a safety factor, F is taken between 8 -10 for alluvial soils.

Water enters the subsoil at the upstream end and has a head H. As it moves from the upstream to the downstream, there is a loss of head. When it emerges at the downstream end, the head becomes zero, because there is no tail water. At the upstream end, the water has a head of H which is completely lost through the passage of flow. At the intermediate of its path, the water has a certain residual head h still to be dissipated in the remaining seepage length up to the downstream end.

f) Starting with a simple case of a horizontal flow with negligibly small thickness, various cases were analysed mathematically.

g) Seeping water below a hydraulic structure does not follow the bottom profile of the impervious floor as stated by Bligh's theory but each particle traces its path along a series of streamlines.

For any given profile of the *apron* (a small area adjacent to another larger area or structure) of a weir, barrage, and any hydraulic structure on pervious foundation. An upstream ***apron*** is used to lengthen the path of the water that is seeping through beneath the structure and to reduce the uplift on the bottom of the structure.

A hydraulic structure consists of a number of elementary forms. Fig.7 shows the cross section of a typical hydraulic structure, consisting of a horizontal floor, three piles, upstream and downstream glacis. For the determination of uplift pressure at the key points (the key points are the junctions of the floor and the pile lines on either side and the bottom point of the pile line) of such a structure, Khosla et al. gave the theory of independent variables. This theory a composite profile is split into a number of simple elementary standard forms.

The uplift pressure obtained from the superposition of the individual forms are to be corrected because the individual pressures have been obtained based on the following assumptions:

1. The floor is of negligible thickness.
2. There is only one pile line.
3. The floor is horizontal.

Because in an actual profile, the above assumptions are not satisfied, the following corrections are needed:

a) Correction for the mutual interference of piles.
b) Correction for the thickness of floor.
c) Correction for the slope of the floor.

Thus the corrected pressures at the key points of all the piles are determined. The uplift pressure at any point on the floor between the two piles is obtained by linear interpolation of the pressures at the key points of these two points.

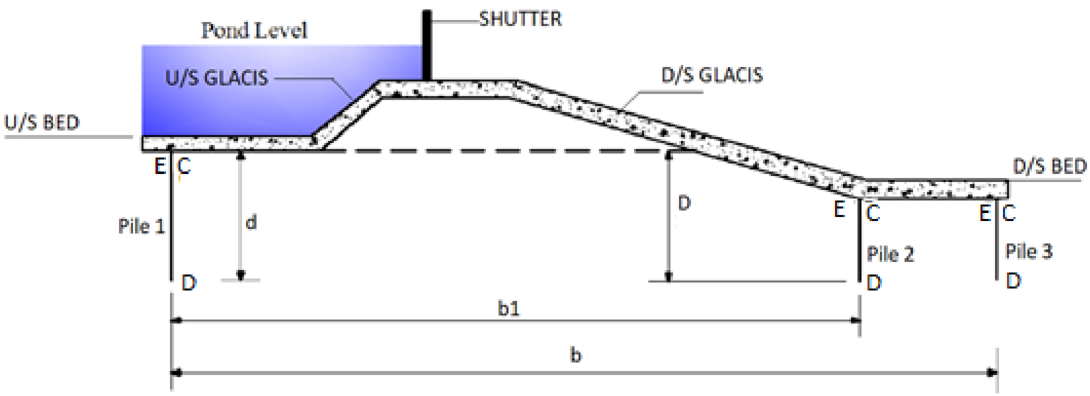


Figure (7): A typical cross section of a hydraulic structure.

In order to know as how the seepage flow below the foundation of a hydraulic structure is taking place, Khosla has evolved a simple, quick and an accurate approach, called method of independent variables. In this method, a complex profile like that a weir is broken into a number of simple profiles, each of which can be solved mathematically and presented in the books of hydraulic structures in the form of curves. These curves help in determining the percentage of pressures at the various key points in the Figure above. The simple profiles are shown in Figure (8).

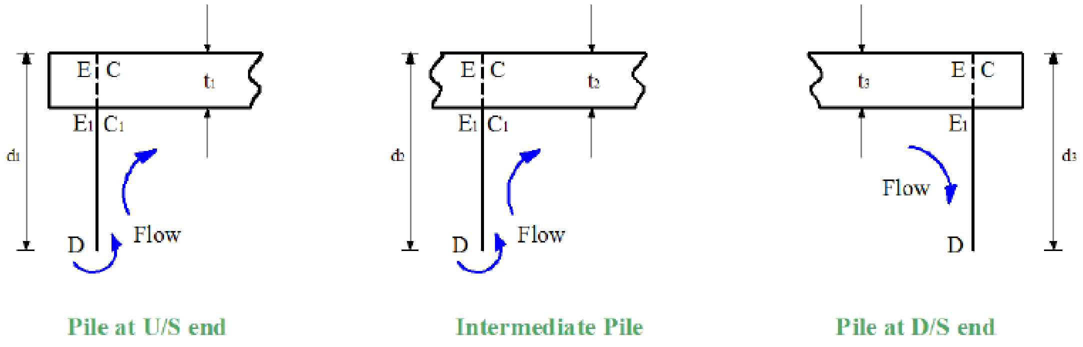


Figure (8): Pile locations.

The different locations of piles (see Fig. 8), are:

a) A straight horizontal floor of negligible thickness with a sheet pile at the u/s end.

b) A straight horizontal floor of negligible thickness with a sheet pile at some intermediate point.

c) A straight horizontal floor of negligible thickness with a sheet pile at the d/s pile.

The percentage pressures at the key points in (Fig. 7) can be determined by divided the complex structure form into the simple forms (a, b, and c) and carrying out the following corrections.

2.3.1 Correction for The Mutual Interference Piles:

The correction C in a percentage at the corner can be written as:

$$C = \mp 19\sqrt{\frac{D}{b_1}}\left(\frac{d+D}{b}\right) \quad \% \text{ of } H_L \quad \ldots \ldots (17)$$

where, H_L = seepage head, b_1 = distance between two pile lines, d = depth of pile on which the effect of pile is required to be determined, D = depth of pile whose influence has to be determined on the neighboring pile of depth (d) , b = total floor length (see Fig.7).

This correction is positive for the effective of D/S pile on U/S pile (+ve) and negative for the effective of U/S pile on D/S pile (–ve). Also, this equation does not apply to the effect of an outer pile on an intermediate pile if the latter is equal to or smaller than the former and is at a distance less than twice the length of the outer line.
The correction C is a % of head due to this effect *H*, this percentage value was "Added" or "subtracted" from the uplift pressure of key-point according to its location relative to "intermediate" pile, in which it should be;
*Added (+C) to the value of calculated uplift pressure of U/S pile.
*Subtracted (− C) to the value of calculated uplift pressure of D/S pile.

Note: *The correction "C" was neglected (has no effect) if* $d \geq D$ & $d > \frac{1}{2} b_1$ *(both conditions must sustained).*

Suppose in Figure (9), we are considering the influence of pile No. (2) on pile No. (1) for correcting the pressure at C_1. Since the point C_1 is in the rear, and hence, this correction shall be positive (+ve). While the correction to be applied to E_2 due to pile No. (1) shall be negative since the point E_2 is in the forward direction of flow. Similarly, the correction at C_2 due to pile No. (3) is positive and the correction at E_3 due to pile No. (2) is negative.

