Chapter 4

Amplitude Modulation

Modulation:

A process by which a property of a signal is varied in proportion to a second signal.

Types of Modulation:

1- Continuous Wave (CW) Modulation:

In which a sinusoidal signal is changed in amplitude, frequency or phase in proportion to a message signal, such as AM, FM and PM.

2- Pulse Modulation:

In which a periodic train pulses is changed in amplitude, position or width in proportion to a message signal. Such as PAM, PPM, PWM, PCM and DM.

Reasons of Modulation:

1- Modulation for frequency location assignment.
2- Modulation for bandwidth alteration.
3- Modulation to increase efficiency of radiation.
4- Modulation to reduce noise and interference.
5- Modulation to overcome equipment limitation.

Amplitude Modulation:

The general sinusoidal signal can be written as:

$$\Phi(t) = a(t)cos[\omega_c t + \gamma(t)]$$

Amplitude Frequency Phase

Angle

In amplitude modulation (AM), $a(t)$ is changed in proportion to the message signal. Frequency is constant, phase $(t) = 0$.

Types of AM:

1- Double-Sideband, suppressed Carrier (AM/DSB-SC).
2- Double-Sideband, Large Carrier (AM/DSB-LC) [AM].
3- Single-sideband, suppressed carrier (AM/SSB-SC) [SSB].
4- Vestigial –sideband (AM/VSB).

1- AM/DSB-SC

The AM/DSB-SC signal, assuming proportionality constant =1, is given by:

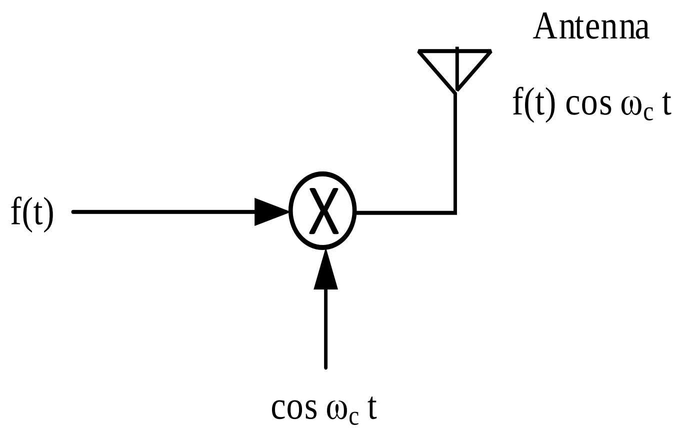
$$\Phi(t)_{DSB/SC} = f(t)cos\omega_c t \qquad \ldots (4\text{-}1)$$

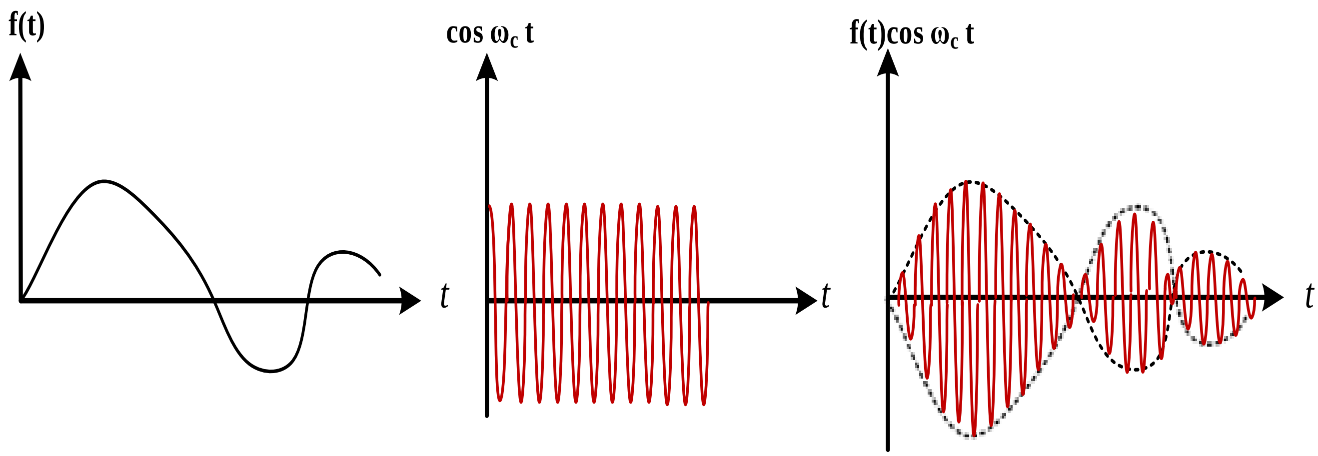
Modulated Signal Modulating Signal Carrier Signal

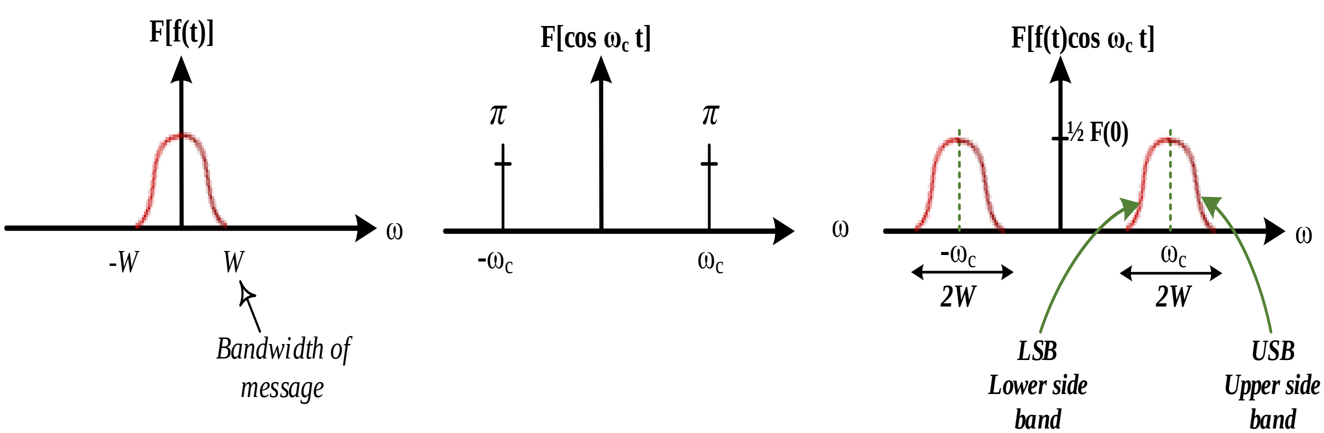
and the spectrum is:

$$\Phi(\omega)_{DSB/SC} = \pi F(\omega - \omega_c) + \pi F(\omega + \omega_c) \qquad \ldots (4\text{-}2)$$

DSB-SC Transmitter







Notes:

1- No carrier term is presents (carrier is suppressed)

2- $BW_{DSB/SC} = 2W$ rad/sec …(4-3)

Where W is the bandwidth of message (modulating signal) i.e. the bandwidth is doubled.

3- Above process (multiplication) is called "Frequency conversion" or "frequency mixing" or Heterodyning.

DSB-SC Receiver

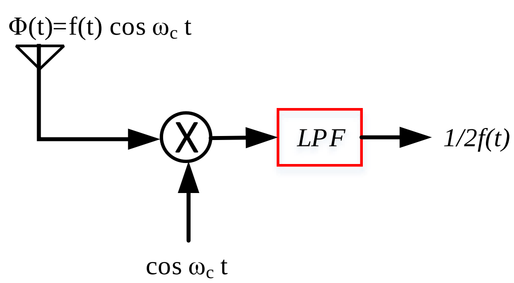
To detect (demodulate) the DSB-SC signal, we multiply it again by $\cos\omega_c t$ as follows:

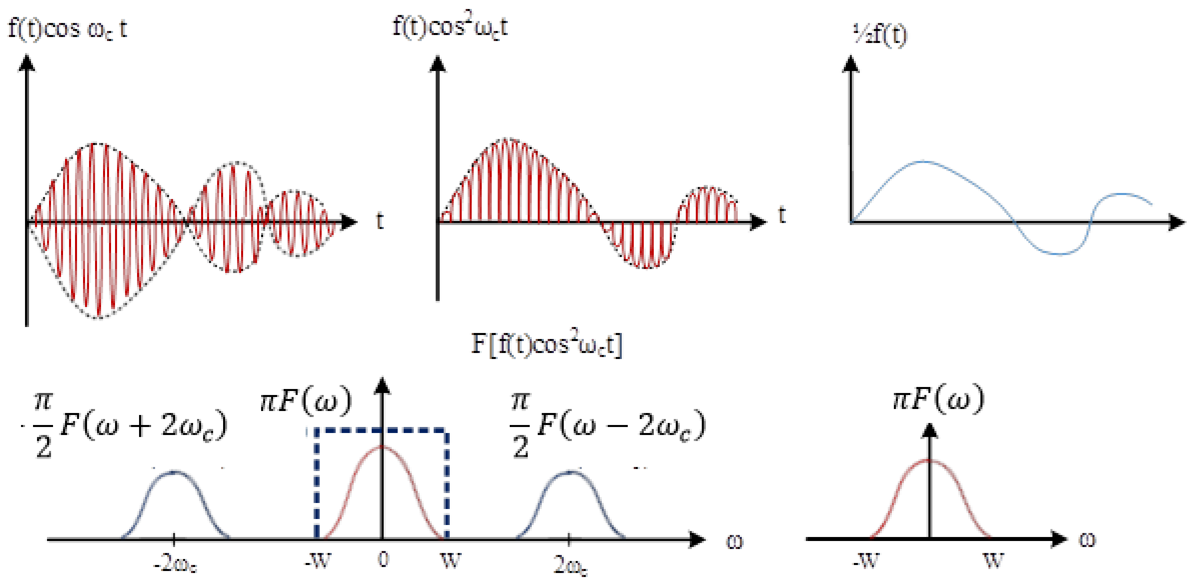
$$\Phi(t)cos\omega_c t = f(t)cos^2\omega_c t$$

$$= \frac{1}{2}f(t) + \frac{1}{2}f(t)cos2\omega_c t$$

$$F[\Phi(t)cos\omega_c t] = \pi F(\omega) + \frac{\pi}{2}F(\omega - 2\omega_c) + \frac{\pi}{2}F(\omega + 2\omega_c)$$

Then using LPF of bandwidth W **rad/sec** we obtain the original signal.





Notes:

1- For LPF will reject the frequency component at $\pm 2\omega_c$.

2- For correct detection it must that:

a) $\omega_c \gg W$

b) Both the local oscillator (cos ω_ct generators) in Tx and Rx are **synchronized.** (Synchronous detection and coherent detection).

Generation of DSB-SC:

1- Using Switching Modulator:

f(t) is multiplied by a periodic signal given by:

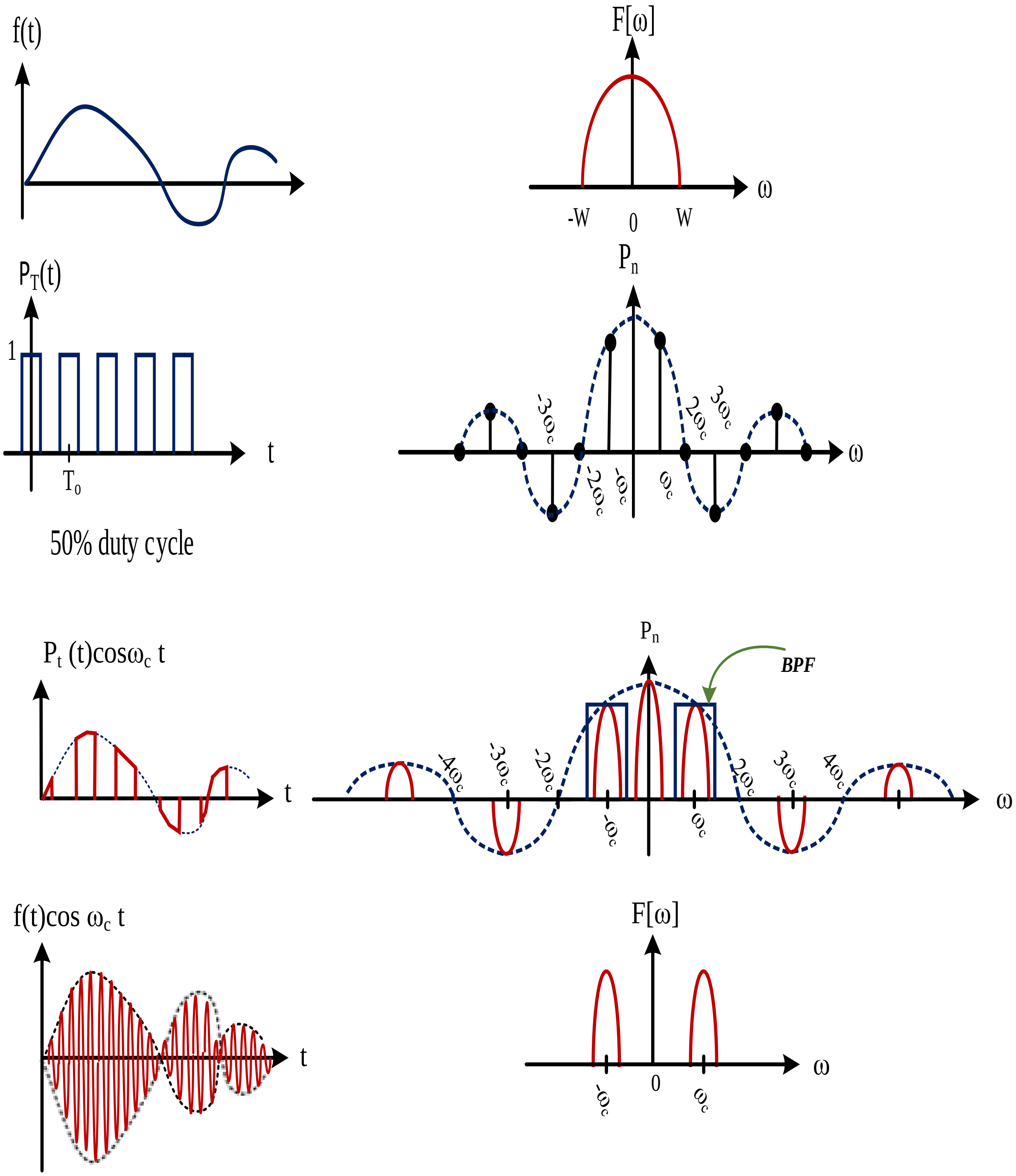
$$P_T = \sum_{n=-\infty}^{\infty} P_n e^{jn\omega_o t} \omega_o = \frac{2\pi}{T_o}$$

Letting $\omega_o = \omega_c$ [speed of switching] the result would be

$$f(t)P_T(t) = \sum_{n=-\infty}^{\infty} f(t) P_n e^{jn\omega_c t}$$

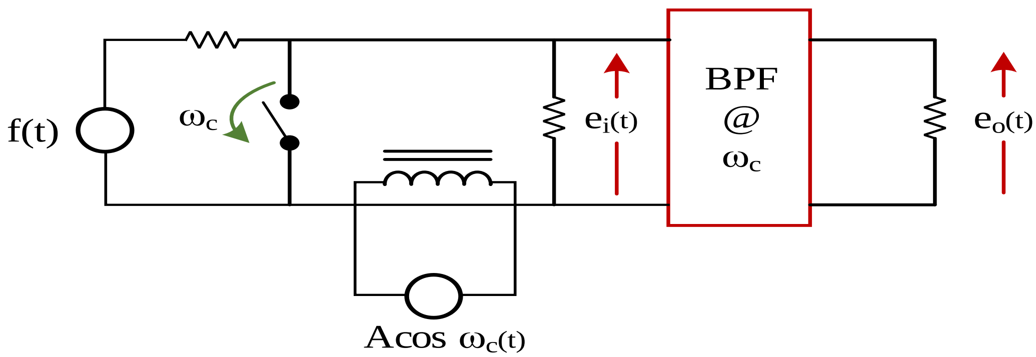
$$F[f(t)P_T(t)] = \sum_{n=-\infty}^{\infty} P_n F(\omega - n\omega_c)$$

