Lec.10 Realization of Digital Filters

10.1 Realization of IIR Filters

Digital filters described by the transfer function H(z) may be generally realized in the following forms:

1. Direct form I realization.
2. Direct form II realization.
3. Cascade realization.
4. Parallel realization.

10.2 Direct-Form I Realization

A digital filter transfer function, H(z), is given by:

$$H(Z) = \frac{B(Z)}{A(Z)} = \frac{b_0 + b_1 Z^{-1} + ... + b_M Z^{-M}}{a_0 + a_1 Z^{-1} + ... + a_N Z^{-N}} \tag{10.1}$$

Let x(n) and y(n) be the digital filter input and output, respectively. Taking z-transform:

$$Y(Z) = H(Z)\, X(Z) \tag{10.2}$$

Where X(z) and Y(z) are the z-transforms of x(n) and y(n), respectively. If we substitute equation (10.1) into H(z) in equation (10.2), we have

$$Y(Z) = \left(\frac{b_0 + b_1 Z^{-1} + ... + b_M Z^{-M}}{a_0 + a_1 Z^{-1} + ... + a_N Z^{-N}} \right) X(Z) \tag{10.3}$$

Taking the inverse of the z-transform of Equation (10.3), then:

$$y(n) = b_0\, x(n) + b_1\, x(n-1) + \ldots + b_M\, x(n-M)$$
$$- a_1\, y(n-1) - a_2\, y(n-2) - \ldots - a_N\, y(n-N) \tag{10.4}$$

This difference equation thus can be implemented by a direct-form I realization shown in Fig. (10.1A). Figure (10.1B) illustrates the realization of the second-order IIR filter (M = N = 2).

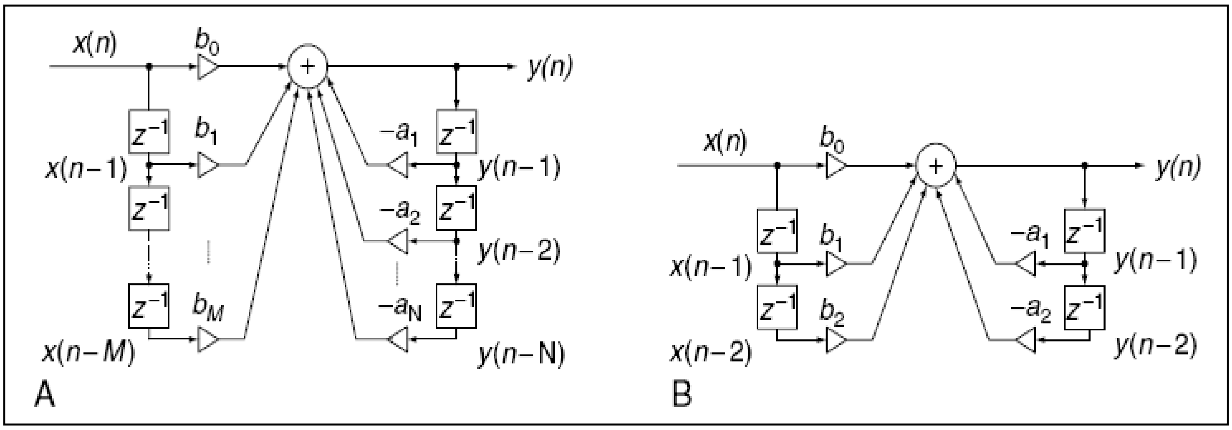


Fig.(10.1) (A) Direct-form I realization. (B) Direct-form I realization with M = 2.

Note that the notation used in Figures (10.1 c) and (10.1d) below;