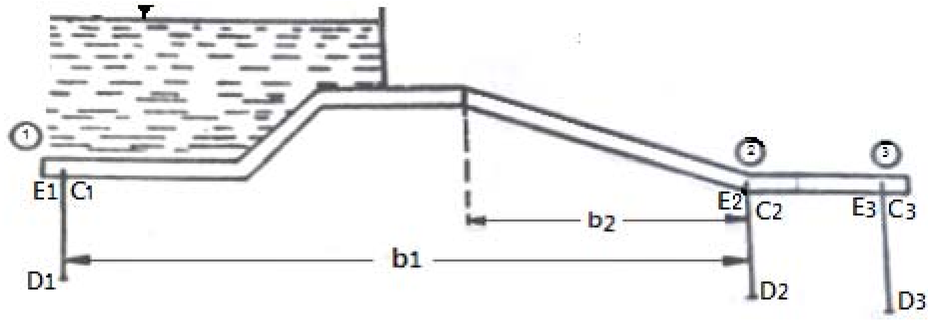


Figure (9): An example of calculation a correction for the slope of the floor.

2.3.2 Correction Due to Floor Thicknesses

For different locations of piles (see Fig. 8), the corrections to be applied are as follows:

a- A straight horizontal floor of negligible thickness with a sheet pile at the u/s end.

Corrected pressure at point C1:

$\emptyset C_1 = \emptyset C + \left(\frac{\emptyset D - \emptyset C}{d1}\right) t_1$ ……………………………………………………..…. (18)

b- A straight horizontal floor of negligible thickness with a sheet pile at some intermediate point.

Corrected pressure at point E1

$\emptyset E_1 = \emptyset E - \left(\frac{\emptyset E - \emptyset D}{d2}\right) t_2$ ……………………………………………………..….. (19)

c- A straight horizontal floor of negligible thickness with a sheet pile at the d/s pile.

Corrected pressure at point E1

$\emptyset E_1 = \emptyset E - \left(\frac{\emptyset E - \emptyset D}{d_3}\right) t_3$ ………………………………………………………. (20)

Where: $\emptyset_{C1}$, $\emptyset_{D1}$, $\emptyset_{E1}$ are uplift pressures at points C_1, D_1 , E_1, and d_1, d_2, and d_3 are depth of piles, t_1, t_2, t_3 are floor thickness respectively.

We can use the following equations to find the uplift pressure ($\emptyset$) at *E*, *C* & *D*:

a) U/S & D/S piles *(see Fig. 10)*

$P_E = \frac{H}{\pi} cos^{-1}\left(\frac{\lambda-2}{\lambda}\right)$ $\qquad$ $P_D = \frac{H}{\pi} cos^{-1}\left(\frac{\lambda-1}{\lambda}\right)$

These equations are usually written in terms of the percentage pressure, $\emptyset_E$ and $\emptyset_D$ such that:

$\% \emptyset_E = (P_E/H) \times 100$ $\qquad$ $\% \emptyset_D = (P_D/H) \times 100$

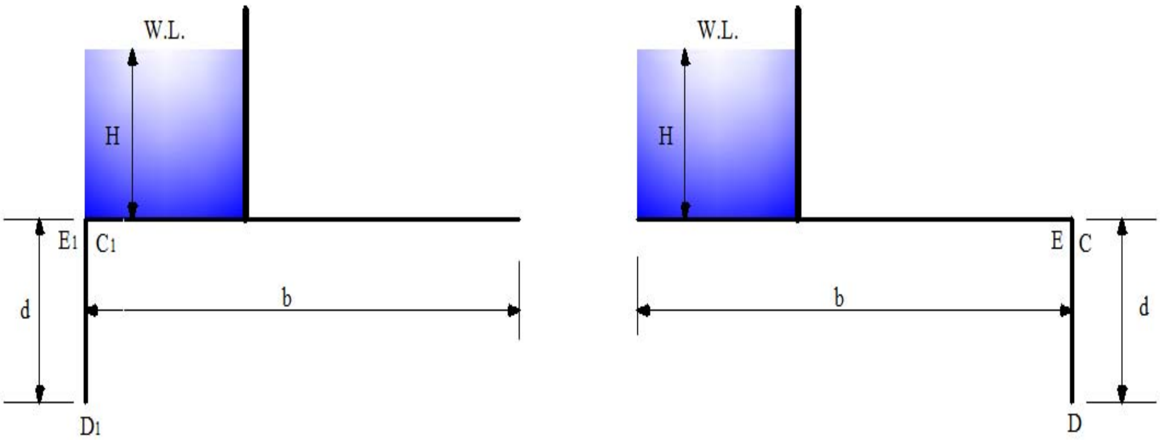


Figure (10): Pile at U/S end (left), pile at D/S end (right)

Thus, for floor with d/s pile (Fig. 11 ,Right)

$\% \emptyset_E = \frac{100}{\pi} cos^{-1}\left(\frac{\lambda-2}{\lambda}\right)$

$\% \emptyset_D = \frac{100}{\pi} cos^{-1}\left(\frac{\lambda-1}{\lambda}\right)$

And hence, for floor with sheet pile at the u/s end (Fig. 11 , Left)

$\% \emptyset_{C_1} = \frac{100}{\pi} cos^{-1}(\frac{2-\lambda}{\lambda}) = 100-\emptyset_E,$

$\% \emptyset_{D_1} = \frac{100}{\pi} cos^{-1}(\frac{1-\lambda}{\lambda}) = 100-\emptyset_D$

Where, $\lambda = \frac{1+\sqrt{1+\alpha^2}}{2}$

$\alpha = \frac{b}{d}$

The values of $\emptyset_D$ and $\emptyset_E$ can also be obtained from the chart (Fig. 11), below:

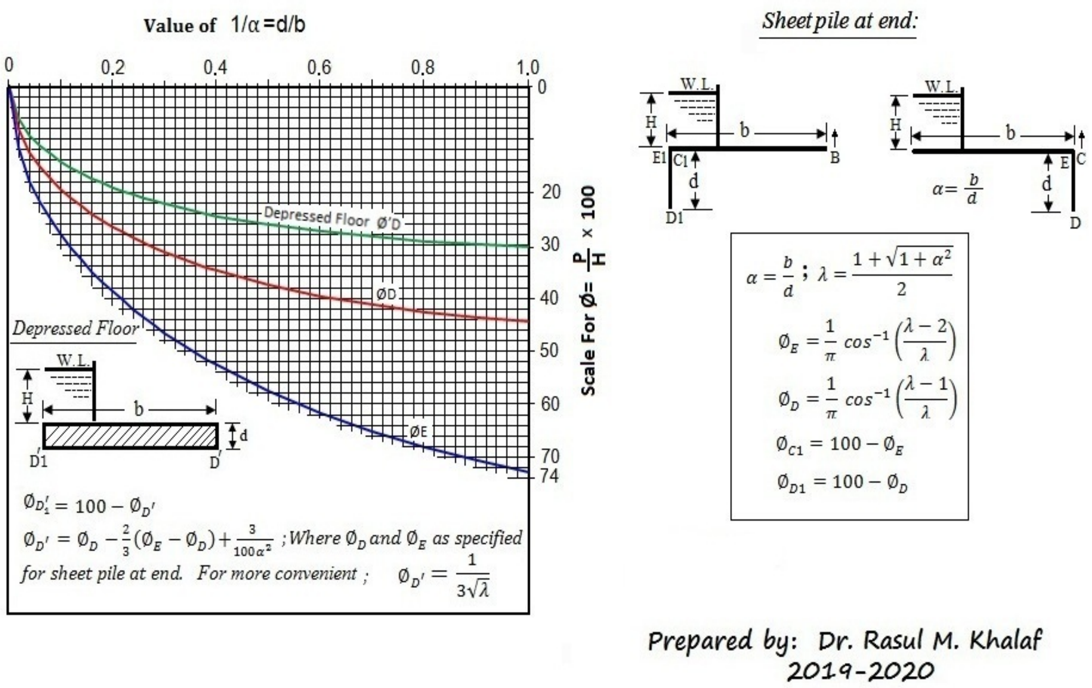


Figure (11): Khosla's chart for Depressed Floor and pile at End.