The desired result at n=1

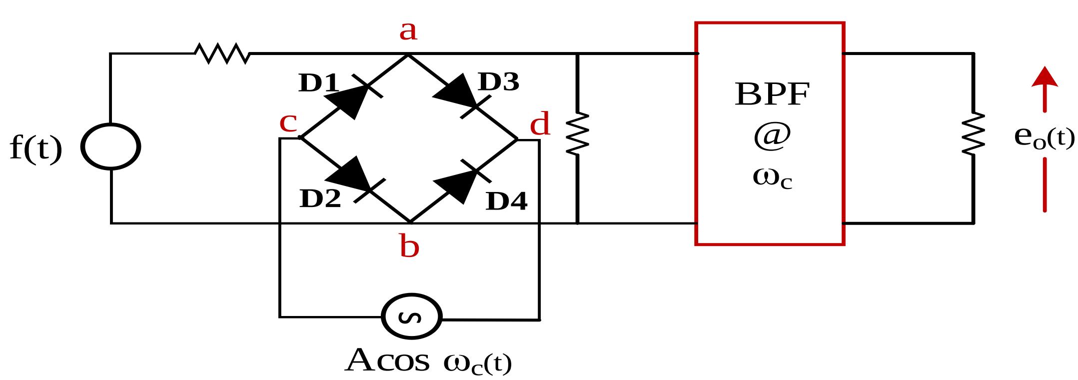


Modulator Implementation:

a) Using Electromechanical Switch:



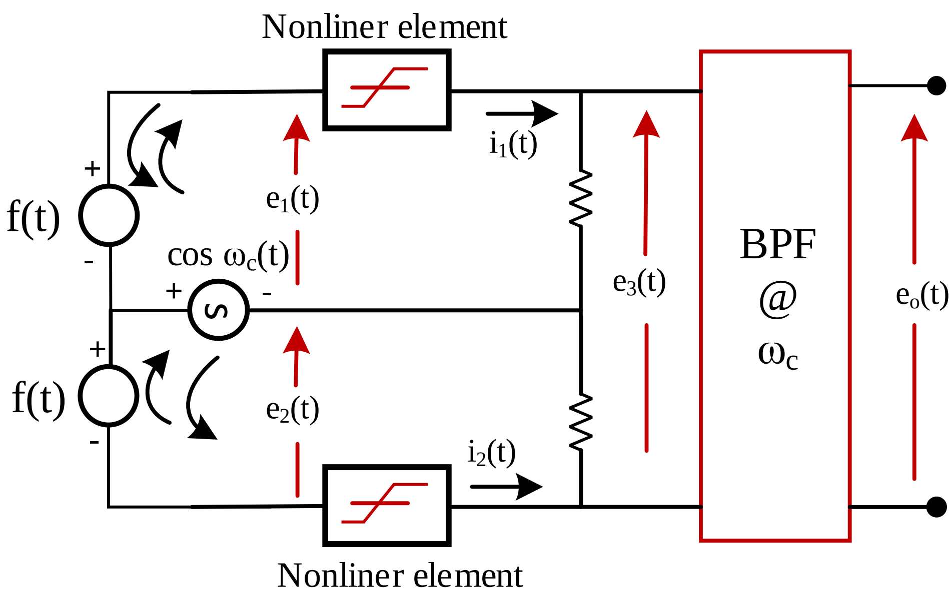
b) Using Diodes and Switches:



2- Using Nonlinear Devices:

A non-liner device such as diode could be used as a balanced modulator. The nonlinearity between voltage and current approximately is given by:

$$i(t) = a_1 e(t) + a_2 e^2(t) + a_3 e^3(t) + \cdots \qquad \ldots(4\text{-}4)$$



From above figure

$$e_1(t) = cos\omega_c t + f(t)$$

$$e_2(t) = cos\omega_c t - f(t)$$

Taking the first two terms only

$$i_1(t) = a_1[cos\omega_c t + f(t)] + a_2[cos\omega_c t + f(t)]^2$$

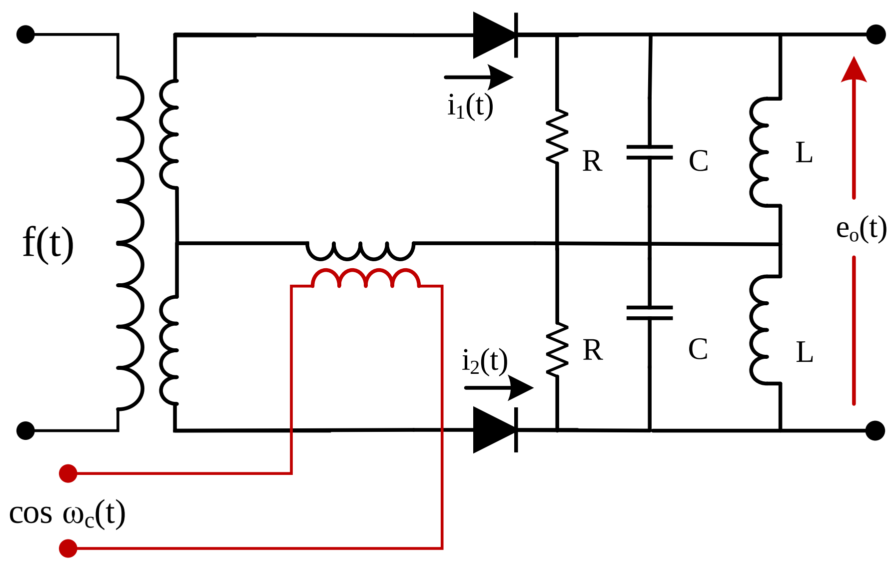
$$i_2(t) = a_1[cos\omega_c t - f(t)] + a_2[cos\omega_c t - f(t)]^2$$

$$e_3(t) = [i_1(t) - i_2(t)].R$$

$$= 4a_2 R\left[\underline{f(t)cos\omega_c t} + \frac{a_1}{2a_2} f(t)\right]$$

Filtered by BPF

The schematic diagram of this modulator is shown below:

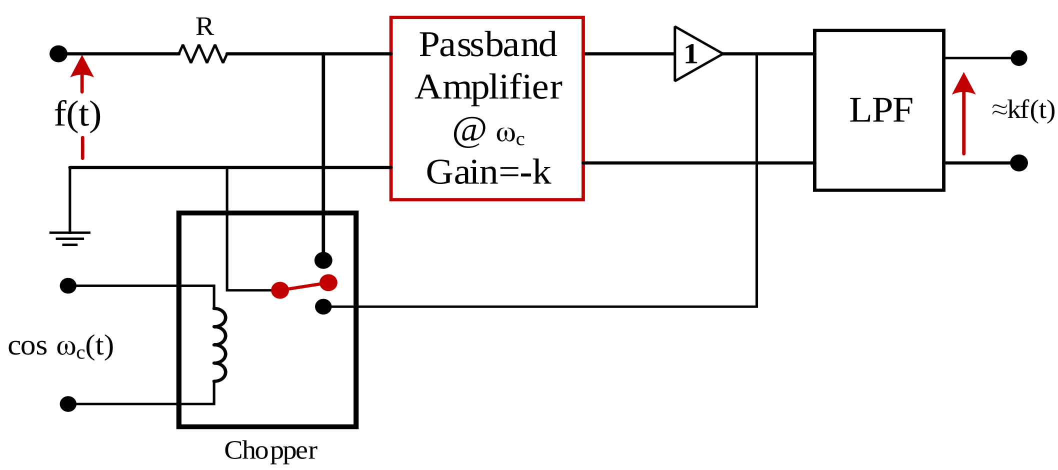


Demodulation (Detection) od DSB-SC Signals:

Since the received and locally generated carrier signals should be synchronized for correct detection, the detection process is accomplished using one of the following schemes:

1- Direct connection (Chopper Amplifier):

It may be used when the modulator and demodulator are near from each other.

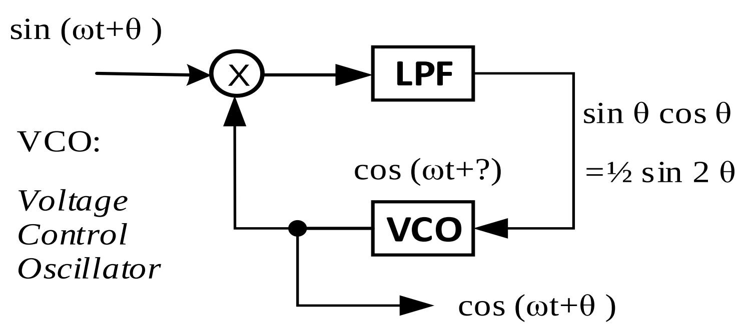


2- <u>Pilot carrier system:</u>

By supplying a sinusoidal signal related to carrier signal in frequency and phase when the demodulator and modulator are far from each other.

- The pilot carrier signal is transmitted outside the baseband of the modulated signal.
- A tuned circuit in the receiver detect this signal and use it to correctly synchronize the carrier signal using a circuit Phase-Locked Loop (PLL).

PLL



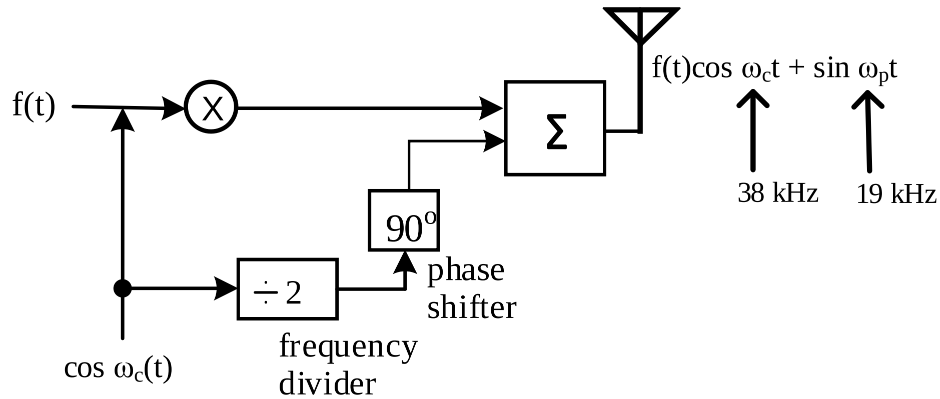
The output of LPF (sign and Magnitude) make VCO increase or decrease the phase of its signal proportional to the magnitude. If the output of LPF is zero, the phase is locked.

Ex 4-1:

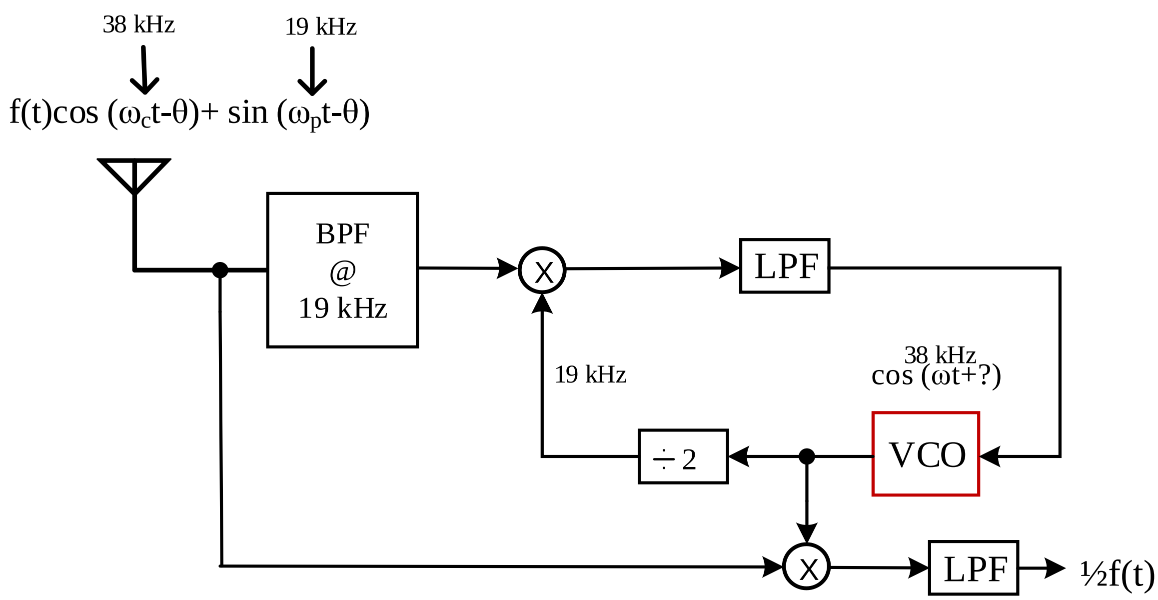
Draw the block diagram for AM/DSB-SC system uses a carrier frequency of 38 kHz and pilot carrier of 19 kHz.

Solution:

<u>Tx:</u>



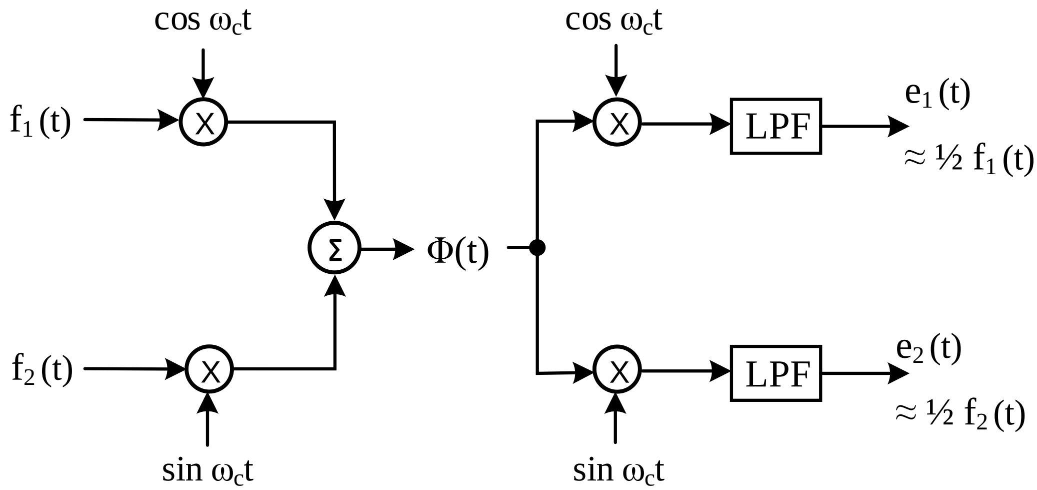
Rx:



- Since the pilot carrier has no actual information, it sends represent losses in the system.
- AM/DSB-SC is useful to obtain good performance in case of point-to-point communication (one Tx for each Rx).