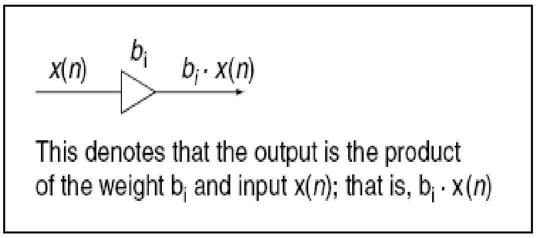


Fig.(10.1c) Notations

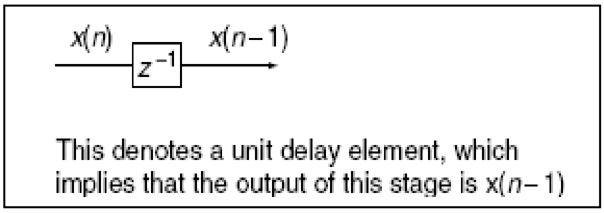


Fig.(10.1d) Notations

10.3 Direct-Form I I Realization

Considering Equations (10.1) and (10.2) with $N = M$, we can express

$$Y(Z) = H(Z)\,X(Z) = \frac{B(Z)}{A(Z)}X(Z) = B(Z)\frac{X(Z)}{A(Z)}$$

$$= (b_0 + b_1 Z^{-1} + ... + b_M Z^{-M})\left(\frac{X(Z)}{a_0 + a_1 Z^{-1} + ... + a_M Z^{-M}}\right) \qquad (10.5)$$

Also, defining a new z-transform function as

$$W(Z) = \frac{X(Z)}{a_0 + a_1 Z^{-1} + ... + a_M Z^{-M}} \qquad (10.6)$$

$Y(Z) = (b_0 + b_1 Z^{-1} + \ldots + b_M Z^{-M}) W(Z)$ (10.7)

$w(n) = x(n) – a_1 w(n–1) – \ldots – a_M w(n–N)$ (10.8)

$y(n) = b_0 w(n) – b_1 w(n–1) – \ldots – b_M w(n–N)$ (10.9)

Realization of equations (10.8) and (10.9) becomes another direct-form II realization, which is demonstrated in Fig. (10.2A). Again, the corresponding realization of the second-order IIR filter is described in Fig.(10.2B).

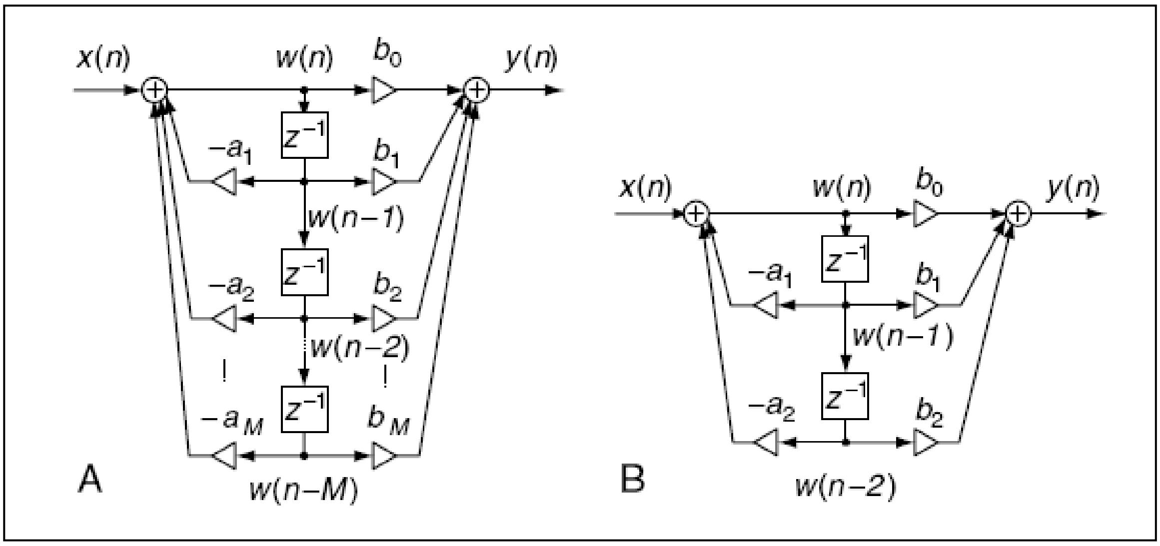


Fig.(10.2) (A) Direct-form II realization. (B) Direct-form II realization with M = 2.