b. **Floor with an intermediate pile** (see Fig. 12)

$$\emptyset_E = \frac{1}{\pi} cos^{-1}(\frac{\lambda_1 - 1}{\lambda})$$

$$\emptyset_C = \frac{1}{\pi} cos^{-1}(\frac{\lambda_1 + 1}{\lambda})$$

$$\emptyset_D = \frac{1}{\pi} cos^{-1}(\frac{\lambda_1}{\lambda})$$

$$In\ which\ ;\ \lambda = \frac{\sqrt{1+{\alpha_1}^2}+\sqrt{1+{\alpha_2}^2}}{2}\ ,\ \lambda_1 = \frac{\sqrt{1+{\alpha_1}^2}-\sqrt{1+{\alpha_2}^2}}{2}$$

$$\alpha_1 = \frac{b_1}{d}, \alpha_2 = \frac{b_2}{d}$$

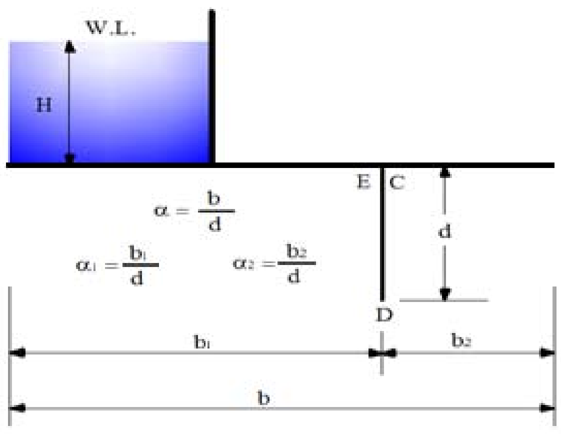
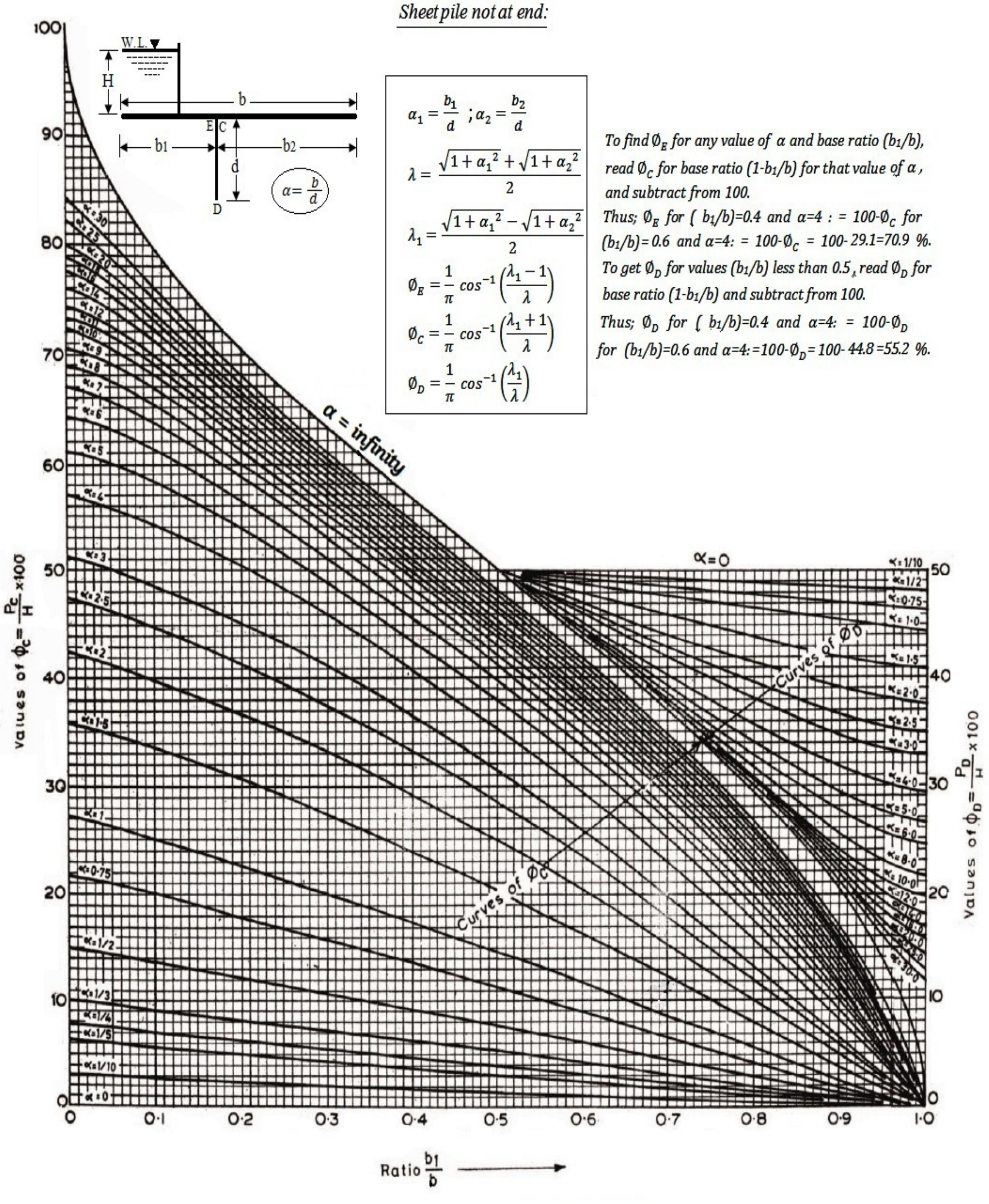


Figure (12): Intermediate pile

The values of $\emptyset_E$, $\emptyset_C$, and $\emptyset_D$ can also be obtained from the charts (Figs. 11 &13):

Khosla's Pressure Curves

Sheet pile not at end:



$$\alpha_1 = \frac{b_1}{d} \; ; \alpha_2 = \frac{b_2}{d}$$

$$\lambda = \frac{\sqrt{1+\alpha_1{}^2} + \sqrt{1+\alpha_2{}^2}}{2}$$

$$\lambda_1 = \frac{\sqrt{1+\alpha_1{}^2} - \sqrt{1+\alpha_2{}^2}}{2}$$

$$\emptyset_E = \frac{1}{\pi} \cos^{-1}\left(\frac{\lambda_1 - 1}{\lambda}\right)$$

$$\emptyset_C = \frac{1}{\pi} \cos^{-1}\left(\frac{\lambda_1 + 1}{\lambda}\right)$$

$$\emptyset_D = \frac{1}{\pi} \cos^{-1}\left(\frac{\lambda_1}{\lambda}\right)$$

To find $\emptyset_E$ for any value of α and base ratio (b_1/b), read $\emptyset_C$ for base ratio ($1-b_1/b$) for that value of α, and subtract from 100.