Quadrature Multiplexing:

Using the orthogonality of sines and cosines make it possible to transmit and receive two different signals simultaneously on the same carrier frequency.



$$\emptyset(t) = f_1(t)cos\omega_c t + f_2(t)sin\omega_c t \qquad \ldots (4\text{-}5)$$

$$\emptyset(t)cos\omega_c t = f_1(t)cos^2\omega_c t + f_2(t)sin\omega_c t\ cos\omega_c t$$

$$= \frac{1}{2}f_1(t) + \frac{1}{2}f_1(t)cos2\omega_c t + \frac{1}{2}f_2(t)sin2\omega_c t$$

$$\emptyset(t)sin\omega_c t = f_1(t)cos\omega_c t\ sin\omega_c t\ + f_2(t)sin^2\omega_c t$$

$$= \frac{1}{2}f_1(t)sin2\omega_c t + \frac{1}{2}f_2(t) - \frac{1}{2}f_2(t)cos2\omega_c t$$

In the low pass filter all terms at $2\omega_c$, are attenuated yielding:

$$e_1(t) = \frac{1}{2}f_1(t)$$

$$e_2(t) = \frac{1}{2}f_2(t)$$

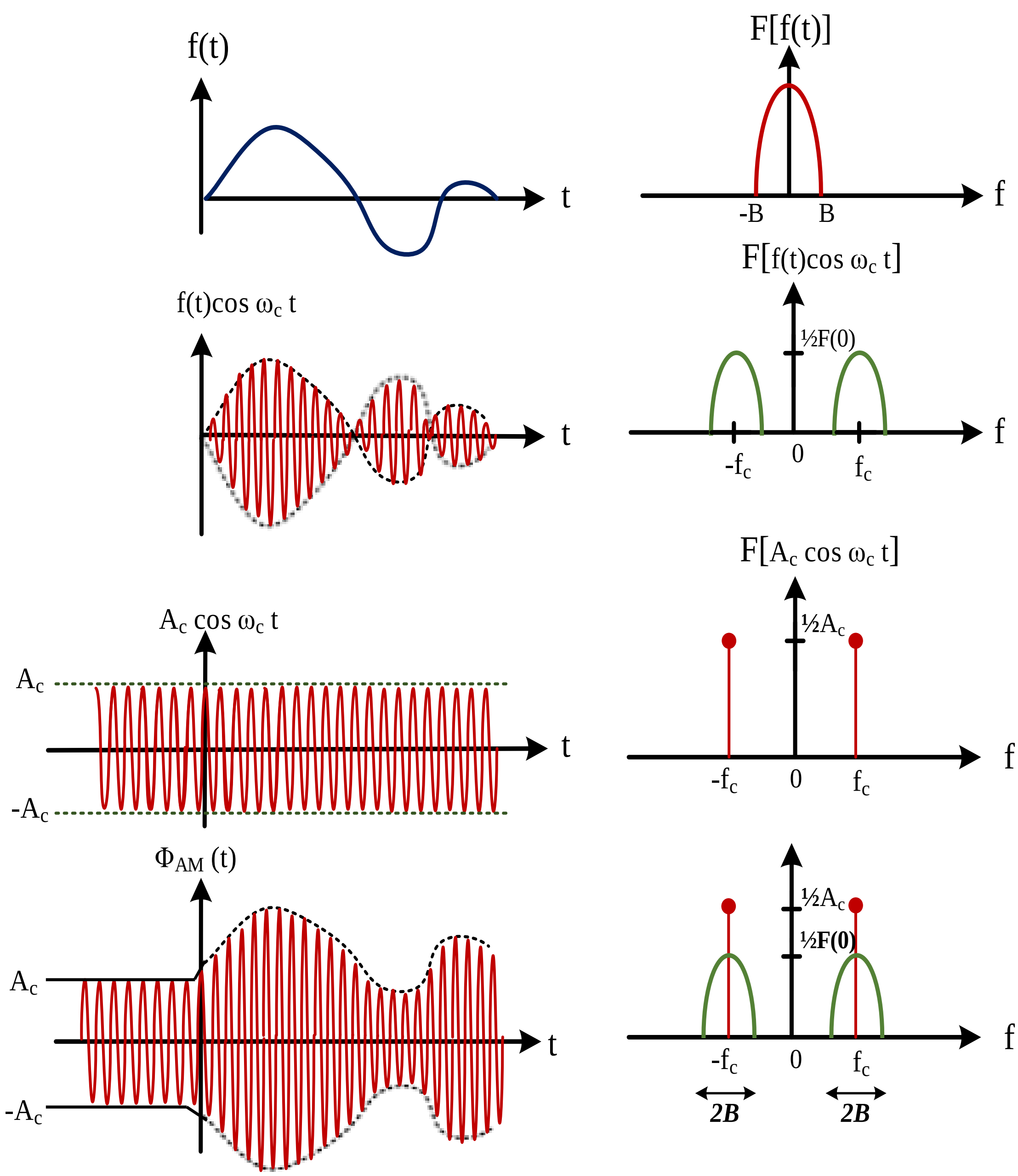
2 - AM/DSB-LC [Standard AM]:

For broadcast system (many Rx for each Tx) it is more economical to obtain less expensive receivers. For such a case, a larger signal is transmitted with the AM/DSB-SC to eliminate the need of local oscillator in Rx.

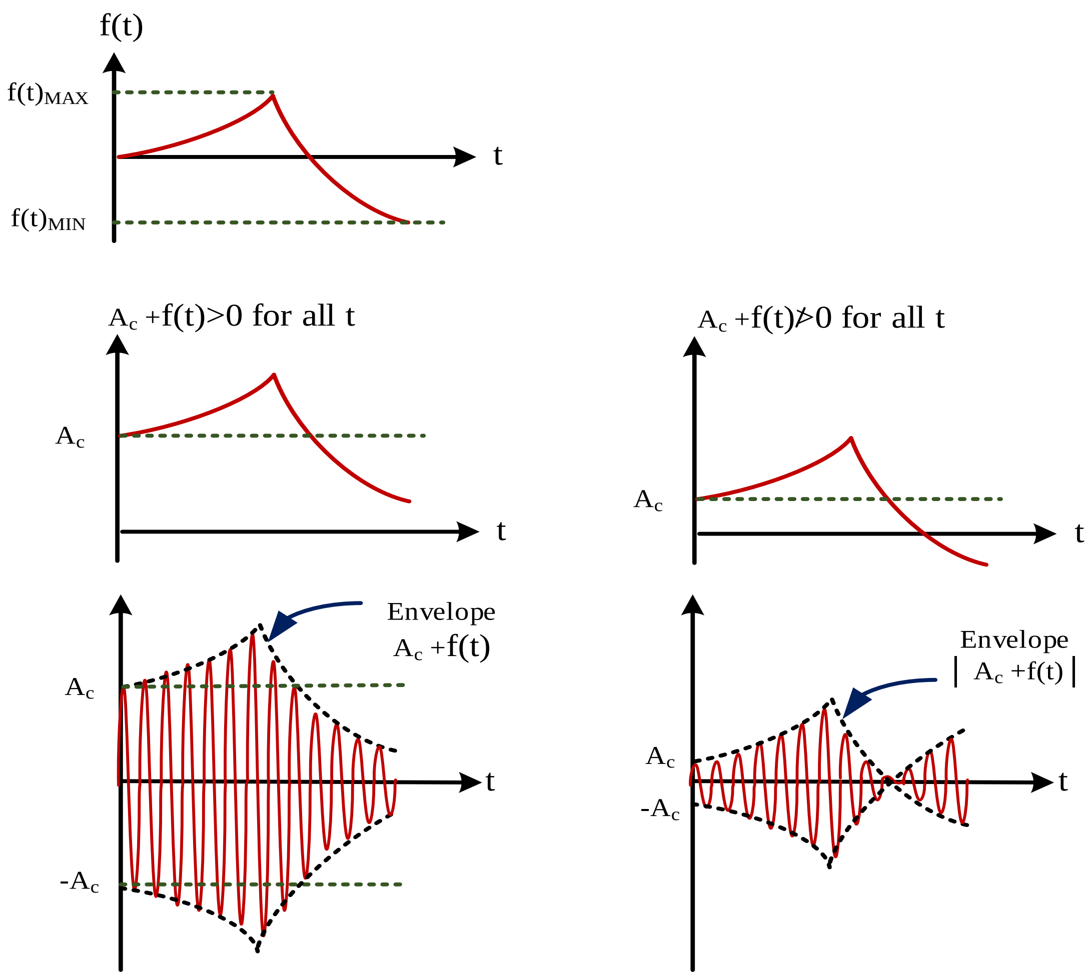
The AM/DSB-LC signal [AM signal] is given by:

$$\Phi_{AM} = f(t)cos\omega_c t + A_c cos\omega_c t$$
$$= [f(t) + A_c]cos\omega_c t \quad \text{…(4-6)}$$

$$\Phi_{AM}(t) = \pi F(\omega + \omega_c) + \pi F(\omega - \omega_c) + \pi A_c \delta(\omega - \omega_c) + \pi A_c \delta(\omega - \omega_c)$$



The envelope of $\Phi_{AM}(t)$ is $A_c + f(t)$, if A_c is large enough to make $A_c + f(t)$positive for all t. the recovery of $f(t)$ from $\Phi_{AM}(t)$ simply reduced to envelope detection:



$if\ A_c + f(t) > 0 \quad for\ all\ \ t$ "Envelope detector" (Low cost)

$if\ A_c + f(t)\ \ \leq 0 \quad for\ all\ \ t$ "Synchronous detector" (High cost)

For envelope detection:`

$$A_c + f(t) > 0 \quad or\ \ A_c \geq -f(t)_{min}$$

We define “modulation index” m as:

$$m = \frac{-f(t)_{min}}{A_c}$$ …(4-7)

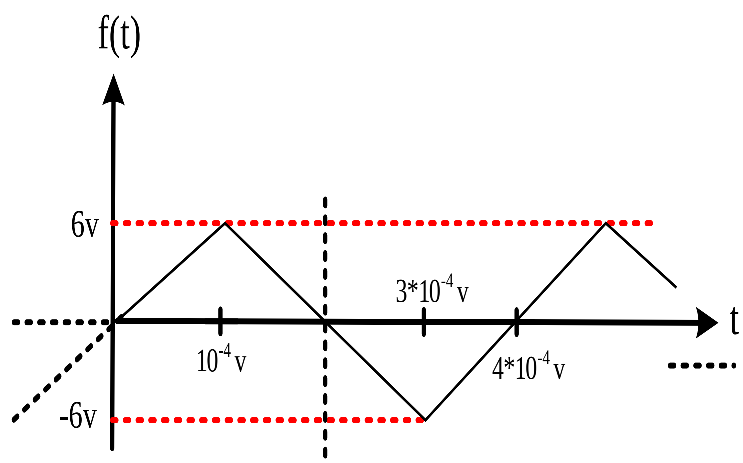
$m \leq 1$ for envelope detection, and modulation depth as:

$$D = m \times 100\%$$ …(4-8)

Ex 4-2:

Find the modulation index and modulation depth if a carrier signal given by $8\cos 2\pi * 10^5 t$ is modulated by the signal shown below using AM/DSB-LC technique.

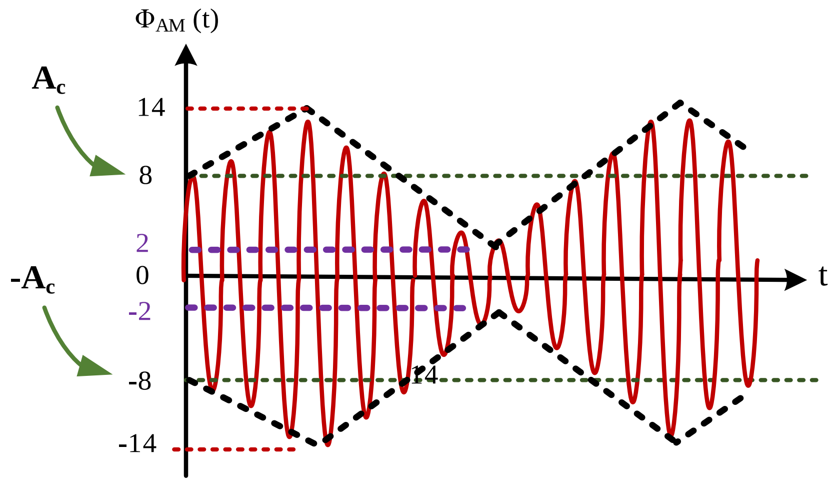
Solution:



$$m = \frac{-f(t)_{min}}{A_c} = \frac{6}{8} = 0.75 f(t)_{min} = -6\ v A_c = 8\ v$$

$D = m \times 100\%$=75%

The modulated signal waveform may be plotted as shown below



H.W:

a) Write the equation of the modulated wave in the previous example.

b) Sketch the waveform of modulated wave and write its equation if the modulation type is AM/DSB- SC.

Single Tone Modulation:

The single tone signal is given by:

$$f(t) = A_m cos\omega_m t \quad \ldots(4\text{-}9)$$

It is an experimental signal commonly used in communication systems. It is a simple signal, since it has only one frequency.

The modulation index and depth for single tone modulation would be:

$$m = \frac{-f(t)_{min}}{A_c} \quad \ldots(4\text{-}10)$$

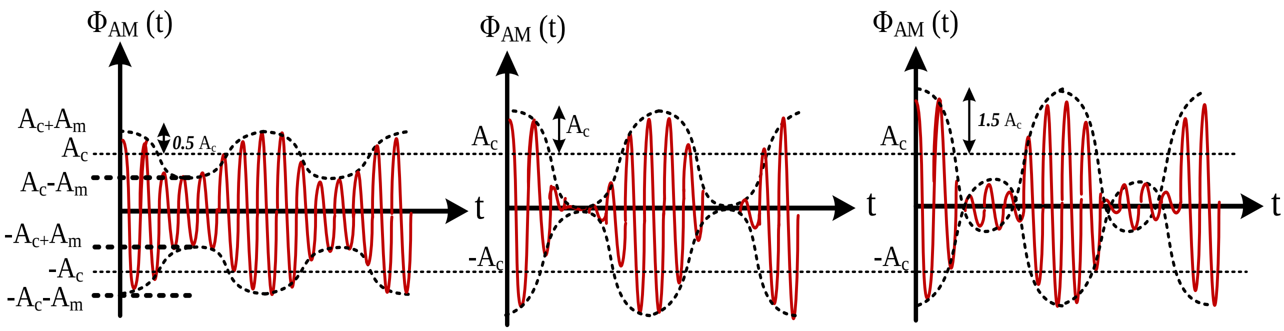
$$D = \frac{A_m}{A_c} * 100\% \qquad \text{… (4-11)}$$

The AM signal for tone modulation is given by:

$$\Phi_{AM} = [A_c + f(t)]cos\omega_c t$$
$$= A_c[1 + mcos\omega_m t]cos\omega_c t$$

For single tone

… (4-12)



m=0.5	m=1	m=1.5
$\left(A_m = \frac{A_c}{2}\right)$	$(A_m = A_c)$	$\left(A_m = \frac{3A_c}{2}\right)$
Correct modulation	Critical modulation	Over modulation

From the previous figure, the modulation index could be obtained using:

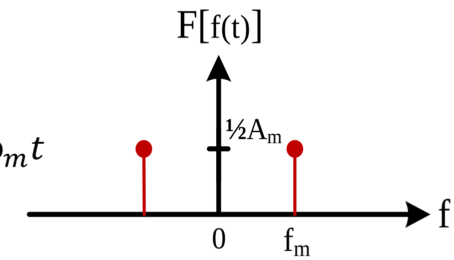
$$m = \frac{A_{max} - A_{min}}{A_{max} + A_{min}} \qquad \text{… (4-3)}$$

Where:

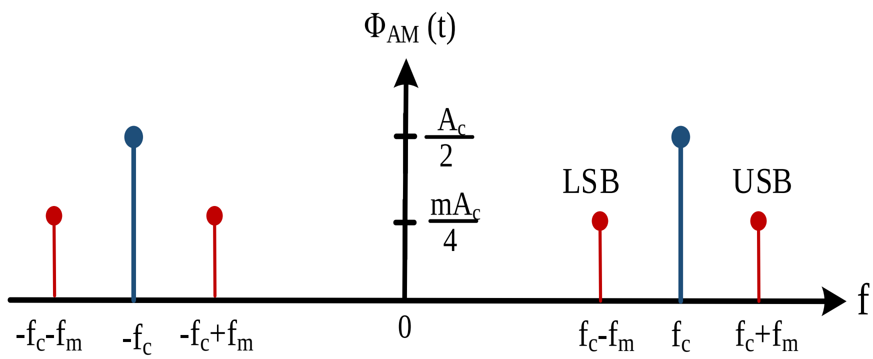
$A_{max} = A_c + A_m$ and $A_{min} = A_c - A_m$

The spectrum of single-tone AM modulation should be:

$$f(t) = A_m cos\omega_m t$$



$$\Phi_{AM_{S.T}}(t) = A_c cos\omega_c t + \frac{mA_c}{2} cos(\omega_c - \omega_m)t + \frac{mA_c}{2} cos(\omega_c + \omega_m)t$$



H.W:

A carrier signal given by $10 \cos 10000\, \pi t \;\; volt$ is AM/DSB-LC modulated by single-tone signal $4 \cos 100\; \pi t \;\; volt$.

1- Calculate the modulation index.
2- Sketch the spectrum of the modulated signal.
3- Calculate transmission bandwidth.

