

Domain and Range function

1. Domain المجال

* يمثل قيم (x) التي تجعل قيمة (y) حقيقية.
 $y = f(x)$

2. Range المجال المقابل

* يمثل قيم (y) التي تجعل قيمة (x) حقيقية.
 $x = f(y)$

حالات Domain and Range

أذا كانت الدالة ليست كسرية أو جذرية (1)
 فإن كل القيم الحقيقية $Df = R = (-\infty, \infty)$ المجال

$Rf = R = (-\infty, \infty)$ المجال المقابل

أذا كانت الدالة كسرية (لا تحتوي متغير بالمقام) (2)
 المجال $\neq 0$

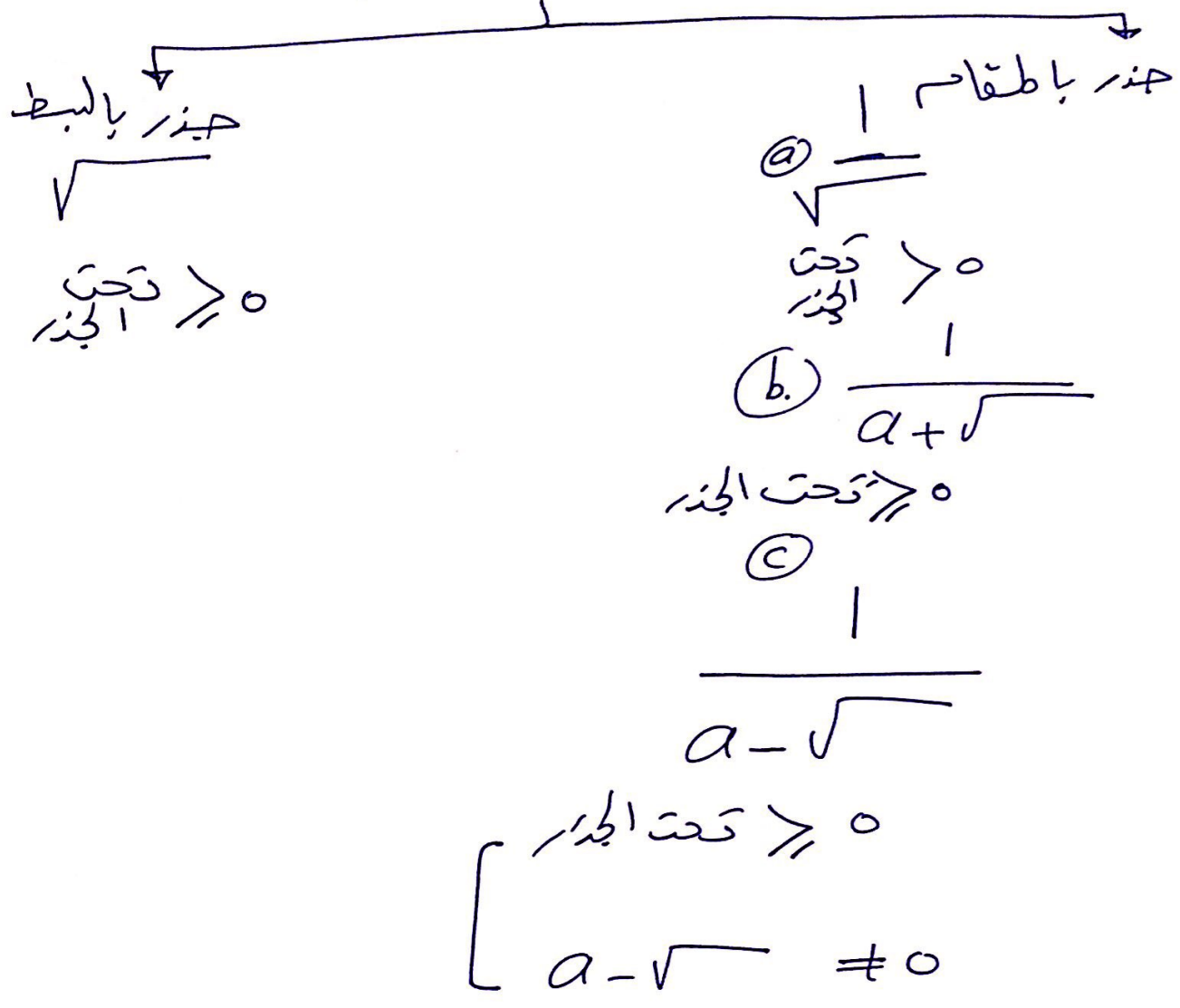
$$\frac{3}{x-2} \Rightarrow x-2 \neq 0 \text{ و } x \neq 2$$

$$\frac{1}{x^2-1} \Rightarrow \begin{aligned} x^2-1 &\neq 0 \\ x &\neq 1 \\ x &\neq -1 \end{aligned}$$

3.