Thus; $\emptyset_E$ for (b_1/b)=0.4 and α=4 : = 100-$\emptyset_C$ for (b_1/b)= 0.6 and α=4: = 100-$\emptyset_C$ = 100- 29.1=70.9 %.

To get $\emptyset_D$ for values (b_1/b) less than 0.5, read $\emptyset_D$ for base ratio ($1-b_1/b$) and subtract from 100.

Thus; $\emptyset_D$ for (b_1/b)=0.4 and α=4: = 100-$\emptyset_D$ for (b_1/b)=0.6 and α=4: =100-$\emptyset_D$= 100- 44.8 =55.2 %.

Figure (13): Khosla's Chart for Intermediate pile

where, $\emptyset$ is the ratio of the residual seepage head (h) to the total seepage head (H_L), thus

$\emptyset = h/H_L$ ……………………………………….……………………… (21)

2.3.3 Correction For The slope of the Floor

A correction is applied for a sloping floor, and is taken as positive (+ve) for the down and negative (-ve) for the up slope following the direction of flow. The slope correction is applicable to the key point of pile line fixed at the *beginning* or *the end* of the slope. The correction factor given below in Table (3) is to be multiplied by the horizontal length of the slope and divided by the distance between the two pile lines between which the sloping floor is located.

Values of correction for standard slopes such as 1:1, 2:1, 3:1, etc. are tabulated in table (3).

Table 3: Correction factor for slope of the floor

Slope Horizontal (H) **:** Vertical(V)	1:1	2:1	3:1	4:1	5:1	6:1	7:1	8:1
Correction factor, (C %)	11.2	6.5	4.5	3.3	2.8	2.5	2.3	2.0

The above table can be represented by a figure as shown below:

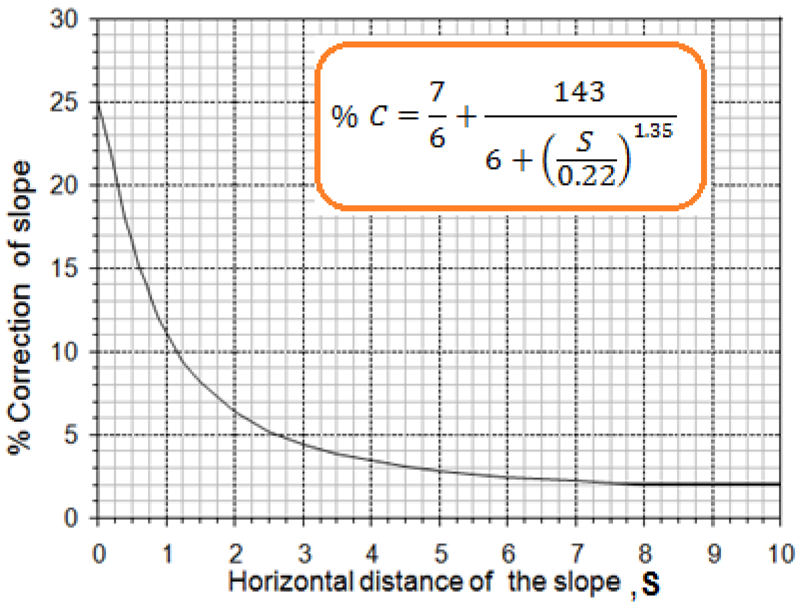


Figure (14): Curve for Correction Due to Slope of Floor.

Referring to Figure (9) , this correction is applicable only to point E_2. Since the slope is down at point E_2 in the direction of flow, hence, the correction shall be (+ve) and will be equal to the correction factor for this slope multiplied by $\frac{b_2}{b_1}$, where b_2 and b_1 are shown in Fig. 11. The slope correction is given in the following equation:

$C_s = \mp \frac{b_2}{b_1} C$ ………………………………………………………………… (22)

where, b_1 = distance between two piles which the sloping floor is located and b_2 = horizontal length of slope (see Fig.14), *Cs* = slope correction, and *C* = coefficient due to slope from table (3)and Figure (14).

2.4 Exit Gradient (GE)

For a floor of length b with a vertical cutoff of depth d, the exit gradient at its downstream end is given by:

$$G_E = \frac{H}{d} \cdot \frac{1}{\pi\sqrt{\lambda}} \quad \text{.......................................} (23)$$

Where,

$$\lambda = \frac{1+\sqrt{1+\alpha^2}}{2} \quad \text{.......................................} (24)$$

and,

$$\alpha = \frac{b}{d} \quad \text{.......................................} (25)$$

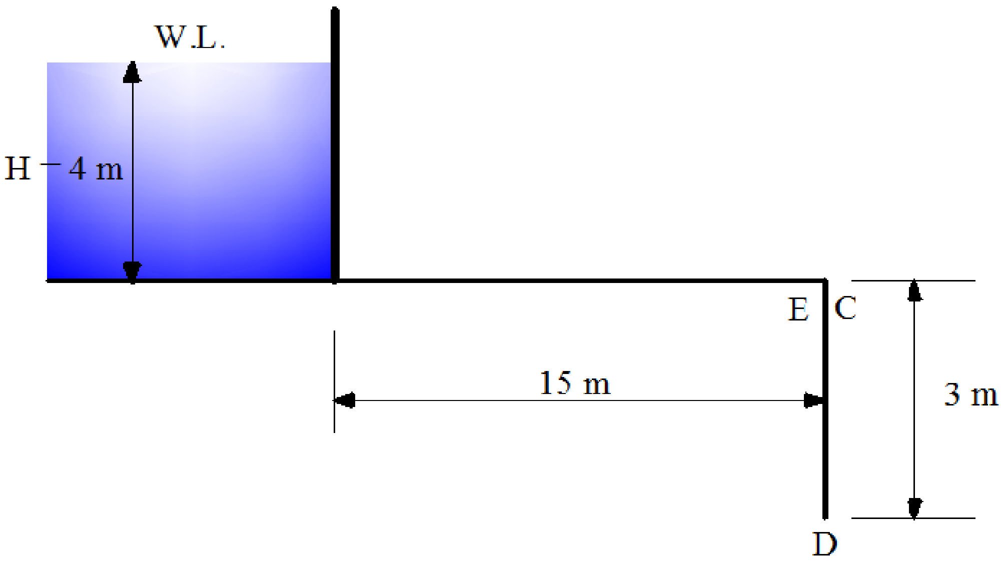
where, G_E = exit gradient, H = maximum static head, d = depth of d/s cutoff, and b = length of floor (horizontal). Safe exit gradients of different soil types is given in Table-4.