Sideband and Carrier Powers:

$$\Phi_{AM} = \boxed{A_c cos\omega_c t} + \boxed{f(t) cos\omega_c t}$$

Carrier (P_c)"Losses"

Sidebands (P_s)

$$P_c = \frac{A_c^2}{2}$$ "Carrier Power" or $$P_c = \frac{A_c^2}{2R}$$ if A_c in volts and R is given

…… (4-14)

$$P_s = \frac{1}{2}\overline{f^2(t)}$$ "Sideband Power" $$\overline{f^2(t)} = \frac{1}{T}\int_0^T |f(t)|^2 dt$$

…(4-15)

$$P_t = P_c + P_s = \frac{1}{2}\left[A_c^2 + \overline{f^2(t)}\right] \quad \text{…(4-16)}$$

$$\eta = \frac{P_s}{P_s} \times 100\%$$ "efficiency of transmission"

$$\eta = \frac{\overline{f^2(t)}}{\overline{f^2(t)} + A_c^2} \times 100\% \quad \text{…(4-17)}$$

For Single tone modulation

$$\Phi_{AM} = A_c[1 + m\cos\omega_m t]\cos\omega_c t$$

$$P_t = \frac{A_c^2}{2} + \frac{m^2A_c^2}{8} + \frac{m^2A_c^2}{8} = \frac{A_c^2}{2} + \frac{m^2A_c^2}{4}, \quad P_t = P_c\left(1 + \frac{m^2}{2}\right) \quad \text{…(4-18)}$$

P_c (Losses), P_{USB}, P_{LSB}, P_c, P_S

$$\eta = \frac{P_s}{P_t} = \frac{m^2}{m^2 + 2}$$ max. efficiency when m=1, …(4-19)

$$\eta_{max} = 33.33\%$$

$$\frac{P_c}{P_t} = \frac{2}{m^2 + 2} \quad \text{…(4-20)}$$

Ex 4-3:

A transmitter transmits an AM/DSB single tone-modulating signal given by $3\cos(2\pi 10^3 t) volt$ with a carrier signal given by $10\cos(2\pi 10^6 t) volt$,Find:

1- Modulation depth.
2- USB & LSB frequencies.
3- Amplitude of sideband frequencies.
4- Efficiency of Transmission.

Solution:

A_m=3 v, A_c=10 v

f_m=103 Hz, f_c=106 Hz

1- $D = \frac{A_m}{A_c} \times 100\% = \frac{3}{10} \times 100\% = 30\%$
2- USB frequency is $f_c + f_m = 10^6 + 10^3 = 1.001\ MHz$, LSB frequency is $f_c - f_m = 10^6 - 10^3 = 0.999\ MHz$
3- Amp. of USB & LSB is $\frac{mA_c}{4} = \frac{0.3\times 10}{4} = 0.75\ volt$
4- $\eta = \frac{P_s}{P_t} = \frac{m^2}{m^2+2} = \frac{0.3^2}{0.3^2+2} = 4.3\ \%$

H.W:

Repeat the previous example assuming that the modulating signal is given by:

$$f(t) = \begin{cases} t-1, & 0 < t < 2*10^{-3} \\ 0\ , & 2*10^{-3} < t < 4*10^{-3} \end{cases}$$

Assume that the signal is band-limited to 1 kHz.

Ex. 4-4:

Broadcast transmitter transmits AM/DSB-LC signal, with total average power of 50 kW and uses a modulation index of 0.707 for a sinusoidal message signal, calculate:

1- carrier signal power (P_c)
2- Efficiency of transmission (η).
3- Maximum carrier signal amplitude if the antenna is represented as a 50Ω resistance (A_c).

Solution:

1- $\frac{P_c}{P_t} = \frac{2}{m^2+2}$

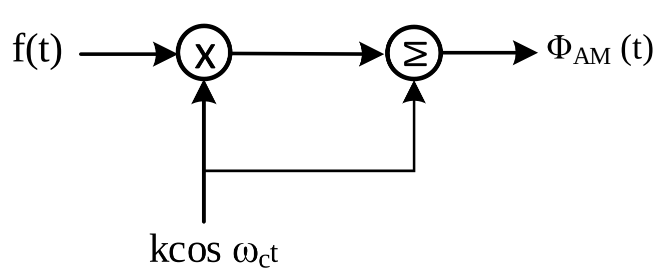
$$P_c = \frac{2}{m^2+2} P_t = \frac{2}{2+0.707^2} \times 50\ kw = 40\ kw$$

2- $\eta = \frac{P_s}{P_t} = \frac{m^2}{m^2+2} = 20\ \%$

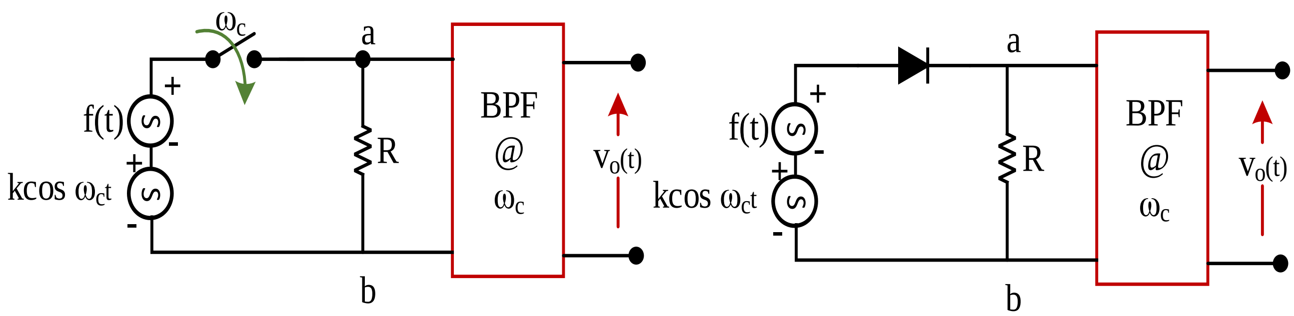
3- $P_c = \frac{A_c^2}{2R}$

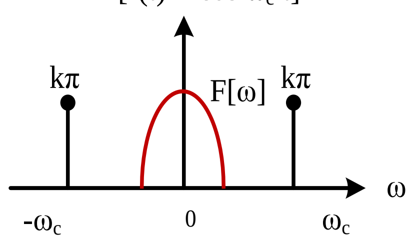
$$A_c = \sqrt{2 * R * P_c} = \sqrt{2 * 50 * 40kw}$$
$$= 2kv$$

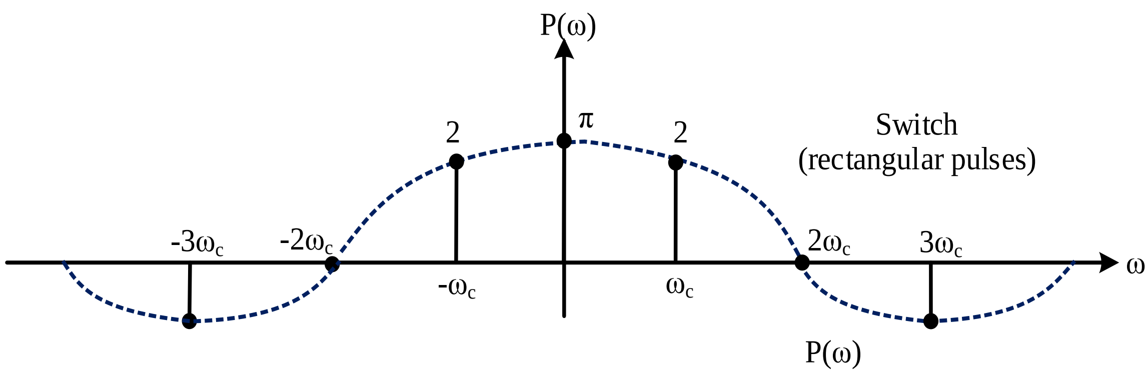
Generation of AM/DSB- LC Signal:

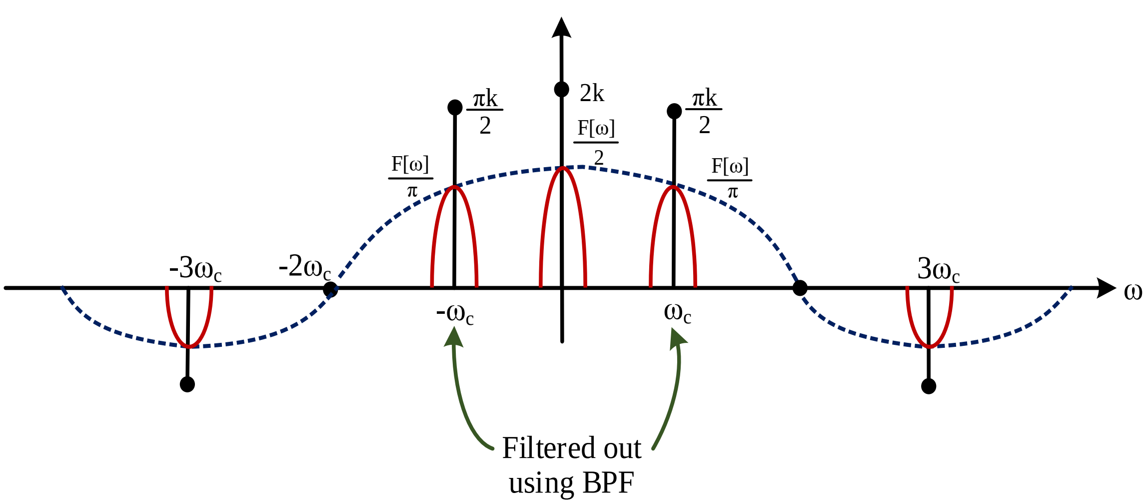


1- Switching (Chopper) modulator [rectifier modulator].









2- Using Nonlinear Devices:

In the previous figure, if the diode is not operated as an ideal switch, the nonlinearities in the diode characteristics may be given as:

$$i(t) = a_1 e(t) + a_2 e^2(t) + a_3 e^3(t) + \cdots$$

Therefore, from the figure:

$$v_{ab}(t) = a_1 R[f(t) + k cos\omega_c t] + a_2 R[f(t) + k cos\omega_c t]^2 + \cdots$$

$$= \boxed{a_1 Rk cos\omega_c t + 2a_2 Rk f(t) cos\omega_c t} + \boxed{other terms}$$

AM suppressed by BPF

$$V_o(t) = a_1 Rk cos\omega_c t + 2a_2 Rk f(t) cos\omega_c t$$

Ex 4-5:

A carrier signal with frequency of 1 MHz and peak-to-peak amplitude of $10\sqrt{2}v$ is added to a single tone message with frequency of 1 kHz and peak-to-peak amplitude of $4\sqrt{2}v$. Then passed through a diode having the following c/c$_s$:

$i(t) = e(t) + 0.5e^2(t)$, find:

1- Modulation depth of DSB signal.
2- Frequency of each component in the diode current.

Solution:

Carrier: $5\sqrt{2} cos 2\pi \times 10^6 t\ volt$

S.T message: $2\sqrt{2} cos 2\pi \times 10^3 t\ volt$

$e(t) = 5\sqrt{2}cos2\pi \times 10^6 t + 2\sqrt{2}cos2\pi \times 10^3 t$

$i(t) = e(t) + 0.5e^2(t)$

$= \left(5\sqrt{2}cos2\pi \times 10^6 t + 2\sqrt{2}cos2\pi \times 10^3 t\right) + 0.5\left(5\sqrt{2}cos2\pi \times 10^6 t + 2\sqrt{2}cos2\pi \times 10^3 t\right)^2$

$= 5\sqrt{2}cos2\pi * 10^6 t + 2\sqrt{2}cos2\pi * 10^3 t + 14.5 + 12.5cos4\pi * 10^6 t + 2cos4\pi * 10^3 t + 10\cos(10^6 + 10^3)\,t + 10\cos(10^6 + 10^3)\,t$

The frequencies in the diode current are

$1\ MHz, 1\ kHz, 0\ Hz, 2\ MHz, 2\ kHz, 1.001\ MHz\ and\ 0.999\ MHz.$

The AM signal is represented by the terms of frequencies, f_c, $f_c \pm f_m$ (other terms are suppressed by a BPF) i.e

$$\underbrace{5\sqrt{2}}_{A_c}cos\underbrace{2\pi * 10^6}_{\omega_c}t + \underbrace{20}_{A_mA_c}cos\ \underbrace{2\pi * 10^6}_{\omega_c}t\ 20cos\ \underbrace{2\pi * 10^3}_{\omega_m}t$$

$\therefore A_m = 2\sqrt{2}\ volt$ and $A_c = 5\sqrt{2}\ volt$

Therefore $D = \frac{2\sqrt{2}}{5\sqrt{2}} \times 100\% = 40\%$

Demodulation of AM/DSB-LC Signals:

1- Synchronous (coherent) detection:

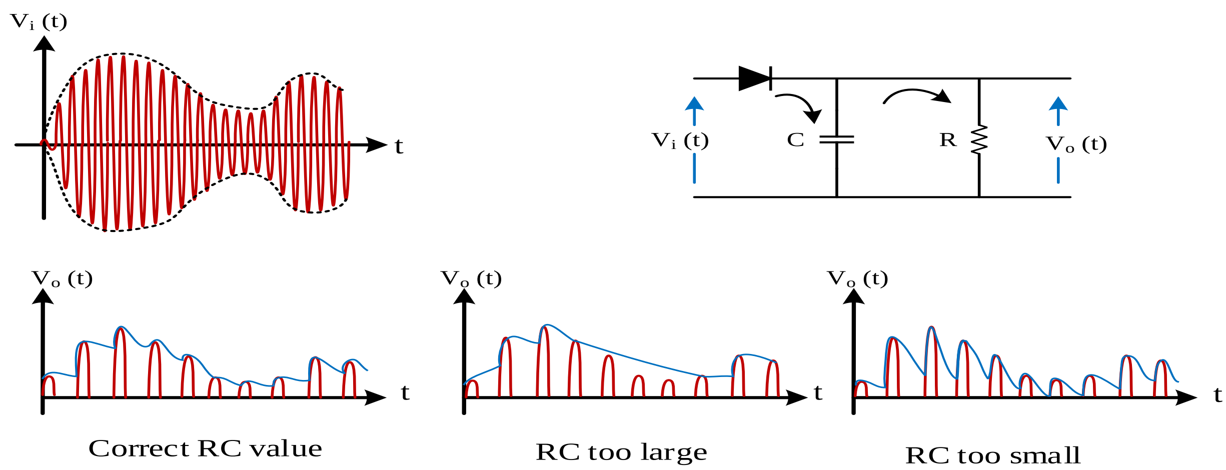
The same as in DSB/SC (i.e direct connection and pilot carrier) [High cost].

2- Asynchronous detection (Noncoherent detection)

It is also called envelope detection since it is extracts the message from its envelope. It is low cost and applicable only when $m \leq 1$

Noncoherent detection:

a) Envelope detector:



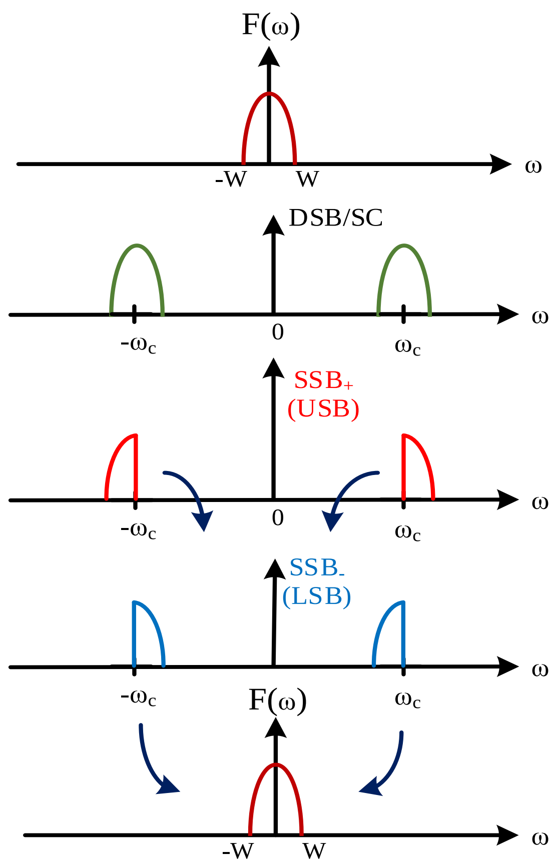
- It is a very simple, efficient, and cheap demodulator
- For correct detection: the charge time should be fast and discharge time [RC] should be slow.
- In P.H.S, C is charged wile in N.H.S, C is discharged through R.
- Usually this circuit is followed by LPF to eliminate unwanted harmonic content.