إذا كانت الدالة جذرية

يجب أن لا يكون داخل الجذر سالب



Ex 1 Find domain and Range

$y = 3x + 6$

Sol: $y = 3x + 6$

$Df = R$

$3x = y - 6 \Rightarrow x = \frac{y-6}{3}$

$\therefore Rf = R$

الدالة ليست كسرية او جذرية

Exo-② find domain and Range

3.

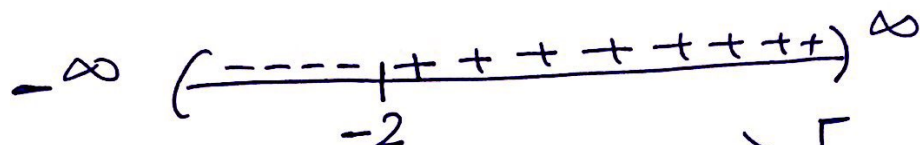
$$y = x^2 - 2$$

Sol: $y = x^2 - 2 \rightarrow Df = \mathbb{R}$ ليست كسرية أو جزئية

$$x^2 = y + 2$$

$$x = \pm \sqrt{y + 2}$$

$$y + 2 \geq 0 \rightarrow y \geq -2$$



$$Rf = [-2, \infty)$$

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Exo-③ Find domain and Range.

الدالة كسرية

$$y = \frac{2x}{x-1}$$

$$x-1 \neq 0 \rightarrow x \neq 1$$

$$Df = \mathbb{R} / \{1\}$$

$$xy - y = 2x$$

$$xy - 2x = y \rightarrow x(y-2) = y$$

$$\therefore x = \frac{y}{y-2} \rightarrow \begin{matrix} y-2 \neq 0 \\ y \neq 2 \end{matrix}$$

$$\therefore Rf = \mathbb{R} / \{2\}$$

Ex 4) $y = \frac{1}{x+1} - \frac{1}{x-1}$

4.

Find domain and Range?

Sol: $y = \frac{(x-1) - (x+1)}{(x+1)(x-1)}$

$$y = \frac{x-1-x-1}{(x+1)(x-1)} = \frac{-2}{(x+1)(x-1)}$$

$$x+1 \neq 0 \rightarrow x \neq -1$$

$$x-1 \neq 0 \rightarrow x \neq 1$$

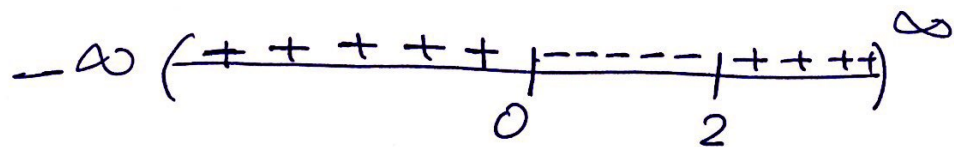
$$Df = R / \{-1, 1\}$$

$$y = \frac{-2}{x^2-1}$$

$$yx^2 - y = -2$$

$$yx^2 = y-2 \Rightarrow x^2 = \frac{y-2}{y}$$

$$\therefore x = \sqrt{\frac{y-2}{y}} \rightarrow \begin{cases} y-2 \geq 0 & \bullet y \geq 2 \\ y > 0 \end{cases}$$



$$Rf = (-\infty, 0) \cup [2, \infty)$$

Exe-5 find domain and Range

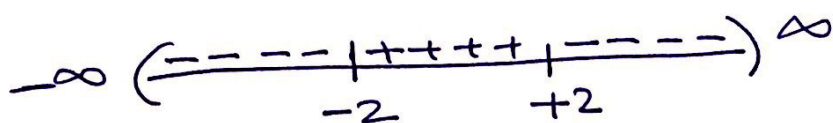
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$$y = \sqrt{4-x^2}$$