Table 4: Safe exit gradient for three types of soil

Type of soil	*Shingle*	*Coarse sand*	*Fine sand*
Safe exit gradient G_E	$\frac{1}{4}$ *to* $\frac{1}{5}$	$\frac{1}{5}$ *to* $\frac{1}{6}$	$\frac{1}{6}$ *to* $\frac{1}{7}$

It is obvious from the equation (23), that if d = 0; G_E = infinite. Hence it becomes essential that a vertical cutoff at the downstream end must be provided.

Example (6): A hydraulic structure with length of horizontal floor in alluvial soil 15 m and 3 m deep vertical sheet pile is attached at its downstream end and the head of water is 4.0 m (see Figure below). Find the thickness of the floor (using Khosla's theory). Is the structure safe against the exit gradient? (F = 8, G_f = 2.45).



Solution:

From equation (7), floor thickness can be calculated as follows:

$t_E = \frac{H_E}{G_f - 1}$, where ($H_E$ *is defined by h in equation 7 and* t_E *by t) and;*

$\emptyset_E = \frac{1}{\pi} cos^{-1}\left(\frac{\lambda-1}{\lambda}\right)$

$\lambda = \frac{1+\sqrt{1+\alpha^2}}{2}$, $\alpha = \frac{b}{d} = \frac{15}{3} = 5$

$\lambda = 3.05$

$\emptyset_E = \frac{1}{\pi} cos^{-1}\left(\frac{3.05-1}{3.05}\right) = 0.265$

$H_E = 0.265 \times 4 = 1.06$ m of water

$t_E = \frac{1.06}{2.45-1} = 0.73$ m of concrete

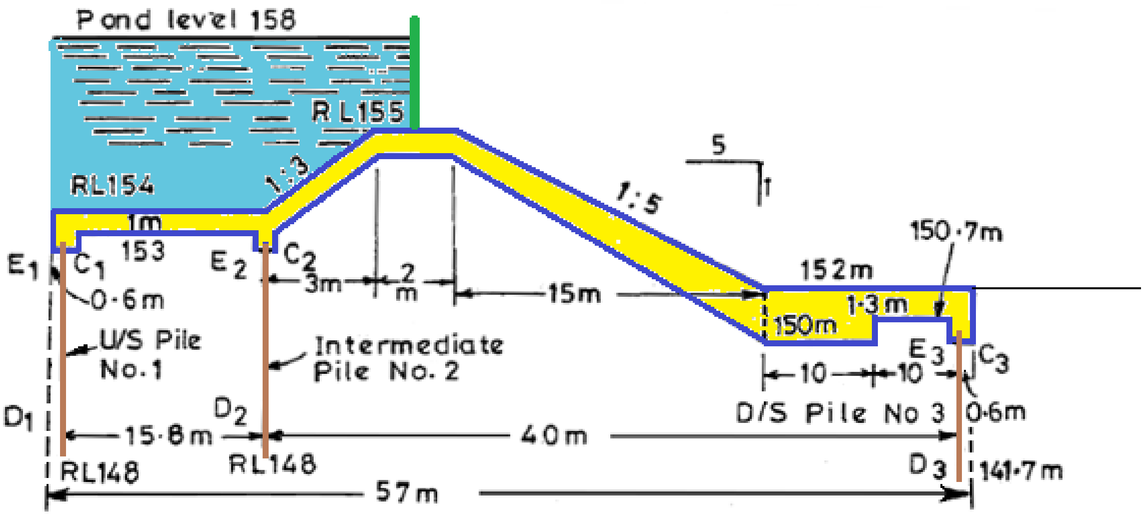
$G_E = \frac{H}{d} . \frac{1}{\pi\sqrt{\lambda}} = \frac{4}{3} . \frac{1}{\pi\sqrt{3.05}} = 0.243$

A safety factor F is defined by equation (16) as follows:

$F = \frac{1}{G_E}$

$= \frac{1}{0.243} = 4.11 < 8$ the structure is unsafe

Example (7): Determine the percentage of pressures at various key points in Figure below. Also determine the exit gradient.



Solution:

<u>Use either fig(11) and figure (13) or use the formulas:</u>

(1) ***Upstream pile line No. (1)***

Total length of the floor = b = 57.0 m

Depth of upstream pile line = d = 154.0 – 148.0 = 6.0 m

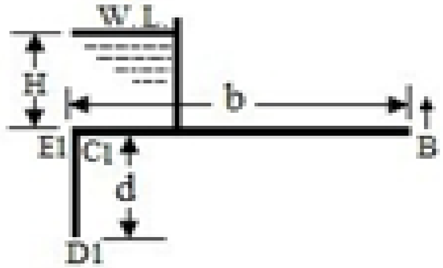
$\alpha = \frac{b}{d} = \frac{57.0}{6.0} = 9.5$

$\frac{1}{\alpha} = \frac{1}{9.5} = 0.105$

From curve of plate (Fig. 11) $\emptyset_E = 29,$ $\emptyset_D = 20$

And for U/s pile :

$\% \, \phi_{D_1} = 100 - \phi_D = 100\text{-}20 = 80\%$
$\% \, \phi_{C1} = 100 - \phi_E = 100\text{-}29 = 71\%$



Or from formulas of u/s pile:

where, $\lambda = \frac{1+\sqrt{1+\alpha^2}}{2} = \frac{1+\sqrt{1+9.5^2}}{2} = 5.276$

$\% \, \phi_D = \frac{100}{\pi} \cos^{-1}\left(\frac{\lambda-1}{\lambda}\right) = \frac{100}{\pi} \cos^{-1}\left(\frac{5.276-1}{5.276}\right) = 20\%$

$\% \, \phi_{D_1} = 100 - \phi_D = 100\text{-}20 = 80\%$

$\% \, \phi_E = \frac{100}{\pi} \cos^{-1}\left(\frac{\lambda-2}{\lambda}\right) = \frac{100}{\pi} \cos^{-1}\left(\frac{5.276-2}{5.276}\right) = 28.6\%$

$\% \, \phi_{C1} = 100 - \phi_E = 100 - 28.6 = 71.4\%$

The value of ϕ_{C1} must be corrected for three corrections as below:

a) Correction at C_1 for mutual interference of piles, ϕ_{C1} is effected by intermediate pile No. 2.

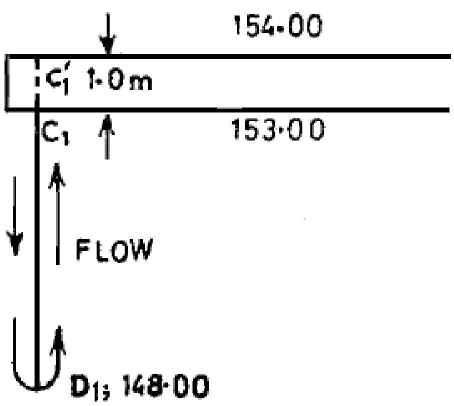
Correction, $C = 19 \sqrt{\frac{D}{b1}} \left(\frac{d+D}{b}\right)$

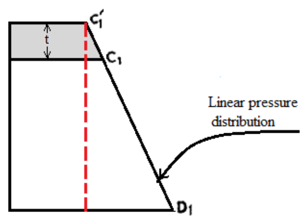
where,
D = Depth of pile No. 2 = 153.0 – 148.0 = 5.0 m
d = Depth of pile No. 1 = 153.0 – 148.0 = 5.0 m
b_1 = Distance between two piles = 15.8 m
b = Total floor length = 57.0 m
Therefore,

$C = 19 \sqrt{\frac{5.0}{15.8}} \left(\frac{5.0+5.0}{57.0}\right) = 1.88\%$

Since the point C_1 is in the rear in the direction of flow, so the correction is +ve.
Correction at C_1 due to pile interference on C_1 = 1.88% (+ve).

b) Correction at C_1 Due to thickness of floor





$\frac{\phi D_1 - \phi C_1'}{\text{Depth from } D_1 \text{ to } C_1'} = \frac{\phi C_1 - \phi C_1'}{\text{Thickness of floor (t)}}$

$\text{Correction} = \phi C_1 - \phi C_1' = \frac{\phi D_1 - \phi C_1'}{\text{Depth from } D_1 \text{ to } C_1'} \times t$

Pressure calculated from the aforementioned Figure is at C_1', but we need the pressure at C_1. Pressure at C_1 shall be more than at C_1' as the direction of flow, so the correction is +ve.