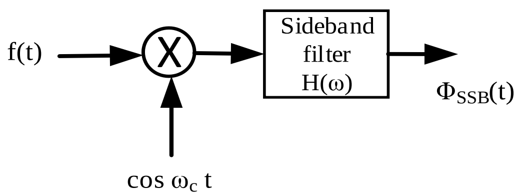
3-AM/SSB:

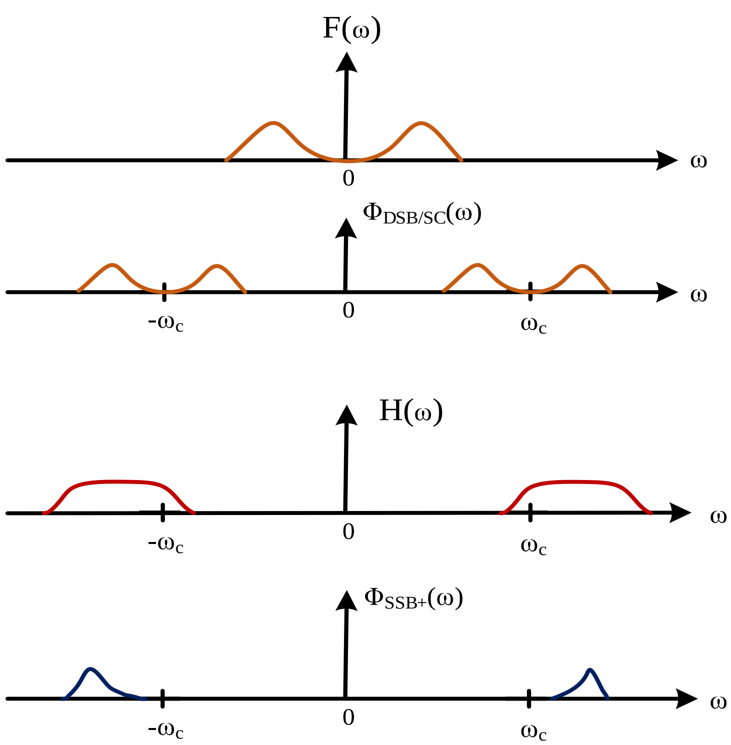
Since either LSB and USB in DSB modulation contains the complete information of the baseband signal. It is more efficient to transmit only one sideband [LSB (-ve) or USB (+ve)] to reduce the required signal bandwidth.

Generation of SSB Signals:

1- Modulation and filtering method:







The filter used should be as sharp as possible (for speech signal,
transmission bandwidth
$\approx$ 600Hz,
stop band attenuation
$\approx$ 40 dB

2- Phase Shift Method:

Using a complex valued carrier signal having one-side spectral density and when taking its real part would have two-sided spectral density

Let $f(t) = e^{j\omega_m t}$

$\therefore$ Upon modulation $f(t)e^{j\omega_c t} = e^{j\omega_m t}e^{j\omega_c t}$

$$Re\{e^{j\omega_m t}e^{j\omega_c t}\} = Re\{e^{j\omega_m t}\}Re\{e^{j\omega_c t}\} - Im\{e^{j\omega_m t}\}Im\{e^{j\omega_c t}\}$$

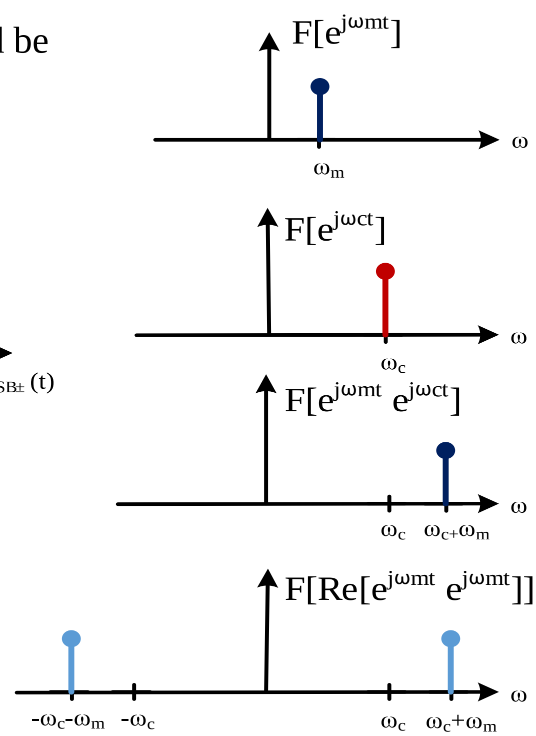
$$= cos\omega_m t cos\omega_c t - sin\omega_m t sin\omega_c t$$

if $f(t) = e^{j\omega_m t} \Longrightarrow \emptyset_{SSB_+} = cos\omega_m t cos\omega_c t - sin\omega_m t sin\omega_c t$

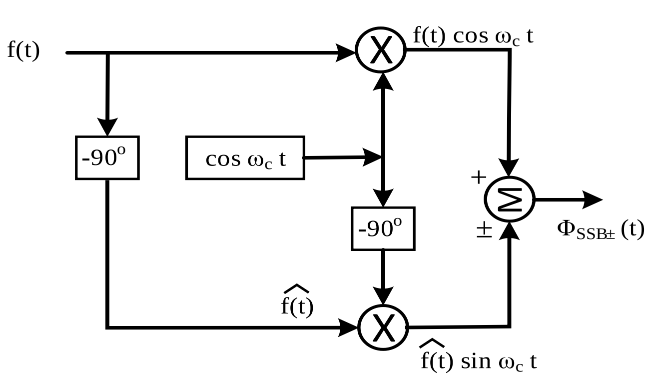
if $f(t) = e^{j\omega_m t} \Longrightarrow \emptyset_{SSB_-} = cos\omega_m t cos\omega_c t + sin\omega_m t sin\omega_c t$

in general:

$$\emptyset_{SSB_\mp} = f(t)cos\omega_c t \pm \widehat{f(t)}sin\omega_c t \quad \text{...(4-25)a}$$



The block diagram of the modulator would be



The major problem in this method is the realization of -90° phase shift for all frequency comonent

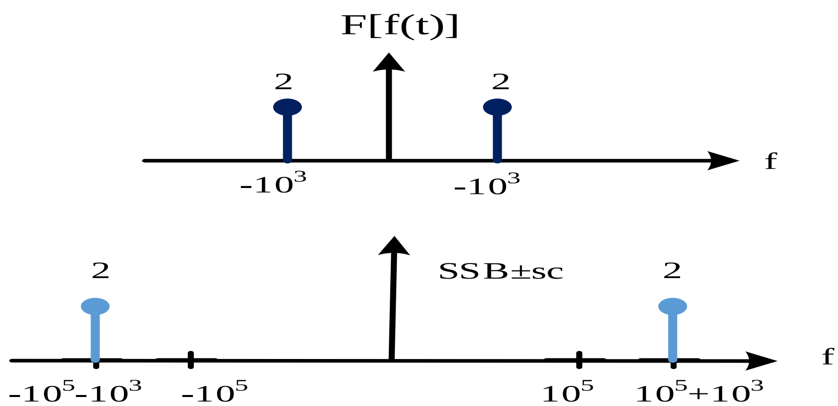
Ex 4-7:

A carrier signal given by $\cos 2\pi \times 10^5 t\ volt$ is AM/SSB-SC modulated by the message signal given by $4\cos 2\pi \times 10^3 t\ volt$:

1- Sketch the spectrum of modulated wave
2- Calculate the sideband power and transmission efficiency
3- Calculate the transmission bandwidth

Solution:

1)



2) $\emptyset_{SSB_+} = f(t)\cos\omega_c t - f\widehat{(t)}\sin\omega_c t$

$P_{SB} = \frac{\overline{f^2(t)}}{2} + \frac{\overline{f^2\widehat{(t)}}}{2}$, since $\overline{f^2\widehat{(t)}} = \overline{f^2(t)}$

$$\therefore P_{SB} = \overline{f^2(t)}$$

For $f(t) = 4\cos 2\pi * 10^3\ t \Longrightarrow P_{SB} = \frac{4^2}{2} = 8\ volt = 8\ watt\ (for\ R = 1\Omega)$

$P_c = 0$ Since suppressed carrier modulation

$\therefore P_T = P_{SSB}$ and $\eta = 100\%$

3) $BW_{tr} = BW_{infor.} = 1\ kHz = f_m$

Demodulation of SSB Signals

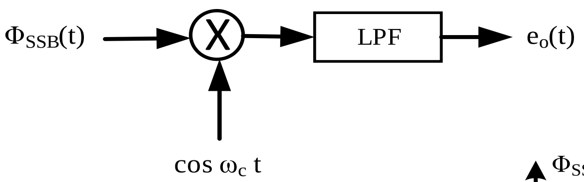
1- SSB-SC:

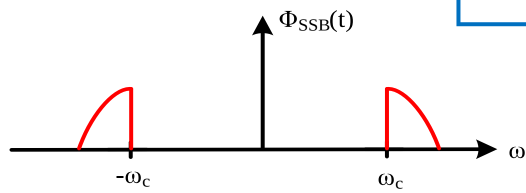
$$\emptyset_{SSB_{\pm}} = f(t)cos\omega_c t \mp f\widehat{(t)}sin\omega_c t$$

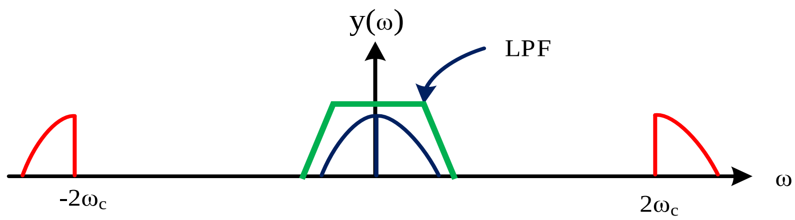
- Using synchronous detection

Direct connection (if mod &demod are near from each

Pilot carrier (if mod &demod are far from each other







$$y(t) = f(t)cos^2\omega_c t \mp f\widehat{(t)}sin\omega_c t cos\omega_c t$$

$$= \frac{1}{2}f(t) + \frac{1}{2}f(t)cos2\omega_c t \mp \frac{1}{2}f\widehat{(t)}sin2\omega_c t$$

$$e_o(t) = \frac{1}{2}f(t)$$

2- SSB-SL:

$$\emptyset_{SSB_{-LC\pm}} = A_c cos\omega_c t + f(t)cos\omega_c t \mp \hat{f}(t)\ sin\omega_c t \quad \ldots(4\text{-}25)b$$

- Using noncoherent detection (if $m \le 1$) → Envelope Detector
- Using coherent detection (if $m > 1$)

Ex 4-8:

A carrier signal given by $cos\omega_c t$ is modulated by a single tone signal given by $f(t) = 0.3cos\omega_m t \quad (\omega_m \ll \omega_c)$. Determine and plot the envelope of the modulated signal in the following modulation types are used:

1- AM/DSB-LC (AM)

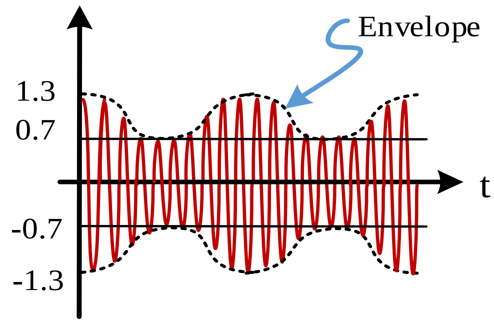
2- AM/SSB-LC (AM)

Solution:

1-

$$\emptyset_{AM}(t) = cos\omega_c t + 0.3\ cos\omega_m t cos\omega_c t$$

$$= \underbrace{(1 + 0.3\ cos\omega_m t)}_{\text{Envelope}}\ cos\omega_c t$$



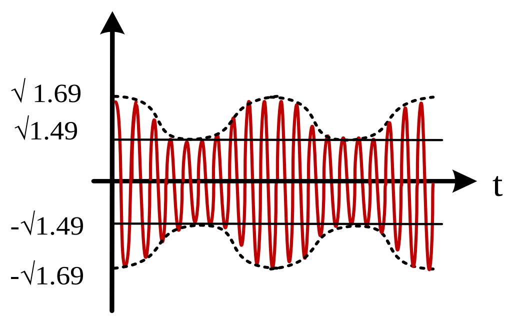
2-

$$\emptyset_{SSB_{\pm LC}}(t) = cos\omega_c t + 0.3\ cos\omega_m t cos\omega_c t - j0.3\ cos\omega_m t sin\omega_c t$$

$$= (1 + 0.3\ cos\omega_m t)cos\omega_c t - j(0.3\ cos\omega_m t)sin\omega_c t$$

$$= \sqrt{(1 + 0.3\ cos\omega_m t)^2 + (1 + 0.3\ cos\omega_m t)^2}\cos(\omega_c t - \theta)$$

$$= \sqrt{\underbrace{1.09 + 0.6cos\omega_m t}_{\text{Envelope}}}\ \cos(\omega_c t - \theta)$$



- In general :

The envelope of $\emptyset_{SSB_{-LC}}(t)$

$$r(t) = \sqrt{[A + f(t)]^2 + [\widehat{f(t)}]^2} \ldots \qquad (4\text{-}26)$$

4 AM/VSB (Vestigial sideband):

- It is used to modulate the video signals which have a large bandwidth (0→4 MHz).
- It represent a compromise between SSB and DSB techniques [DSB requires large BW and SSB requires sharp filtering].
- In this type of modulation, a special filter $H_V(\omega)$ is used to pass one sideband and a vestige of other one.

$$\emptyset_{VSB\_SC}(\omega) = \left[\frac{1}{2}F(\omega-\omega_c)\right] + \left[\frac{1}{2}F(\omega+\omega_c)\right] H_V(\omega) \quad \ldots (4\text{-}27)$$

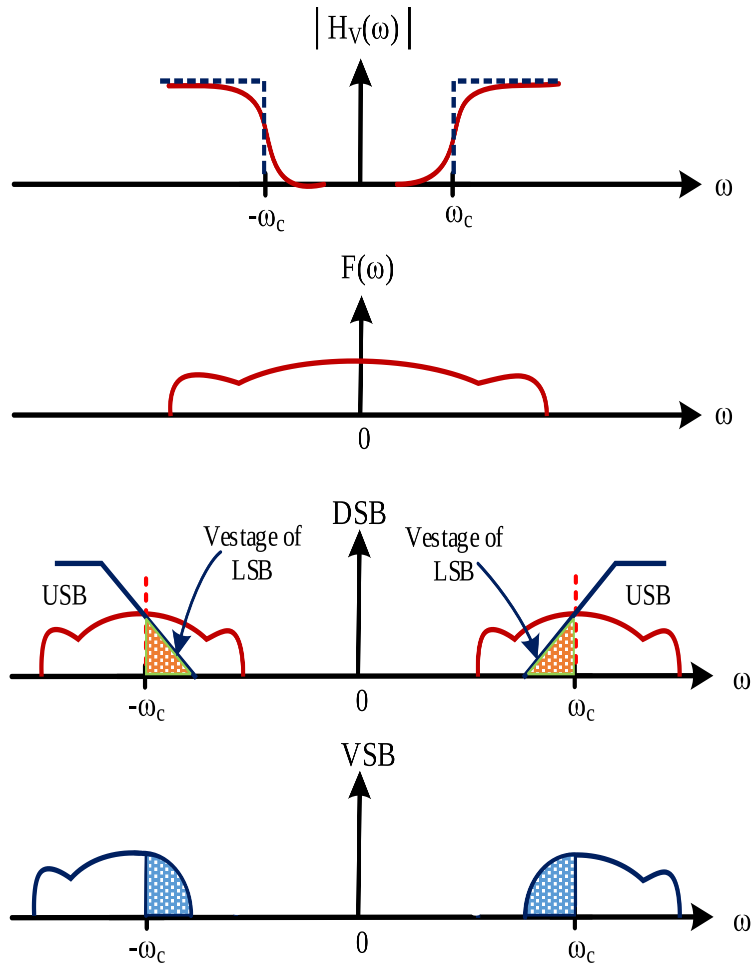
Generation of VSB Signals:

Modulation and filtering

(VSB/SC and VSB/LC)

Demodulation of VSB Signals:

- Synchronous detection for (VSB/SC)
- Asynchronous detection for (VSB-LC)

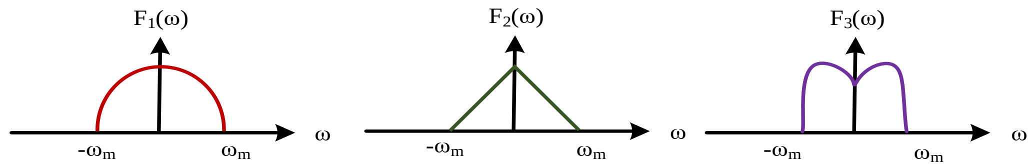


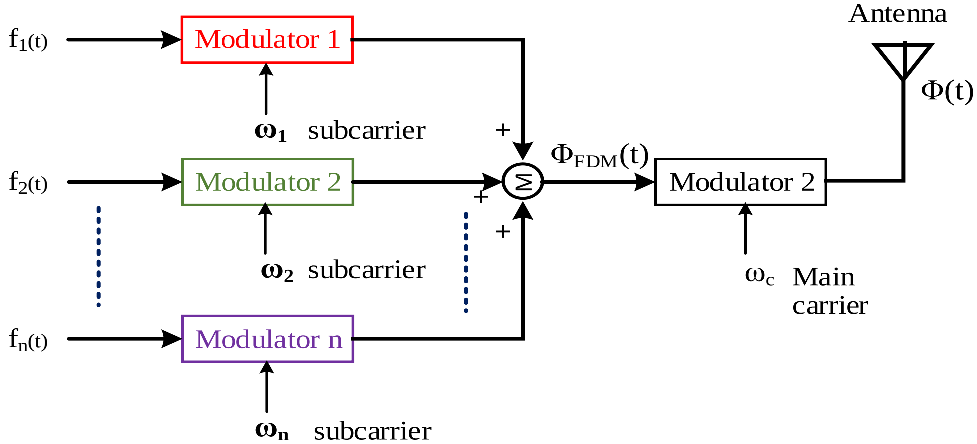
$$2W > BW_{VSB} > W \quad \ldots (4\text{-}28)$$

Frequency Division Multiplexing (FDM):

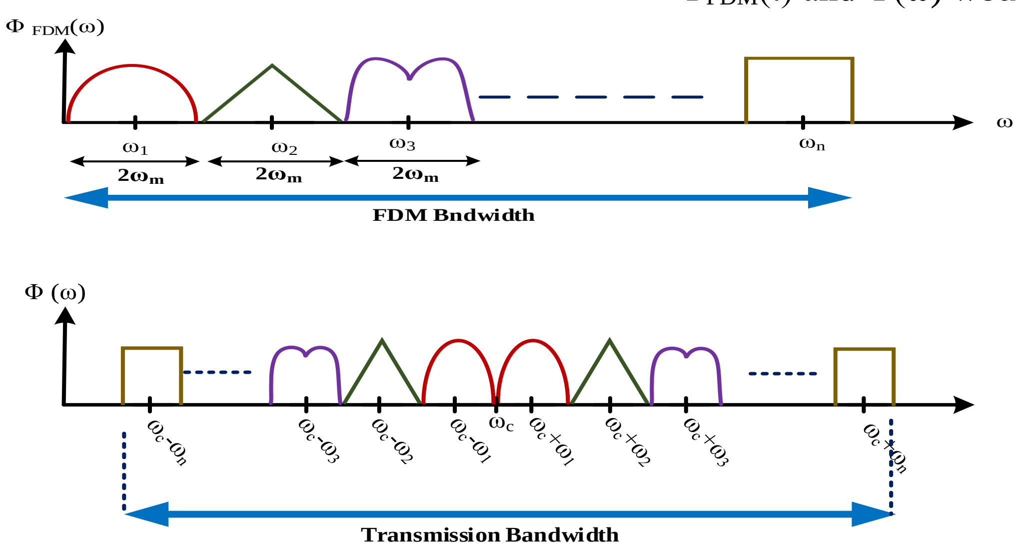
A mode of transmission by which several signals may be send simultaneously by positioning their spectra such that each signal spectrum can be separated out from all others by filtering.

Transmission:



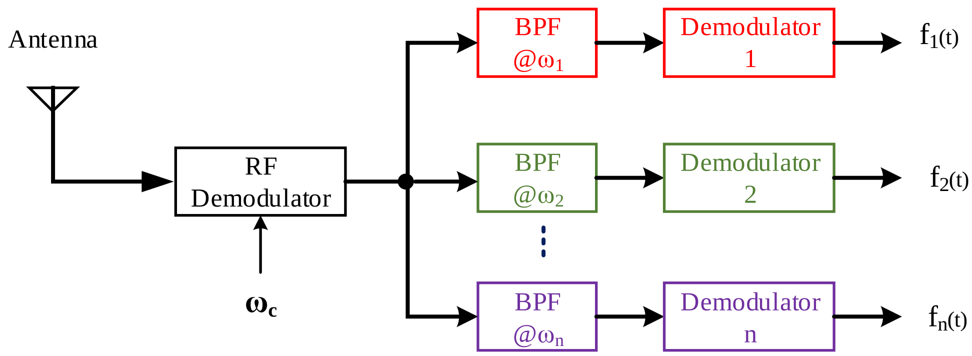


If the modulators are of type AM/DSB-SC, then the spectrum of $\Phi_{FDM}(t)$ and $\Phi(\omega)$ would be:

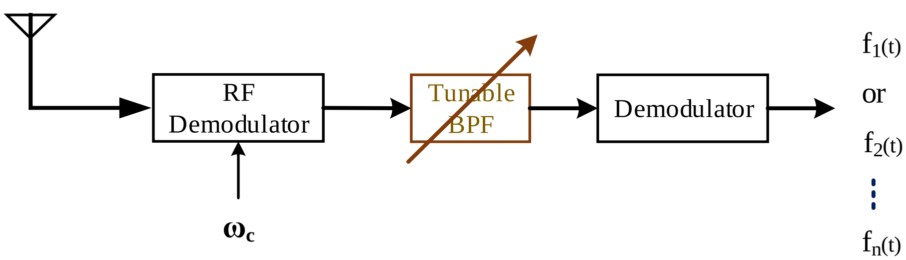


Reception:

1- Simultaneous Reception:



2- Selective Reception:



Ex 4-9:

Twenty speech signal each bandlimited to 4 kHz are FDM/SSB multiplexed, then RF modulated by a main carrier using AM modulator. Calculate the bandwidth of multiplexing (minimum) and final transmission bandwidth (minimum).

Solution:

Since the multiplexing scheme using SSB modulation

$\therefore\ BW_{FDM_{min}} = n * 4\ kHz$ (n:no. of signals)

$$= 20 * 4\ kHz$$

$$= 80\ kHz$$

Since the RF modulation is AM(DSB-LC)

$$\therefore\ BW_{FDM_{min}} = 2 * BW_{FDM_{min}}$$

$$= 160\ kHz$$

Ex 4-10:

Repeat the previous example if there is a 0.7 kHz guard band between each two signals and below the first signal during multiplexing.

Solution:

$$BW_{FDM} = 20 * 4\ kHz + 20 * 0.7\ kHz = 94\ kHz$$

$$BW_{tr} = 2 * 94\ kHz \quad = 188\ kHz$$

H.W:

Thirty signals, twenty of them have $4kHz$ bandwidth, the other have bandwidth of 3 kHz are FDM/SSB multiplexed then modulated by an RF carrier of 800 kHz using AM modulator:

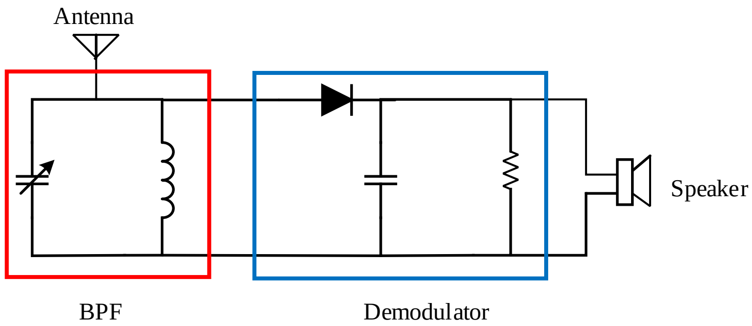
1- Calculate minimum multiplexing and transmission bandwidths.
2- Calculate multiplexing and transmission bandwidths if 0.7 kHz guard band is allowed between each two signals and below the first signal.

Commercial AM Broadcast Transmitters:

- Commercial AM broadcast Tx sends out DSB-LC signals with unique carrier frequency.
- Carrier frequencies are assigned at 10 kHz spacing from 540 kHz to 1600 kHz (MW).

Commercial Receivers:

1- Easier Receivers:

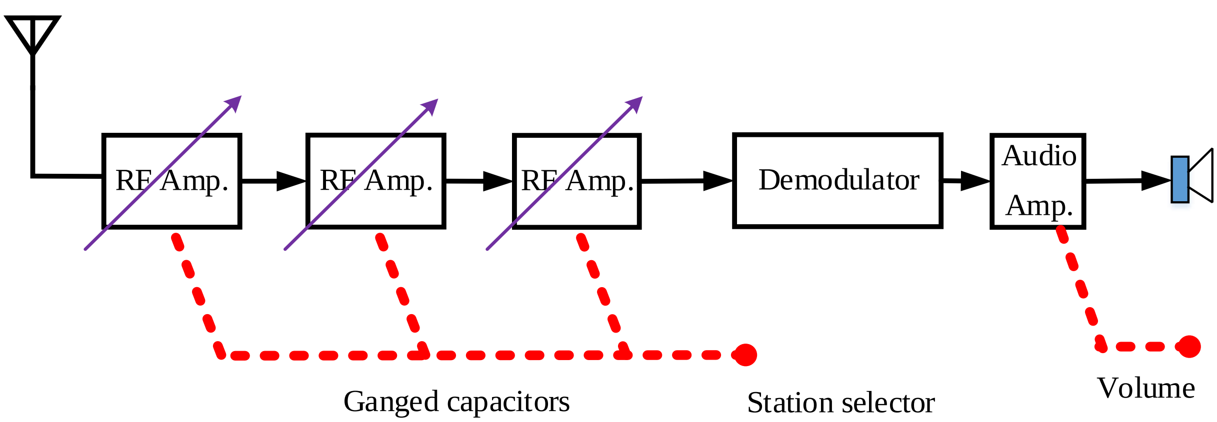


Advantages: simple and low cost.

Disadvantages:

- Less sensitivity for weak signals from far stations
- Less selectivity due to simple filtering.

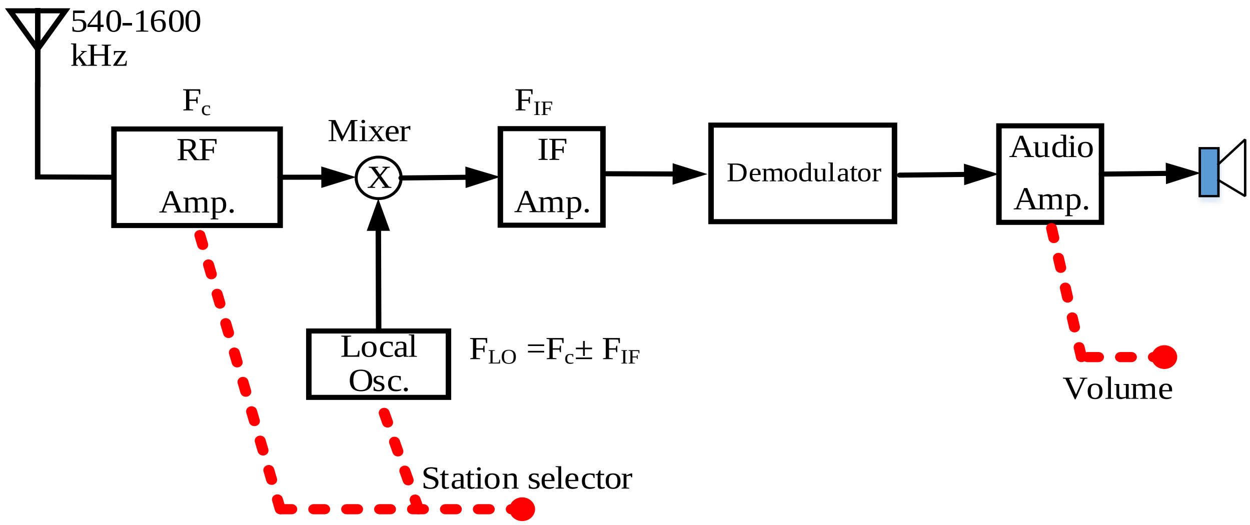
2- Tuned Radio-Frequency (TRF) receiver:



Advantages: Amplifies the weak radio frequencies (3 stage of amplification) i.e. have high sensitivity.

Disadvantages: Not all stages always changed at the same time unless the bandwidth of each stage increases reducing selectivity and sensitivity.

3- Superhytrodyne receiver:



The incoming modulated signal (with carrier frequency f_c) is translated to a new center frequency called Intermediate Frequency F_{IF} (for Am:455 kHz,(MW) for FM:10.7 MHz, for TV: 44 MHz.) by multiplying the incoming modulated signal by variable local carrier $F_{LO} = F_C \pm F_{IF}$. For example if AM station at 600 kHz is designed and if the local oscillator operates above the incoming frequency, the local frequency would be $600 + 455 = 1055\,kHz$. Mixing $1055\,kHz$ with $600\,kHz$ produces two frequencies $1655\,kHz$ and $455\,kHz$, the IF amplifier would pass the frequency $455\,kHz$ and thus the incoming frequency is translated to intermediate frequency.

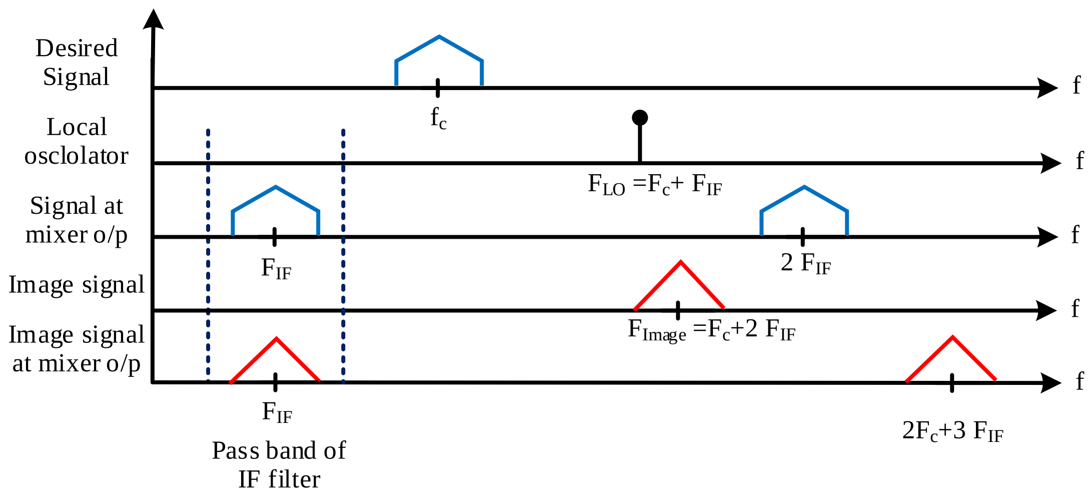
Advantages:

- The amplification and filtering is performed at a fixed frequency regardless of station selection.