Soln-

$$4-x^2 \geq 0 \rightarrow -x^2 \geq -4 \quad *-1$$

$$x^2 \leq 4 \rightarrow x \leq \pm 2$$



$$Df = [-2, 2]$$

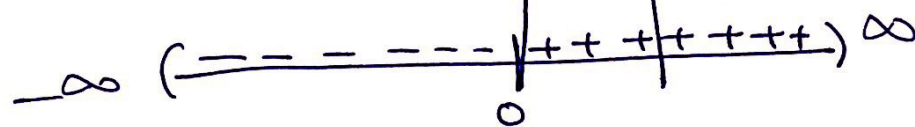
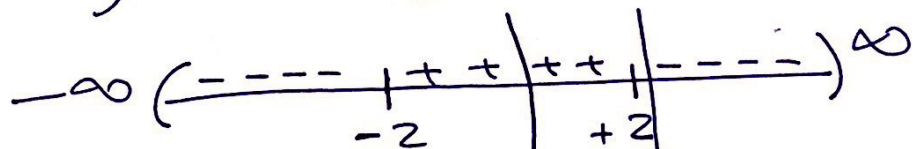
Range ① $y \geq 0$

$$② y^2 = 4-x^2$$

$$x^2 = 4-y^2 \rightarrow x = \pm \sqrt{4-y^2}$$

$$4-y^2 \geq 0 \rightarrow -y^2 \geq -4 \quad *-1$$

$$y^2 \leq 4 \rightarrow y \leq \pm 2$$

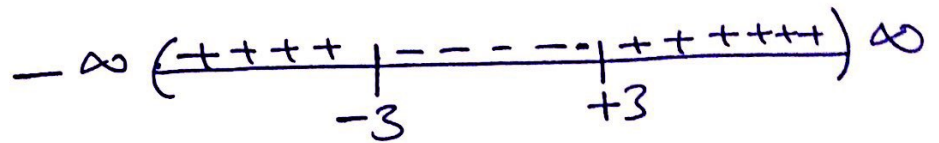


$$\therefore Rf = [0, 2]$$

Ex: ⑥ find domain and Range? [6.]

$$y = \sqrt{x^2 - 9}$$

Sol: $x^2 - 9 \geq 0 \rightarrow x^2 \geq 9 \rightarrow x \geq \pm 3$



$$D_f = (-\infty, -3] \cup [3, \infty)$$

Range

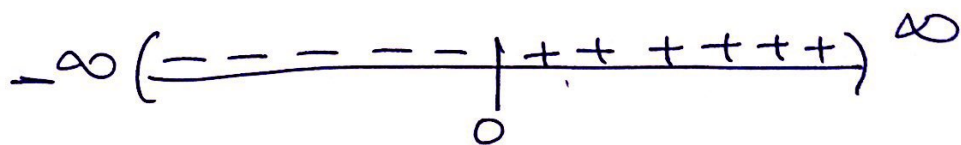
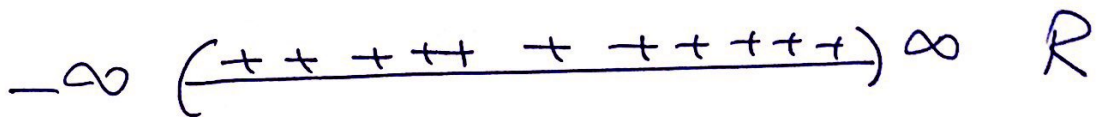
① $y \geq 0$

② $y^2 = x^2 - 9$

$$x^2 = y^2 + 9$$

$$x = \pm \sqrt{y^2 + 9}$$

Note $\sqrt{(\text{مربع}) + (\text{مربع})} \rightarrow R$



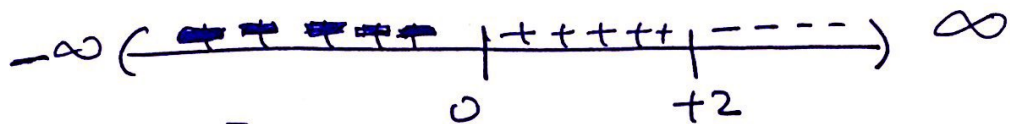
∴ $R_f = [0, \infty)$

Ex: 7 Find domain and Range?

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$$y = \sqrt{2x - x^2}$$

Sol: $2x - x^2 \geq 0 \rightarrow x(2-x) \geq 0$
 $x \geq 0$
 $2-x \geq 0 \rightarrow x \leq 2$