Correction due to thickness = $\frac{\emptyset D_1 - \emptyset C_1}{Depth\ from\ D_1 to\ C_1'} \times$ *Thickness of floor* (t)

Correction due to thickness = $\frac{80\% - 71\%}{154.0 - 148.0} \times (154.0 - 153.0)$

$= \frac{9}{6} \times 1 = 1.5\ (+ve)$

c) *Correction due to slope at C_1 is nil as this point is neither situated at the start nor at the end of the slope.*

so, corrected $\emptyset C_1$ *= 71% + 1.88% + 1.5% = 74.38%*

Hence , corrected $\emptyset C_1$ *= 74.38%*

and $\emptyset D_1$ = 80%

(2) Intermediate pile line No. (2)

d = 154.0 – 148.0 = 6.0 m

b = 57.0 m

$\alpha = \frac{b}{d} = \frac{57.0}{6.0} = 9.5$

Using curve in (Fig.13), we have b_1 in this case

b_1 = 0.6 + 15.8 = 16.4 m (see Fig. 12 for b_1 definition)

b = 57.0

$\frac{b_1}{b} = \frac{16.4}{57.0} = 0.298$;

$\emptyset_{C_2}$ = 57 % (for a base ratio 0.298 and $\alpha = 9.5$)

and thus;

$1 - \frac{b_1}{b} = 1 - 0.298 = 0.702$

($\emptyset_C = 30\%$ for a base ratio of 0.702 and $\alpha = 9.5$), hence:

$\emptyset_{E_2} = 100 - 30\% = 70\%$

Since $\frac{b_1}{b} < 0.5$ then find $\emptyset_D$ for $(1 - \frac{b_1}{b})$;

($\emptyset_D$ =37% for a base ratio of 0.702 and $\alpha = 9.5$)

$\emptyset_{D_2}$ = 100 – 37 = 63%

All above can be calculated by formulas as follow:

$$\alpha_1 = \frac{b_1}{d} = \frac{16.4}{5} = 3.28,\ \alpha_2 = \frac{b_2}{d} = \frac{40.6}{5} = 8.12$$

$$\lambda = \frac{\sqrt{1+\alpha_1{}^2} + \sqrt{1+\alpha_2{}^2}}{2} = \frac{\sqrt{1+3.28^2} + \sqrt{1+8.12^2}}{2} = 5.805$$

$$\lambda_1 = \frac{\sqrt{1+\alpha_1{}^2} - \sqrt{1+\alpha_2{}^2}}{2} = \frac{\sqrt{1+3.28^2} - \sqrt{1+8.12^2}}{2} = -2.376$$

$$\emptyset_E = \frac{100}{\pi} \cos^{-1}\left(\frac{\lambda_1 - 1}{\lambda}\right) = \frac{100}{\pi} \cos^{-1}\left(\frac{-2.376-1}{5.805}\right) = 69.8\%$$

$$\emptyset_C = \frac{100}{\pi} \cos^{-1}\left(\frac{\lambda_1 + 1}{\lambda}\right) = \frac{100}{\pi} \cos^{-1}\left(\frac{-2.376+1}{5.805}\right) = 57.6\%$$

$$\emptyset_D = \frac{1}{\pi} \cos^{-1}\left(\frac{\lambda_1}{\lambda}\right) = \frac{100}{\pi} \cos^{-1}\left(\frac{-2.376}{5.805}\right) = 63.4\%$$

$\emptyset_{E2}$ should be corrected

(a) *Correction at E_2 for sheet pile lines*. Pile No. (1) will affect the pressure at E_2 and since E_2 is in the forward direction of flow and hence, this correction shall be –ve. The amount of this correction is given by equation:

$$C = \mp 19 \sqrt{\frac{D}{b_1}} \left(\frac{d+D}{b}\right)$$

Where D = depth of pile No. 1, the effect of which is considered = 153.0 – 148.0 = 5.0 m.

d = depth of pile No. 2, the effect on which is considered = 153.0 – 148.0 = 5.0 m.

b = total floor length = 57.0 m.

b_1 = distance between two piles = 15.8 m.

$$\text{Correction} = 19 \sqrt{\frac{5.0}{15.8}} \left(\frac{5+5}{57}\right) = 1.88\% \text{ (-ve)}$$

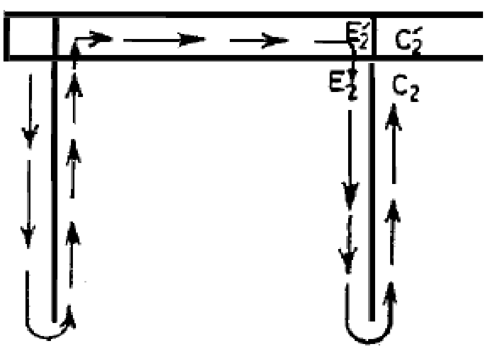
(b) *Correction of E_2 due to floor thickness*

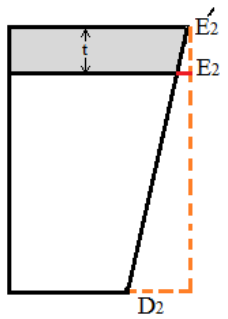
$$= \frac{\emptyset'_{E2} - \emptyset_{D2}}{Depth\ from\ E_2 to\ D_2} \text{ x Thickness of floor}$$

$$= \frac{70\% - 63\%}{154.0 - 148.0} \text{ x } 1$$

$$= \frac{7}{6} \text{ x } 1 = 1.17\%$$

In Figure below since the pressure observed is at E_2' and not at E_2 and by looking at the direction of flow, it can be stated easily that the pressure at E_2 shall be less than that at E_2', hence, this correction is negative.





$$\text{Correction} = \emptyset'_{E_2} - \emptyset_{E_2}$$

$$= \frac{\emptyset'_{E_2} - \emptyset_{D_2}}{Depth\ from\ E_2 to\ D_2} \text{ x t}$$

So, correction at E_2 due to floor thickness = 1.17% *(-ve).*

(c) *Correction at E_2 due to slope is nil as the point E_2 is neither situated at the start of a slope nor at the end of a slope.*

(d) *Hence, corrected percentage pressure at E_2 :*

$$= \text{Corrected } \emptyset_{E2} = 70\% - 1.88\% - 1.17\%$$
$$= 66.95\%$$

$\emptyset_{C2}$ should be corrected

(a) *Correction at C_2 due to pile interference*. Pressure at C_2 is affected by pile No. 3 and since the point C_2 is in the back water in the direction of flow and hence, this correction is +ve. The amount of this correction is given by equation (17):

$$\text{Correction, C} = 19\sqrt{\frac{D}{b1}}\left(\frac{d+D}{b}\right)$$

Where D = depth of pile No. 3, the effect of which is considered = 153.0 – 141.7 = 11.3 m.

d = depth of pile No. 2, the effect on which is considered = 153.0 – 148.0 = 5.0 m.

b = total floor length = 57.0 m.

b_1 = distance between pile 2 and pile 3 = 40.0 m.

$$\text{Correction} = 19\sqrt{\frac{11.3}{40.0}}\left(\frac{11.3+5}{57}\right) = 2.89\% \ (+ve).$$

(b) *Correction at C_2 due to floor thickness*.