Disadvantages:

- High gain IF stages are tuned outside the assigned frequency band.
- Image frequency problem:

If the desired station at $F_C = 600\,kHz$, the local carrier would be $F_{LO} = F_C + F_{IF} = 1055\,kHz$, if there is another station at $F_{image} = 1510 = (F_C + 2F_{IF})$ it would be also received (since $1510 - 1055 = 455\,kHz$).



Notes:

- $F_{Image} = F_C + 2F_{IF}$ when $F_{LO} = F_C + F_{IF}$ …(4-29)
- $F_{Image} = F_C - 2F_{IF}$ when $F_{LO} = F_C - F_{IF}$ …(4-30)
- To avoid image frequency problems, we choose F_{IF} such that:

  $2\,F_{IF} \geq F_{max} - F_{min}$ …(4-31)

Where F_{max}& F_{min} are the maximum and minimum allowed operating frequencies of the receiver.

Ex 4.11:

A given radar receiver operating at a frequency of 2.8 GHz and using a superhytrodyne principle has local oscillator frequency of 2.86 GHz. A second radar receiver operate at the image frequency of the first and interference results.

a) Determine the intermediate frequency of the first radar receiver.

b) What is the carrier frequency of the second receiver?

c) If you were to design a radar receiver, what is the minimum intermediate frequency you would choose to prevent image frequency problems in the range 2.8- 3 GHz radar band?

Solution:

a) $F_{IF} = F_{LO} - F_C = 2.86GHz - 2.8GHz = 60\ MHz$

b) $F_{Image} = F_C + 2\ F_{IF} = 2.80GHz + 0.12GHz = 2.92\ GHz$

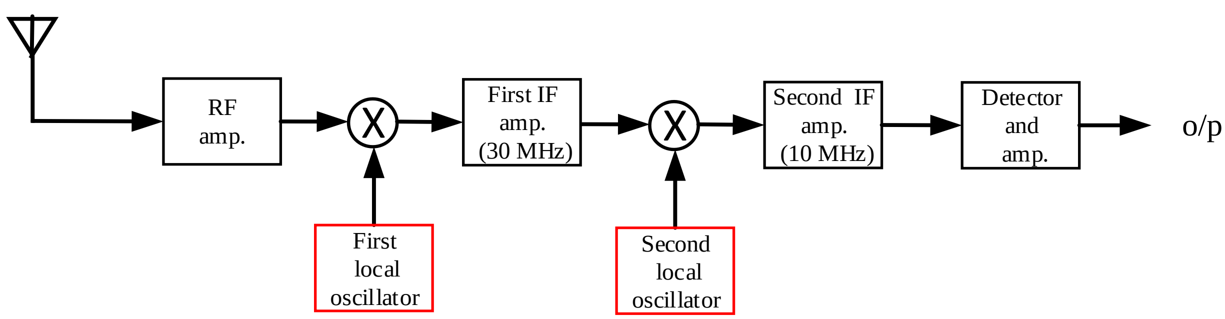
c) $2\ F_{IF} \geq F_{max} - F_{min} = 3.0\ GHz - 2.8\ GHz$

$$= 0.2\ GHz$$

$$\therefore\ F_{IF} \geq 100\ MHz$$

H.W:

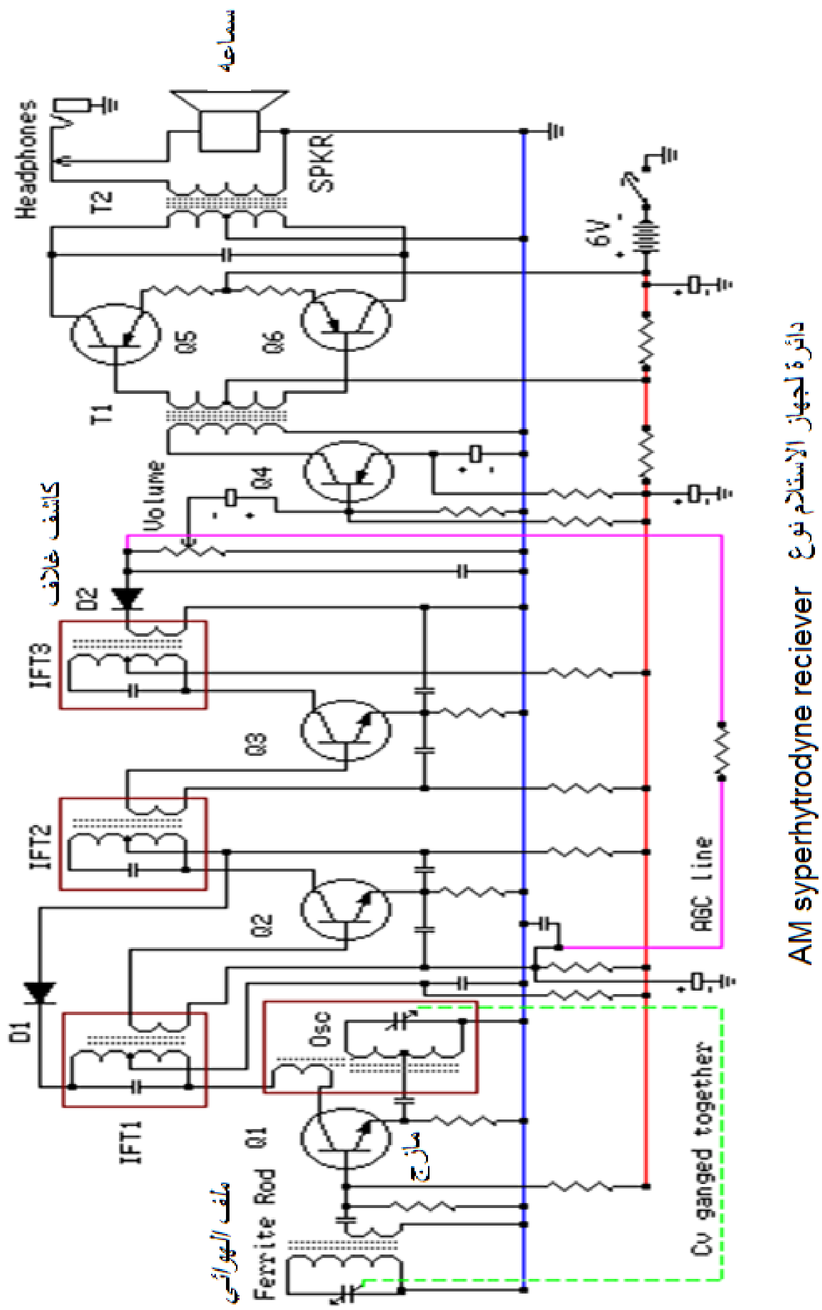
The figure below shows a satellite receiver uses two hytrodyne operations. It is used to receive transmissions at 136 MHz. the first local oscillator operates below the incoming carrier frequency, while the second operates above the first IF frequency. Find all possible image frequencies.



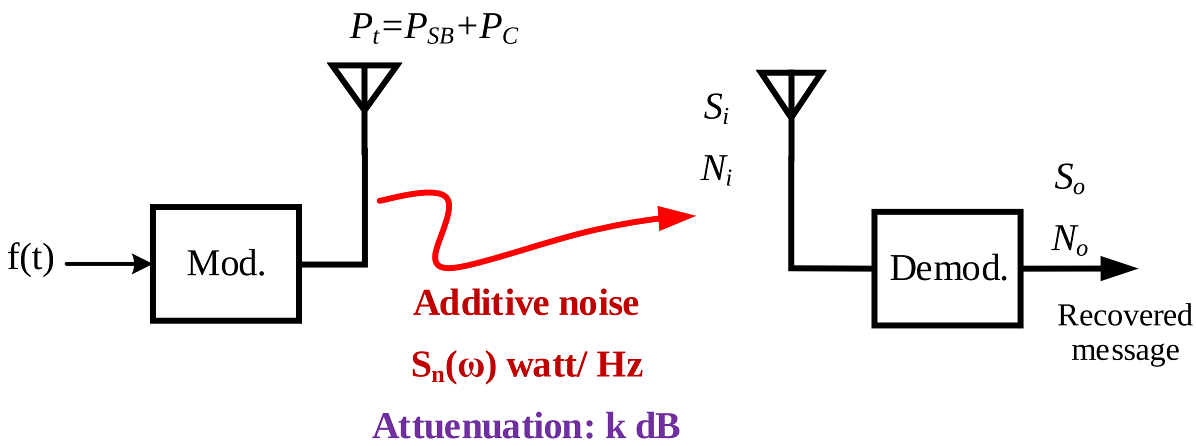
Ans: 76 MHz

156 MHz

56 MHz


AM syperhytrodyne reciever
دائرة لجهاز الاستلام نوع
Headphones
سماعه
SPKR
T2
T1
Q5
Q6
Q4
Volume
6V
كاشف غلاف
D2
IFT3
Q3
IFT2
Q2
AGC line
D1
IFT1
Osc
Q1
مازج
Ferrite Rod
ملف الهوائي
Cv ganged together

Noise in AM Systems:



Above diagram shows a simplified model of AM system with noisy channel. The modulator produces a total power of P_t watt. Due to the path losses (attenuation) usually measured in dB. The received signal power S_i would be:

$$S_i\big|_{dB} = P_t\big|_{dB} - k\big|_{dB} \qquad \ldots(4\text{-}30)$$

The received noise power could be computed if the noise power spectral density and transmission BW are known using:

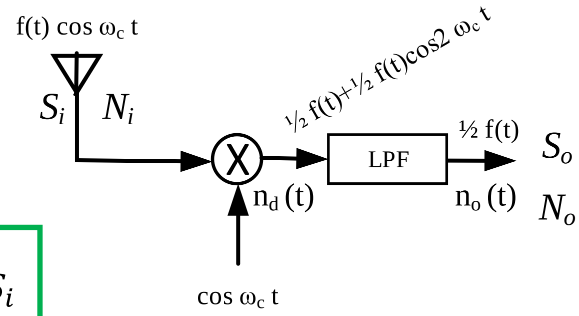
$$N_i = \frac{1}{2\pi}\int_{Bw_{tr}} S_n(\omega)d\omega \quad watt \qquad \ldots(4\text{-}31)$$

The output signal to noise ratio $\frac{S_o}{N_o}$ depend on the structure of the demodulator and the modulation type used.

1- DSB-SC

- Synchronous detector:

$$S_i = \overline{[f(t)cos\omega_c t]^2} = \frac{1}{2}\overline{f(t)^2}$$



$$S_o = \overline{\left[\frac{1}{2}f(t)\right]^2} = \frac{1}{4}\overline{f^2(t)} \quad \boxed{= \frac{1}{2}S_i}$$

$$n_d(t) = \frac{1}{2}n_c(t) + n_c(t)cos2\omega_c t - \frac{1}{2}n_s(t)sin2\omega_c t$$

$$n_o(t) = \frac{1}{2}n_c(t)$$

Defining $\overline{n_i^2(t)} = N_i$

We have $N_o = \overline{n_o^2(t)} = \frac{1}{4}\overline{n_c^2(t)} = \frac{1}{4}\overline{n_i^2(t)} = \boxed{\frac{1}{4}N_i}$

$$\therefore \boxed{\frac{S_o}{N_o} = 2\frac{S_i}{N_i}} \qquad \text{…(4-32)}$$

∴ In DSB-SC system, the detector improves the signal to noise ratio by factor of two.

2- SSB-SC

$$\emptyset_{SSB_\pm} = f(t)cos\omega_c t \mp f\widehat{(t)}sin\omega_c t$$

$$S_i = \overline{\emptyset^2(t)} = \frac{1}{2}\overline{f^2(t)} + \frac{1}{2}\overline{f^2\widehat{(t)}}$$

Since $\overline{f^2(t)} = \overline{f^2\widehat{(t)}}$

$$\therefore S_i = \overline{f^2(t)}$$

The output signal is $\frac{1}{2}f(t)$

$$S_o = \overline{\left[\frac{1}{2}f(t)\right]^2} = \frac{1}{4}\overline{f^2(t)} = \boxed{\frac{1}{4}S_i}$$

$$\therefore \quad \boxed{\frac{S_o}{N_o} = \frac{S_i}{N_i}} \qquad \ldots(4\text{-}33)$$

3- <u>SSB-LC</u>

- Envelope detector

$$\emptyset(t) = [f(t) + A_c]cos\omega_c t$$

$$S_i = \overline{[[f(t) + A_c]cos\omega_c t]^2} = \frac{1}{2}A_c^2 + \frac{1}{2}\overline{f^2(t)}$$

$$S_i(t) + n_i(t) = [f(t) + A_c]cos\omega_c t + n_c(t)cos\omega_c t - n_s(t)sin\omega_c t$$

The envelope of the signal is

$$r(t) = \sqrt{\{[f(t) + A_c] + n_c(t)\}^2 + \{n_s(t)\}^2}$$

$$\cong A_c + \underline{f(t)} + n_c(t) \qquad \text{for High SNR}$$

$$\therefore \quad \boxed{\frac{S_o}{N_o} = \frac{2\overline{f^2(t)}}{A_c^2 + \overline{f^2(t)}} \cdot \frac{S_i}{N_i}} \qquad \ldots(4\text{-}34)$$

For single tone $f(t) = mA_c cos\omega_c t$, $\qquad \overline{f^2(t)} = \frac{m^2A_c^2}{2}$

$$\therefore \quad \boxed{\frac{S_o}{N_o} = \frac{2m^2}{2 + m^2} \cdot \frac{S_i}{N_i}} \qquad \ldots(4\text{-}34)$$

<u>H.W</u>

Derive the relation between output signal to noise ratio and input signal to noise ratio for SSB-LC system, what would be the relation for S.T case?

Ex 4-12:

In a SSB system, the message signal is a single tone, of frequency 3 kHz, and peak amplitude 14 volt, the receiver input impedance is 50 Ω, the equivalent noise spectral density at the input is 10-14 watt/Hz (two sided) and the path losses is 30 dB. If the input noise power is considered in the entire signal bandwidth, find:

1- SNR at receiver input.

2- SNR at receiver output.

Solution:

1) $P_t = \overline{f^2(t)} = \frac{A_m^2}{2R} = \frac{14^2}{2\times 50} = 1.96\, watt \cong 2.92\, dB$

$$S_i = 2.92 - 30 = -27.1\, dB \equiv 1.95 * 10^{-3}\, watt$$

$$N_i = \frac{2}{2\pi}\int_0^{2\pi*3*10^3} S_n(\omega)d\omega = \frac{1}{\pi}\int_0^{6\pi*10^3} 10^{-14}d\omega = 6 * 10^{-11}\, watt$$

$$SNR_i = \frac{S_i}{N_i} = \frac{1.95 * 10^{-3}}{6 * 10^{-11}} = 3.25 * 10^7$$

$$\equiv 75.11\, dB$$

2) In SSB (synchronous detector)

$$\frac{S_o}{N_o} = \frac{S_i}{N_i} \quad \Longrightarrow \therefore \;\; SNR_o = 75.11\, dB$$

Problem Sheet for Amplitude Modulation

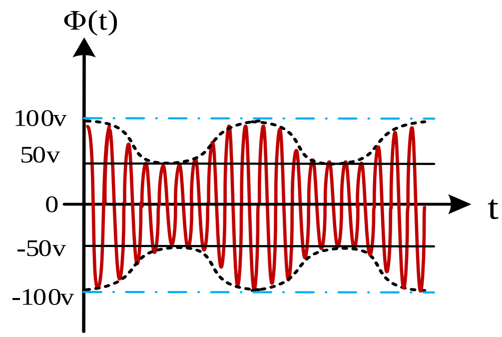
Q1: For the sinusoidaly modulated DSB/LC waveform shown in Fig. below.

a- Find the modulation index.

b- Sketch a line spectrum.

c- Calculated the ratio of average power in the sidebands to that in the carrier.

d- Determine the amplitude of the additional carrier, which must be added to obtain a modulation index of 10%



Ans: (a) 0.33

(b)0.055.

(d) 17s v.

Q2: A given AM (DSB – LC) transmitter develops an unmodulated power output of 1 kW. Across 50Ω resistive load. When a sinusoidal test tone with a peak amplitude of $5v$ is applied to the input of the modulator, it is found that the spectral line for each sideband in the magnitude spectrum for the output is 40% of the carrier line. Determine the following quantities in the output signal:

a-The modulation index.

b-The peak amplitude of the lower sideband.

c-The ratio of total sideband power to carrier power.

d-The total output power.