$$Df = [0, 2]$$

Range ① $y \geq 0$

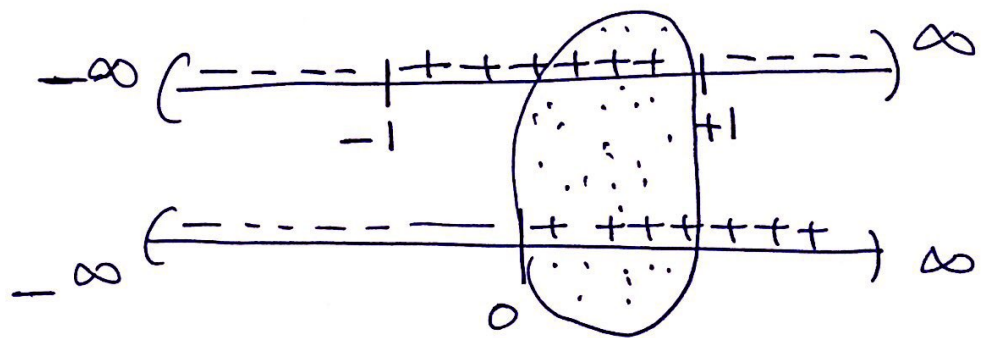
$$y^2 = 2x - x^2 \rightarrow x^2 - 2x + y^2 = 0$$

$$ax^2 + bx + c = 0 \text{ دستور } x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{2 \pm \sqrt{(2)^2 - 4 \times y^2}}{2} = 1 \pm \frac{\sqrt{4 - 4y^2}}{2}$$

$$4 - 4y^2 \geq 0 \div 4 \rightarrow -y^2 \geq -1 \quad * -1$$

$$y^2 \leq 1 \rightarrow y \leq \pm 1$$



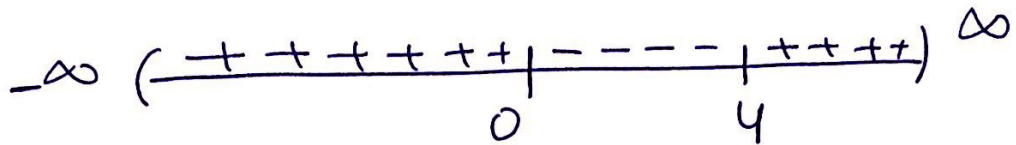
$$\therefore Rf = [0, 1]$$

Ex 8 find domain and Range

8.

$$y = \sqrt{x^2 - 4x}$$

Solve $x^2 - 4x \geq 0 \rightarrow x(x-4) \geq 0$
 $x \geq 0, x \geq 4$



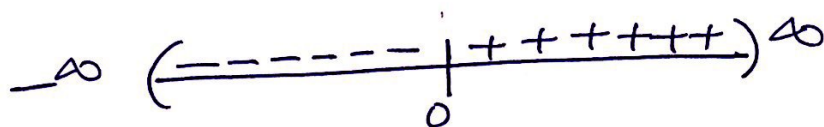
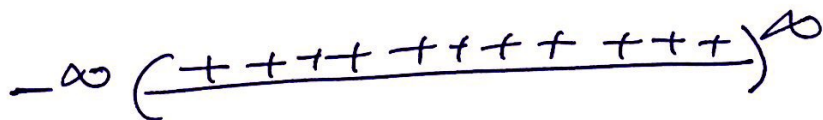
$$Df = (-\infty, 0] \cup [0, 4)$$

Range $y \geq 0$

$$y^2 = x^2 - 4x \rightarrow x^2 - 4x - y^2 = 0$$

$$x = \frac{4 \pm \sqrt{(-4)^2 - 4(-y^2)}}{2}$$

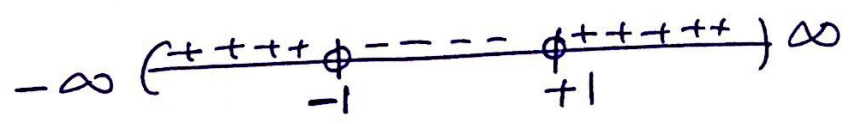
$$x = 2 \pm \frac{\sqrt{4 + 4y^2}}{2} \rightarrow R$$



$$Rf = [0, \infty)$$

Ex: (10) $y = \frac{1}{\sqrt{x^2 - 1}}$

Sol: $x^2 - 1 > 0 \rightarrow x^2 > 1 \rightarrow x > \pm 1$
لا توجد مساواة لان الجذر بالمقام



$D_f = (-\infty, -1) \cup (1, \infty)$

① $y \geq 0$

② $y^2 = \frac{1}{x^2 - 1} \Rightarrow x^2 - 1 = \frac{1}{y^2}$

$x^2 = \frac{1}{y^2} + 1$

$x = \sqrt{\frac{1}{y^2} + 1}$

$\therefore y > 0$

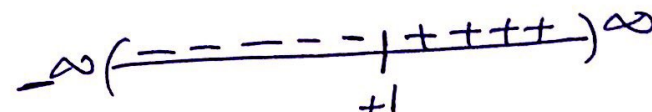
$\therefore R_f = (0, \infty)$

Ex: (11) Find domain and Range

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$$y = \frac{2}{3 + \sqrt{x-1}}$$