From Fig. above, it can be easily stated that the pressure at C_2 shall be more than that at C_2', this correction shall be +ve and its amount is the same as was calculated for the point E_2 = 1.17%.

(c) *Correction at C_2 due to slope*.

The correction for the sloping floor is given by:

$\% \text{ Cs} = \pm\left(\frac{b_s}{b_1}\right)C$ % of H_L ; as defined previously by Eq. (22).

Where,

H_L = Seepage head

bs = the horizontal length of the sloping floor

b_1 = the distance between the two piles

C= the correction factor of slope taken from table 3.or fig (14).

Correction factor for 3:1 slope = 4.5

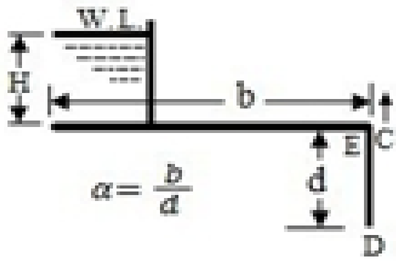
Horizontal length of the slope, bs = 3.0 m.

Distance between two pile lines among which the sloping floor is located = 40 m.

Actual correction, % Cs = 4.5 x $\frac{3}{40}$ = 0.34% (-ve)

Hence, corrected $\emptyset_{C2}$ = 56% + 2.89% + 1.17% − 0.34% = 59.72%

(3) Downstream pile line No. (3)



d = 152.0 – 141.70 = 10.3 m

b = 57.0 m

$\alpha = \frac{b}{d} = \frac{57.0}{10.3} = 5.534$

$\lambda = \frac{1+\sqrt{1+\alpha^2}}{2} = \frac{1+\sqrt{1+5.534^2}}{2} = 2.812$

$\% \, \emptyset_D = \frac{100}{\pi} cos^{-1}(\frac{\lambda-1}{\lambda}) = \frac{100}{\pi} cos^{-1}(\frac{2.812-1}{2.812})$ =27.7%

$\% \, \emptyset_E = \frac{100}{\pi} cos^{-1}(\frac{\lambda-2}{\lambda}) = \frac{100}{\pi} cos^{-1}(\frac{2.812-2}{2.812}) = 40.6\%$

Or from curves of plate (Fig.11), read $\emptyset_D$and $\emptyset_E$ in which $\emptyset_D = \emptyset_{D3}$ and $\emptyset_E = \emptyset_{E3}$ we get :

$\frac{1}{\alpha} = \frac{d}{b} = \frac{10.3}{57.0} = 0.18$

$\emptyset_{D3} = 27\%$

$\emptyset_{E3} = 39\%$

$\emptyset_{E_3}$ should be corrected

(a) *Correction due to mutual of piles*. The point E_3 is affected by pile No. (2) and since E_3 is in the forward direction of flow from pile No. (3), the correction is negative and its amount is given by equation (17):

Correction, C = $19\sqrt{\frac{D}{b1}}\left(\frac{d+D}{b}\right)$

Where,

D = Depth of pile No. 2, the effect of which is considered = 150.7 – 148.0 = 2.7 m.

d = Depth of pile No. 3, the effect on which is considered = 150.7 – 141.7 = 9.0 m.

b_1 = Distance between piles = 40.0 m.

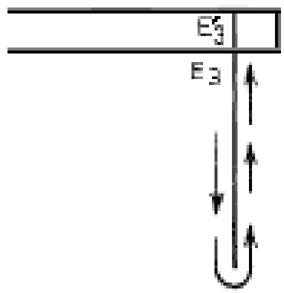
b = Total floor length = 57.0 m.

The correction = $19\sqrt{\frac{2.7}{40.0}}\left(\frac{9.0+2.7}{57.0}\right)$= 1.01 % *(-ve)*

(b) *Correction due to floor thickness*

From Figure below, it can be stated easily that the pressure at E_3 shall be less than at E_3'; thus correction shall be negative (-ve) and its amount

$= \frac{39\%-27\%}{152.0-141.7}$ x 1.3 = 1.52 % *(-ve)*



(c) *Correction due to slope at E_3 is nil as the point E_3 is neither situated at the start nor at the end of any slope*.

Hence, corrected $\emptyset_{E3}$= 39% $-$1.01% $-$ 1.52% = 36.47%

The corrected pressures at various key points are tabulated in Table below:

Upstream pile No.1	*Intermediate pile No.2*	*Downstream pile No.3*
$\emptyset_{E1}$= 100 %	$\emptyset_{E2}$= 66.95 %	*$\emptyset_{E3}$= 36.47 %*
$\emptyset_{D1}$= 80.0 %	$\emptyset_{D2}$= 63.0 %	*$\emptyset_{D3}$=27.0 %*
$\emptyset_{C1}$= 74.38 %	$\emptyset_{C2}$= 59.72 %	*$\emptyset_{C3}$=0.0 %*

Plotting Hydraulic grade lines

The percentage of pressures in above table can be used to work out the elevations of H.G. line above the datum, as given in table 5.However the subsoil H.G. line is then plotted in figure 15.

Table-5 calculation of H.G. Grade lines.

Flow Condition	Upstream water level in meters	Down-stream water level in meters	Head in meters	Height / Elevation of Sub-soil H.G. Line above Datum								
				Upstream Pile Line			Intermediate Pile Line			Upstream Pile Line		
				$Ø_{E1}$	$Ø_{D1}$	$Ø_{C1}$	$Ø_{E2}$	$Ø_{D2}$	$Ø_{C2}$	$Ø_{E3}$	$Ø_{D3}$	$Ø_{C3}$
				100%	80.0%	74.38%	66.95%	63.0%	59.72%	36.47%	27. 0%	0.0%
Pond level with no flow d/s	158.00	152.00	6.00	6.00 158.0	4.80 156.8	4.46 156.46	4.02 156.02	3.78 155.78	3.58 155.58	2.19 154.19	1.62 153.92	0.00 152