10.4 Cascade (Series) Realization

An alternate way to filter realization is to cascade the factorized H(z) in the following form:

$$H(Z) = H_1(Z) \cdot H_2(Z) \dots H_k(Z) . \quad (10.10)$$

Where $H_k(z)$ is chosen to be the first- or second-order transfer function (section), which is defined by

$$H_k(Z) = \frac{b_{k0} + b_{k1} Z^{-1}}{1 + a_{k1} Z^{-1}} \quad (10.11)$$

$$H_k(Z) = \frac{b_{k0} + b_{k1} Z^{-1} + b_{k2} Z^{-2}}{1 + a_{k1} Z^{-1} + a_{k2} Z^{-2}} \quad (10.12)$$

Respectively. The block diagram of the cascade, or series, realization is depicted in Fig.(10.3)

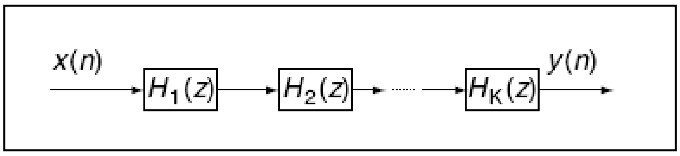


Fig.(10.3) Cascade realization.

10.5 Parallel Realization

Now we convert H(z) into the following form:

$$H(Z) = H_1(Z) + H_2(Z) + \dots + H_k(Z) \quad (10.13)$$

Where $H_k(z)$ is defined as the first- or second-order transfer function (section) given by

$$H_k(Z) = \frac{b_{k0}}{1 + a_{k1} Z^{-1}} \quad (10.14)$$

Or

$$H_k(Z) = \frac{b_{k0} + b_{k1} Z^{-1}}{1 + a_{k1} Z^{-1} + a_{k2} Z^{-2}} \quad (10.15)$$

Respectively. The resulting parallel realization is illustrated in the block diagram in Fig.(10.4).

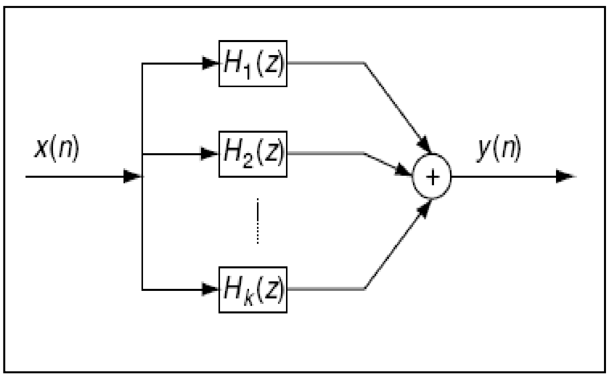


Fig.(10.4) Parallel realization

Example (1): Given a second-order transfer function

$$H(Z)=\frac{0.5(1-Z^{-2})}{1+1.3Z^{-1}+0.36Z^{-2}}$$

Perform the filter realizations and write the difference equations using the following realizations:

1. Direct form I and direct form II.
2. Cascade form via the first-order sections.
3. Parallel form via the first-order sections.

Solution:

1. To perform the filter realizations using the direct form I and direct form II

$$H(Z)=\frac{0.5(1-Z^{-2})}{1+1.3Z^{-1}+0.36Z^{-2}}$$

Where, a_1 = 1.3, a_2 = 0.36, b_0 = 0.5, b_1 = 0, and b_2 = − 0.5. Fig.(10.5 a) shows the direct-form I realization .

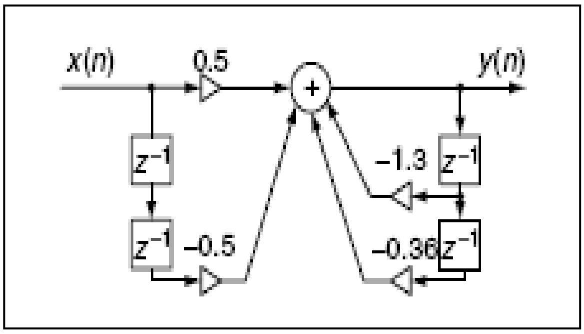
The difference equation for the direct- form I realization is given by

y(n) = 0.5 x(n) – 0.5 x(n–2) – 1.3 y(n-1) –0.36 y(n–2)

The direct-form II realization shown in Fig.(10.5 b) where, the difference equations for the direct-form II realization are expressed as:

w(n) = x(n) – 1.3 w(n–1) – 0.36 w(n–2)

y(n) = 0.5 w(n) – 0.5 w(n-2)