Sol: $x-1 \geq 0 \Rightarrow x \geq 1$

Df = $[1, \infty)$ 

(1) $y \geq 0$

(2) $3 + \sqrt{x-1} = \frac{2}{y}$

$$\sqrt{x-1} = \frac{2}{y} - 3$$

$$x-1 = \left(\frac{2}{y} - 3\right)^2$$

$$\therefore x = \left(\frac{2}{y} - 3\right)^2 + 1$$

$y \neq 0$

$\therefore Rf = (0, \infty)$

H.W Find domain and Range

(1) $(x-2)(y+3) = x-1$

(2) $y = \sqrt{x^2 + 3x}$

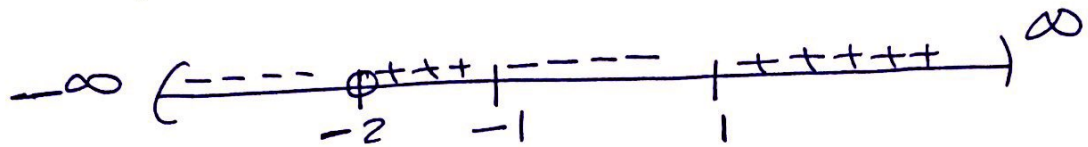
(3) $y = \frac{x-1}{x^2-4}$

Exo-12 find domain only

$$① y = \sqrt{\frac{x^2-1}{x+2}}$$

$$x^2-1 \geq 0 \rightarrow x^2 \geq 1 \rightarrow x \geq \pm 1$$

$$x+2 > 0 \rightarrow x > -2$$



$$D_f = (-2, -1] \cup [1, \infty)$$

Exo-13

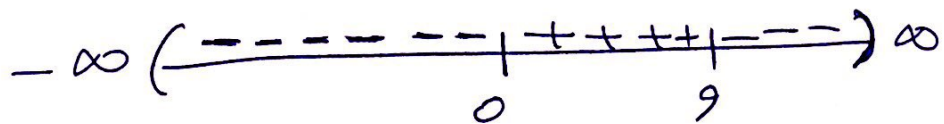
$$② y = \sqrt{3-\sqrt{x}}$$

$$x \geq 0$$

$$3-\sqrt{x} \geq 0 \Rightarrow -\sqrt{x} \geq -3 \quad * -1$$

$$\sqrt{x} \leq 3$$

$$x \leq 9$$



$$D_f = [0, 9]$$

Ex: (14) Find domain only

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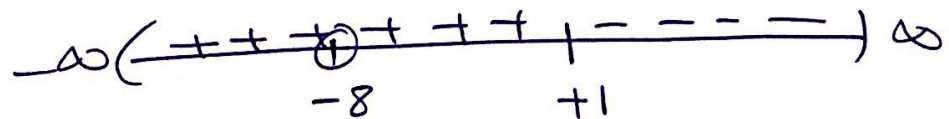
$$y = \frac{x-2}{3-\sqrt{1-x}}$$

$$1-x \geq 0 \Rightarrow x \leq 1$$

$$3-\sqrt{1-x} \neq 0 \quad 3 \neq \sqrt{1-x}$$

$$9 \neq 1-x$$

$$x \neq -8$$



$$Df = (-\infty, +1] / \{-8\}$$

Ex: (15) $y = \sqrt{-|x|}$

$$-|x| \geq 0 \rightarrow |x| \leq 0$$

$$Df = (-\infty, 0]$$

① $y \geq 0$

② $y^2 = -|x| \Rightarrow |x| = -y^2$ لا يوجد مقلوب
بإشارة

∴ Range $y \geq 0$

$$Rf = [0, \infty)$$

Ex:- (16) Find domain and Range

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$$\sqrt{x} + \sqrt{y} = 1$$

Sol:- $\sqrt{x} \rightarrow x \geq 0$

$\sqrt{y} \rightarrow y \geq 0$

$$\sqrt{y} = 1 - \sqrt{x} \Rightarrow 1 - \sqrt{x} \geq 0$$

$$-\sqrt{x} \geq -1$$

$$\sqrt{x} \leq 1$$

$$x \leq 1$$

$$Df = [0, 1]$$

$$\sqrt{x} = 1 - \sqrt{y}$$

$$\therefore 1 - \sqrt{y} \geq 0$$

$$-\sqrt{y} \geq -1$$

$$\sqrt{y} \leq 1$$

$$y \leq 1$$

$$\therefore Rf = [0, 1]$$