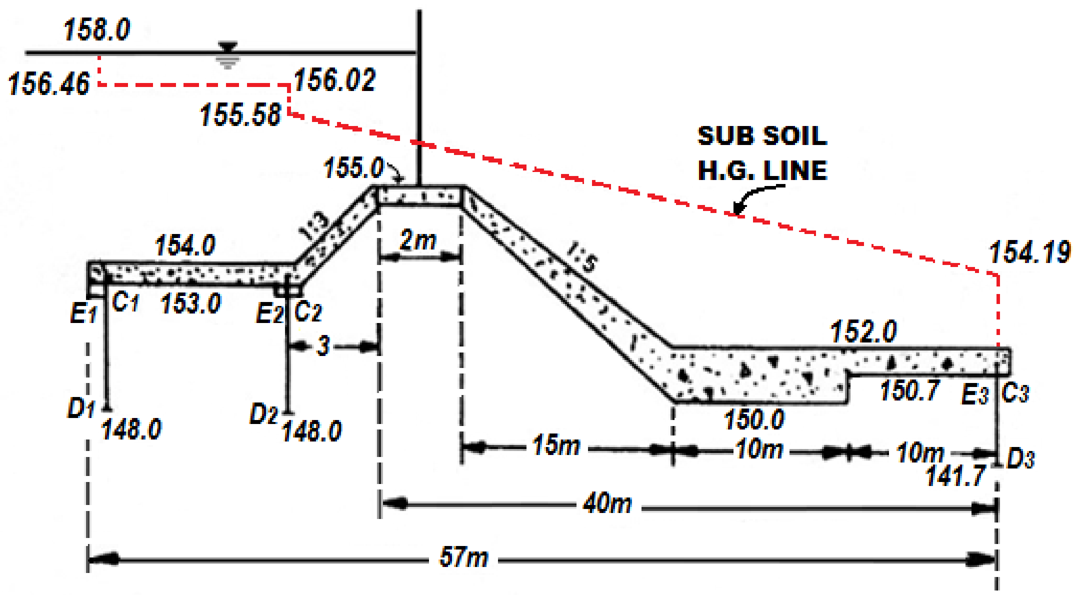


Figure (15): Subsoil H.G. lines distribution

Calculation of Exit Gradient

Let the water be headed up to pond level, i.e. an RL 158.0 m on the upstream side with no flow downstream.

The maximum seepage head = H = 158.0 – 152.0 = 6.0 m.

The depth of the d/s cut-off = d = 152.0 – 141.70 = 10.3 m.

Total floor length = b = 57.0 m.

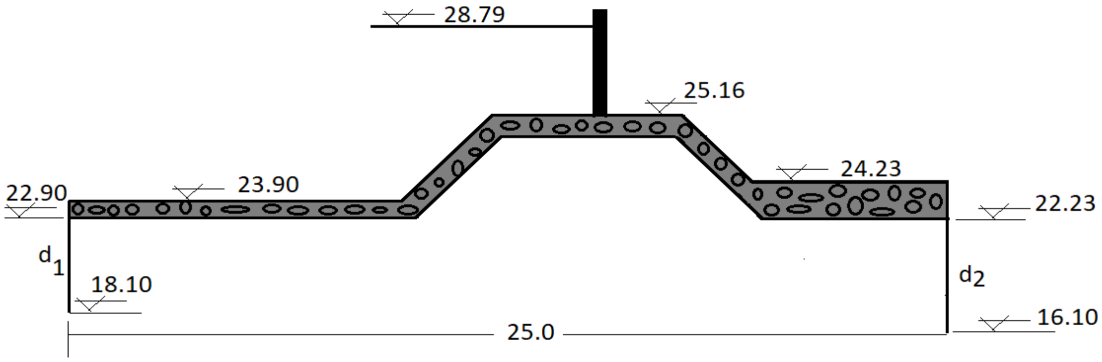
$\alpha = \frac{b}{d} = \frac{57.0}{10.3} = 5.53$

For a value of α = 5.53, $\frac{1}{\pi\sqrt{\lambda}}$ from curves of plate (Fig.11) is equal to 0.18.

Hence, $G_E = \frac{H}{d}\frac{1}{\pi\sqrt{\lambda}} = \frac{6.0}{10.3} \times 0.18 = \frac{1}{9.53}$

Hence, the exit gradient shall be equal to $\frac{1}{9.53}$ which is very much safe.

***Example 8*:** Given the following Figure. According to Khosla's theory, is the structure safe against piping? Use $G_E = \frac{1}{6}$



Solution:

One can solve the problem either by the formula or using Fig.(16)shown below:

$G_E = \frac{H_L}{d_2}\frac{1}{\pi\sqrt{\lambda}}$

$H_L = 28.79 - 24.23 = 4.26$ m

d_2 = D/S cutoff = 24.23 – 16.1

= 8.13 m

b = 25.0 m

$\alpha = b/d_2 = \frac{25.0}{8.13} = 3.075$

$\lambda = \frac{1+\sqrt{1+\alpha^2}}{2}$

$\lambda = \frac{1+\sqrt{1+3.075^2}}{2}$

$\lambda = 2.116$ m

$G_E = \frac{H_L}{d_2}\frac{1}{\pi\sqrt{\lambda}}$

$G_E = \frac{4.56}{8.13}\frac{1}{\pi\sqrt{2.116}}$

$= 0.122 = \frac{1}{8.14} < \frac{1}{6}$

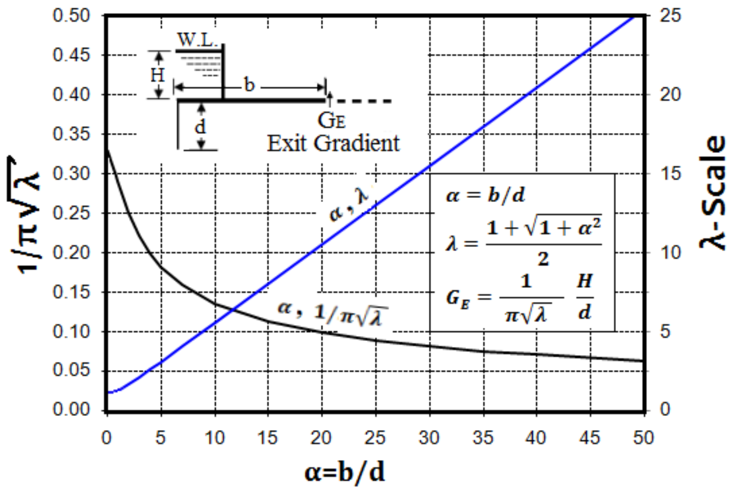


Figure (16) Exit gradient as a function of α and H/d

The structure is safe against piping. (OK)

Or use Fig.(16) to find λ from α and getting $\left(\frac{1}{\pi\sqrt{\lambda}}\right)$, and the just multiply this paraeter,by $\left(\frac{H_L}{d_2}\right)$ to get Exit Gradient GE.

2.5 Depth of Cutoff

The sheet piles at the ends must go below the deepest anticipated scour level.

a) D/S cutoff

The depth d of the cutoff can be obtained from the following equation;

D/S *cutoff = (1.25 to 1.5) R* …………………….……………………………….. (26)

The normal depth of scour (R) is given by Lacey's equation as:

$R = 1.35\ (\frac{q^2}{f})^{\frac{1}{3}}$ ……………………………………………………………… (27)

R = scour depth, m

f = silt factor = $1.76\ \sqrt{D_{mm}}$ ……………………………………………………… (28)

D_{mm} = Diameter of particle of soil in mm.

$q = \frac{Q}{B}$ (discharge per unit widthm^3/s/m)

b) U/S cutoff

U/S cutoff = (1 to 1.25) R …………………………………………………….. (29)