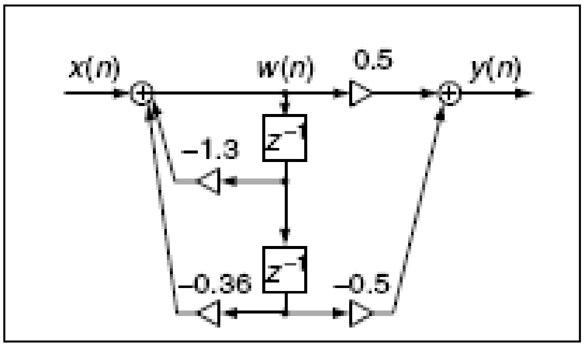


Fig.(10.5) (a) Direct-form I realization **(b) Direct-form II realization**

2. To achieve the cascade (series) form realization, we factor H(z) into two first-order sections to yield

$$H(Z)=\frac{0.5(1-Z^{-2})}{1+1.3Z^{-1}+0.36Z^{-2}}=\frac{0.5-0.5Z^{-1}}{1+0.4Z^{-1}}\,\frac{1+Z^{-1}}{1+0.9Z^{-1}}$$

$$H_1(Z) = \frac{0.5 - 0.5Z^{-1}}{1 + 0.4Z^{-1}}$$

$$H_2(Z) = \frac{1 + Z^{-1}}{1 + 0.9Z^{-1}}$$

Using the $H_1(Z)$ and $H_2(Z)$, and with the direct-form II realization, we achieve the cascade form depicted in Fig.(10.5 c).

The difference equations for the direct-form II realization have two cascaded sections, expressed as

Section 1:

$w_1(n) = x(n) - 0.4\, w_1(n-1)$

$y_1(n) = 0.5\, w_1(n) - 0.5\, w_1(n-1)$

Section 2:

$w_2(n) = y_1(n) - 0.9\, w_2(n-1)$

$y(n) = w_2(n) - w_2(n-1)$

Notice that the obtained $H_1(Z)$ and $H_2(Z)$ *are not unique selections* for realization. For example, there is another way of choosing them to yield the same H(Z).

$$H_1(Z) = \frac{0.5 - 0.5\, Z^{-1}}{1 + 0.9\, Z^{-1}}$$

$$H_2(Z) = \frac{1 + Z^{-1}}{1 + 0.4\, Z^{-1}}$$

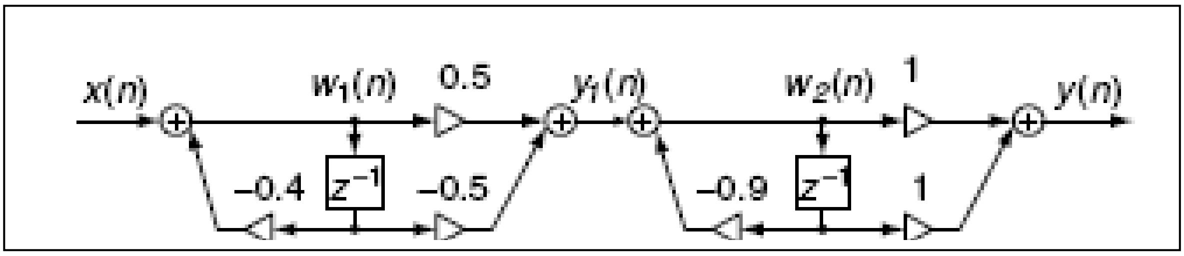


Fig. (10.5 c) Cascade realization

3. In order to yield the parallel form of realization, we need to make use of the partial fraction expansion,

$$\frac{H(Z)}{Z} = \frac{0.5(Z^2 - 1)}{Z(Z + 0.4)(Z + 0.9)} = \frac{A}{Z} + \frac{B}{Z + 0.4} + \frac{C}{Z + 0.9}$$

$$H(Z) = -1.39 + \frac{2.1Z}{Z + 0.4} + \frac{-0.21Z}{Z + 0.9} = -1.39 + \frac{2.1Z}{1 + 0.4Z^{-1}} + \frac{-0.21}{1 + 0.9Z^{-1}}$$

Again, using the direct form II for each section, we obtain the parallel realization in Fig. (10.5d) The difference equations for the direct-form II realization have three parallel sections, expressed as:

$y_1(n) = -1.39\, x(n)$

$w_2(n) = x(n) - 0.4\, w_2(n-1)$

$y_2(n) = 2.1\, w_2(n)$

$w_3(n) = x(n) - 0.9\, w_3(n-1)$

$y_3(n) = -0.21\, w_3(n)$

$y(n) = y_1(n) + y_2(n) + y_3(n)$

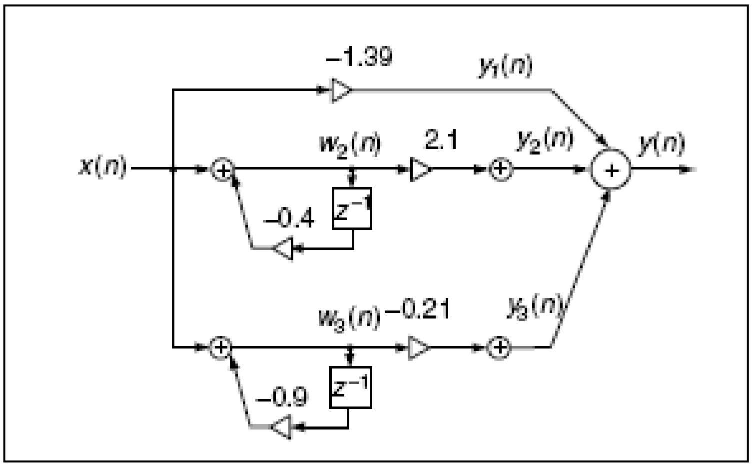


Fig.(10.5 d) Parallel realization

H.W: use direct form I, Direct form II, cascade realization and parallel realization to realize

$$H(Z) = \frac{8\,Z^3 - 4\,Z^2 + 11\,Z - 2}{(Z - 0.25)\,(Z^2 - Z + 0.5)}$$

10.6 Realization of FIR Filters

A causal FIR filter is characterized by:

$$H(Z) = \sum_{k=0}^{M} b_k\, Z^{-k} \qquad (10.16)$$

$$y(n) = \sum_{k=0}^{M} b_k\, x(n-k) \qquad (10.17)$$

The output is simply a weighted sum of present and past input values, as shown in Fig.(10.6)

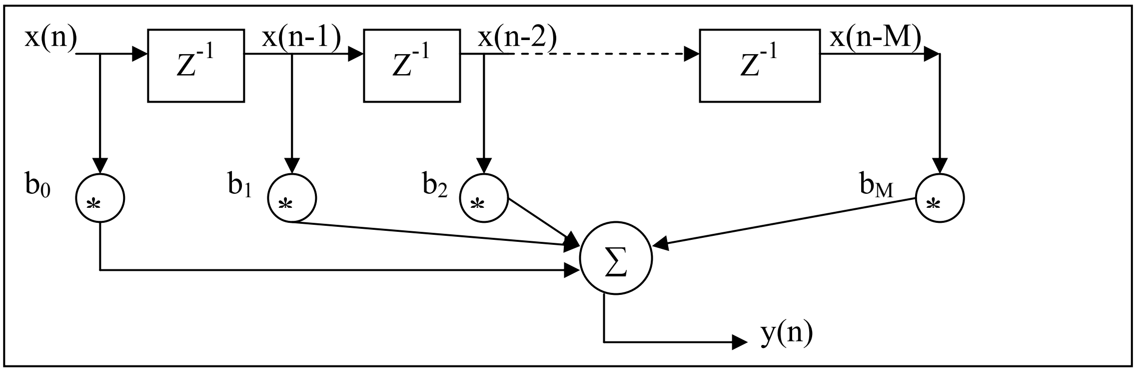


Fig.(10.6) Direct form I realization of FIR filter

H.W:

1- For FIR filters, prove that:

$$H(Z)=\sum_{n=0}^{(N/2)-1} h(n)\,\{Z^{-n}+Z^{-(N-1-n)}\} \qquad \text{for N even} \qquad (10.18)$$

$$H(Z)=\sum_{n=0}^{(N-3)/2} h(n)\,\{Z^{-n}+Z^{-(N-1-n)}\}+h\left(\frac{N-1}{2}\right)Z^{-(N-1)/2} \qquad \text{for N odd} \qquad (10.19)$$

2- Realize the FIR filters given in equations (10.18) and (10.19).