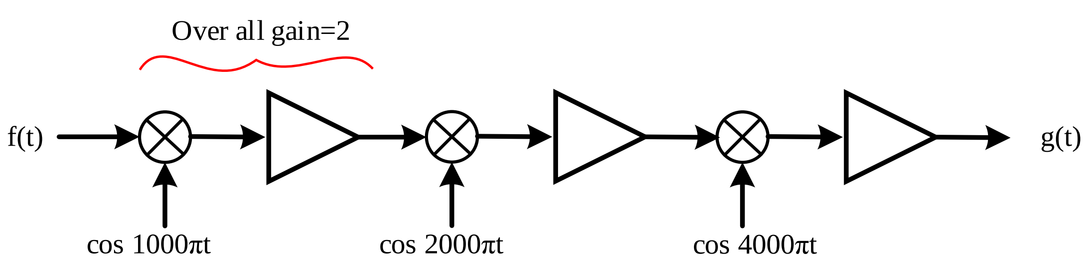
e-The total average power in thr output if the peak amplitude of the modulation sinusoid is reduced to 4 volt.

Q3: The modulating signal $f(t) = 2\cos 100\,\pi t + cos\ 400\pi t$ is applied at the input of a DSB/ SC modulator operating at a carrier frequency of 1 kHz. Sketch the spectral density of $f(t)$ and the resulting AM-DSB/SC waveform identifying the upper and lower sidebands.

Q4: The spectral density of the input $f(t)$ to the system shown in fig. below is band – limited to 100Hz.

a- Sketch the spectral density of the output for an assumed input spectral density.

b- Write an expression for the output spectral density in terms of the input spectral density if the sequence of sinusoidal generators and mixers extended indefinitely:

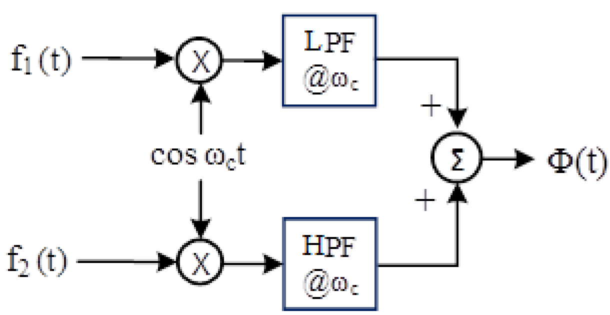


Ans: $G(\omega) = \sum_{n_{odd}=-\infty}^{\infty} F(\omega - 1000\pi n)$

Q5: The system shown in fig. below can be used for sending two messages on one carrier.

(a) If $f_1(t) = cos\omega_1 t$ and $f_2(t) = cos\omega_2 t$ derive an expression for $\phi(t)$.

(b) Devise block diagram for suitable demodulator for $\phi(t)$.



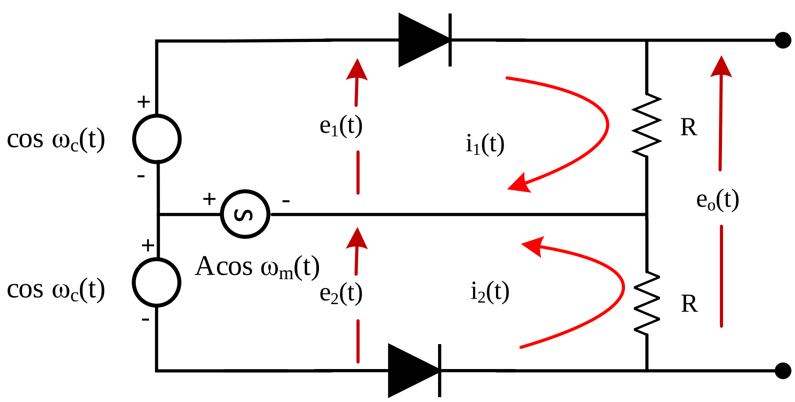
Ans: $\phi(t) = \frac{1}{2}\cos(\omega_{c1} - \omega_1)\,t + \frac{1}{2}\cos(\omega_c - \omega_2)$

Q6: When the input to a given system audio is $(4\cos 800\,\pi t + cos\ 2000\pi t)mv$, the measured frequency component at 600 kHz is 1mv. Represent the amplifier output – input characteristic by $e_o = a_1 e_i + a_2 e_i^2$ and calculate the numerical values of a_1 and a_2 from the data given.

Ans: $a_1 = 1000$; $a_2 = 250\ v^{-1}$.

Q7: The balanced modulator in fig. below is to be investigated for the possible generation of AM – DSB signal with $m \leq 1$ each diode has the characteristic:

$$i(t) = a_1 e(t) + a_2 e^2\,(t)$$



(a) Determine the maximum allowable value of A.

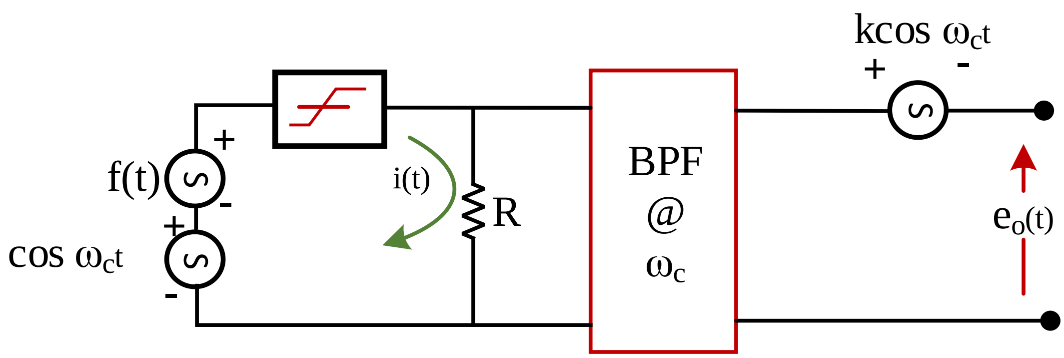
(b) Determine the maximum and minimum bandwidth of an ideal BPF required on the output if one of the diodes is open – circuited.

Ans: (a) $\frac{a_1}{2a_2}$ (b) $B_{max} = 2(f_c - 2f_m)$; $B_{min} = 2f_m$.

Q8: The model of a possible DSB/ SC modulator is shown in fig below. Determine the required value of the constant k if the bandpass filter has unity gain at $+\,\omega_c$.

Ans: $k = a_1 R$

Nonlinear element $i(t) = a_1 e(t) + a_2 e^2(t)$



Q9: Determine the power in the sidebands as a percentage of the total power of a modulated signal in the case of a carrier – amplitude – modulated by two sinusoidal signals of different frequencies, with individual modulation depths of 0.3 and 0.4.

Ans: 11.1%

Q10: Determine the saving, in signal power, in the case of 50% modulated AM signal, if the carrier is suppressed before transmission.

Ans: 88.9%

Q11: An AM modulation waveform signal:

$$\phi(t) = (1 + 0.5\cos 2000\pi t + 0.5\cos 4000\pi t)\cos 20000\pi t$$

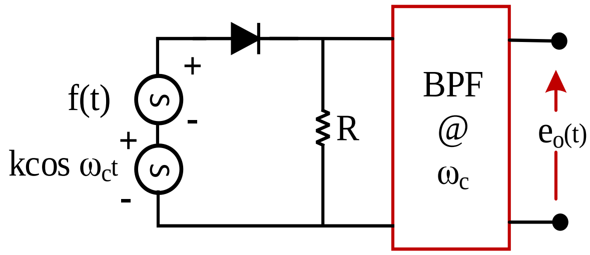
(a) Sketch the amplitude spectrum of $\phi(t)$.

(b) Find total power, sideband power and power efficiency.

(c) Find the average power containing of each sideband.

(d) What is the modulation index?

Q12: The system shown in fig. besides can be used to generate AM signal even if the diode is not operated as an ideal switch.



In this case the nonlinearities in the diode characteristics may be approximated with the power series of the form:

$$i(t) = a_1 e(t) + a_2 e^2(t) + \cdots \quad , i(t)R \ll e(t)$$

Retaining only first two terms, let $(t) = cos\omega_m t$, and if $a1 = 0.01$, $a2 = 0.001$. Determine the modulation index of the resulting DSB signal when all terms except the near the carrier frequency are shifted out.

Ans: m=0.2

Q13: A DSB/ SC and an SSB/ SC transmissions are ach sent at $1\,MHz$ in the presence of noise. The modulating signal in each case is band – limited to3 kHz. The received signal power in each case is $1\,mw$ and the received noise is assumed to white with a (two – sided) power spectral density of $10^{-3}\,\mu w/Hz$. The receiver consists of a bandpass filter (BPF) whose bandwidth matches synchronous detector.

a- Compare the SNR_s at detector input.

b- Compare the SNR_s at detector output.

Q14: The DSB–LC signal $\phi(t) = 3\cos 10000\pi t + \cos 1000\pi t\ \cos 10000\pi t$ Volt is present with additive band-limited white noise whose two-sided power spectral density is $1\,\mu w/Hz$ up to $10\ kHz$ and zero at higher frequencies. The signal – plus – noise is passed through an ideal BPF with a bandwidth of $100\ Hz$ centered at $5500\ Hz$. Assume all resistance level are one ohm.

a- Compute the average SNR at the input of BPF.

b- If the desired output signal is that portion of input signal with spectral components within the passband of the filter, what is the average S/N ratio at the output of the filter?

c- Assume that the desired output signal is the amplitude of the signal in item (b) above. Compute S/N of the synch. Detector with an output bandwidth of $50Hz$ that can make this measurement.

Ans: (a) 23.8 dB (b) 28dB (c) 31dB.

Q15: A certain station uses DSB – SC with an average transmitter power of P watts. If SSB – SC were used instead, what must be the average transmitted power for:

a) The same received signal strength.

b) The same received S/ N ratio.

Assume the synchronous Detection with the same local oscillator Signal strength for both cases.

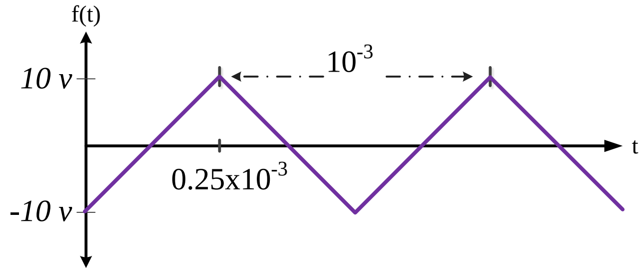
Q16: A FDM system uses SSB – SC modulation and AM main carrier modulation. There are forty (40) equal amplitude voice input channels; each bandlimited to 3.3 kHz. A 0.7 kHz guard band has allowed between channels and below the first channels.

(a) Determine the final transmission BW.

(b) Compute the degradation in signal- to- noise of input No. 40 when compared to input No.1, assuming that PSD of noise (two – sided) is f^2 $\mu w/Hz$.

Ans: 160 kHz, 36 dB.

Q17: A modulating signal f (t) shown besides AM modulates a carrier with modulation index = 0.5, find:



a- Total power.

b- Sideband power.

c- Carrier power.

Q18: A certain message having $\frac{\overline{f^2(t)}}{|f(t)_{min}|^2} = 0.09$ is transmitted through a channel having PSD $(10^{-7} w/Hz$ watt/Hz.

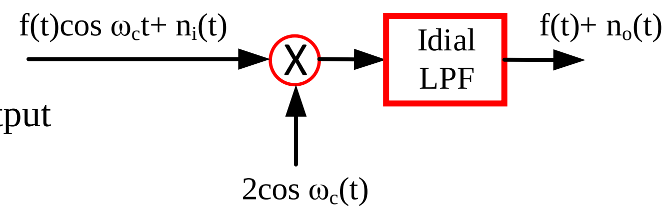
If the signal power at receiver input is 0.1 watt and the signal bandwidth is 3 kHz, find:

(a) SNR_{dB} at Rx input if AM/ DSB – SC Mod. Is used.

(b) 2-SNR_{dB} at Rx output if standard AM Mod. With $m = 0.5$ is used.

Ans: 28.23 dB, 11.66 dB.

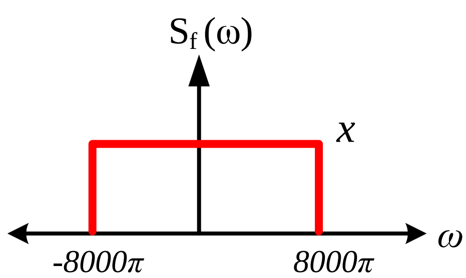
Q19: A DSB – SC receiver shown in figure below. The channel noise PSD is $\eta/2$ with $\eta = 10^{-7}$. The PSD of the modulating signal $f(t)$ is shown in the other figure:



(a) Determine the receiver output noise power S.

(b) Determine the receiver noise output power No.

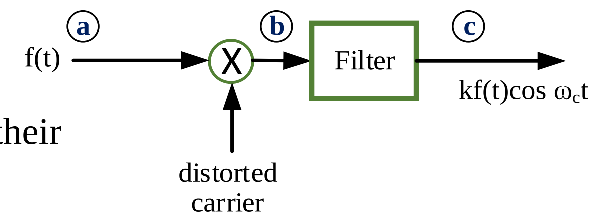
(c) If the output SNR is required to be at least30dB, determine the minimum value of (x) and corresponding power of (t).



Q20: For an AM channel with noise PSD $S_n(\omega) = 10^{-8}\, watt/Hz$. (two- sided), if the receiver output SNR is required to be at least $33dB$. Determine the transmitted power if the modulating signal is a single tone with peak amplitude of 0.1 volt and with frequency of 1 kHz.

Ans: 9.9 *m* watt.

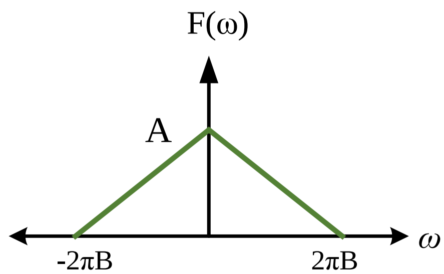
Q21: The figure below shows a DSB –SC modulator. The carrier available at the multiplier is a distorted sinusoid given by $a_1 cos\omega_c t + a_2 cos^2\omega_c t$. The spectrum of f(t) is shown in fig. shown:



(a) Determine the signals and sketch their spectra at points b&c.

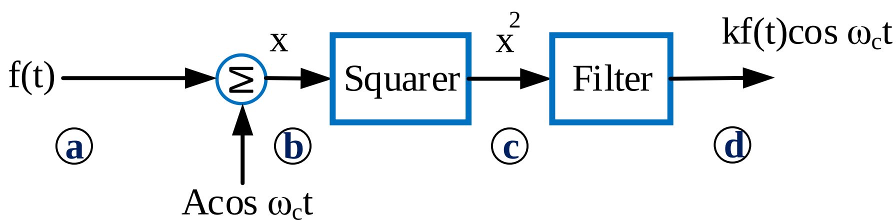
(b) What kind of filter is required in fig?

(c) What is minimum value of ω_c required for this scheme to work



Ans: $4\pi B$ rad/sec.

Q22: Show that the scheme shown in fig. Below can be used to generate DSB-SC signal. Assuming that $f(t)$ is a signal bandlimited to $B\ Hz$, find the signals and sketch their spectra at points a, b, c and d. Explain the nature of filter in this figure.



Q23: An Am communication system with $m = 0.707$ operates in the presence of white noise with two- sided power spectral density $0.25 \times 10^{-14}\, watt/Hz$, and with total path losses of $90\ dB$. The input bandwidth is $10\ kHz$. Calculate the minimum required carrier amplitude of the transmitter for a 10 kHz sinusoidal input and $40\ dB$ output SNR. Assume that resistance level is one ohm.

Ans: 63.254 volt.

Q24: A signal $f(t)$ is bandlimited to ω_m. If it is frequency translated by multiplying it by the signal $cos\omega_c t$. Find ω_c so that the bandwidth of the transmitted signal is 1 percent of the carrier frequency ω_c.