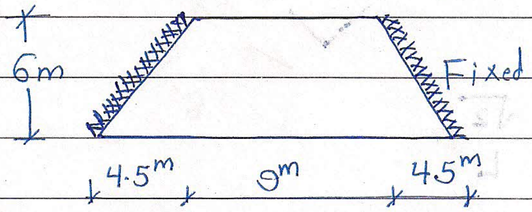
To get Critical $\beta \longrightarrow \dfrac{d(mP_1/w)}{d\beta} = 0$

$$3\mu\beta^2 + 4\alpha^2\beta - 3\alpha^2 = 0$$

$$\therefore \beta = \pm \frac{\alpha^2}{\mu}\left[\sqrt{\frac{4}{3} + \frac{\mu^2}{\alpha^2}} - \frac{2}{3}\right]$$

Home Work

Ex① Find w

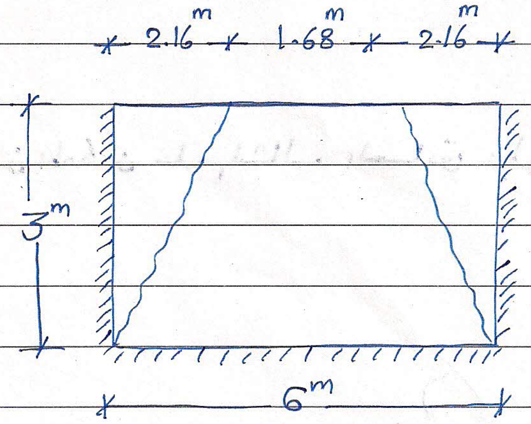


$\bar{m_n} = 60$ kN.m/m

$m_n = 45$ kN.m/m

Answer: $W_n =$

Ex② Find w



Answer: $w = 0.617\, mp$

Assist prof Dr. Ali H. Aziz

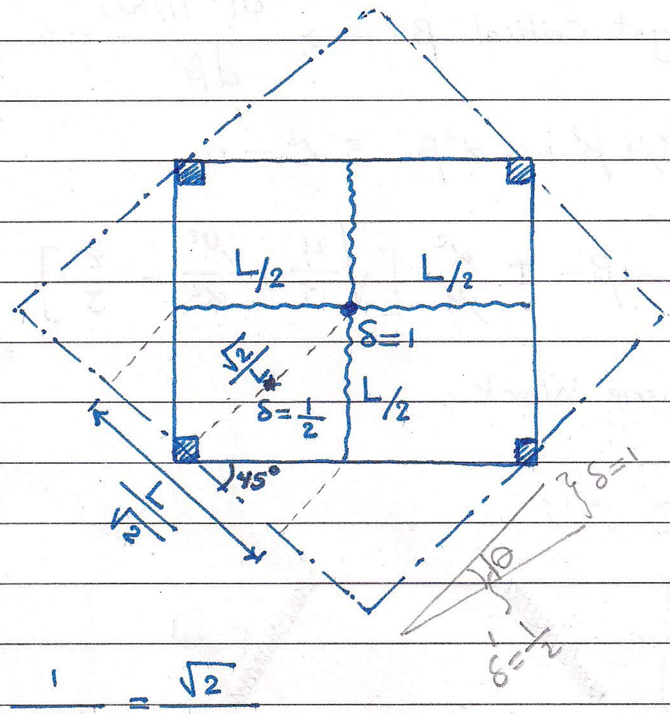
2014

Y.L

Example-11

Find w

Solution



$$W_E = 4\left(\frac{L}{2} \times \frac{L}{2} \times w\right) \times \frac{1}{2}$$

$$= \frac{wL^2}{2}$$

$W_E = W_I$

$W_I = 4(mp \times l \times \theta)$

$$\theta = \frac{1}{\sqrt{\left(\frac{L}{2}\right)^2 + \left(\frac{L}{2}\right)^2}} = \frac{1}{\sqrt{\frac{L^2}{2}}} = \frac{1}{\frac{\sqrt{L^2}}{\sqrt{2}}} = \frac{\sqrt{2}}{L}$$

$$\therefore \quad W_I = 4 \times mp \times \cancel{\frac{L}{\sqrt{2}}} \times \cancel{\frac{\sqrt{2}}{L}} = 4\ mp$$

$$W_E = W_I \Rightarrow \frac{wL^2}{2} = 4\,mp \Rightarrow w = \frac{8\,mp}{L^2}$$

Note

Rotation about inclined axis   من الممكن حل المثال السابق بطريقة

Example _ 12 _ ok

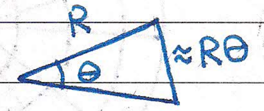
Analyze and Design the following slab.

$W_u = 15\ kN/m^2$, $d = 112\ mm$, $R = 3^m$

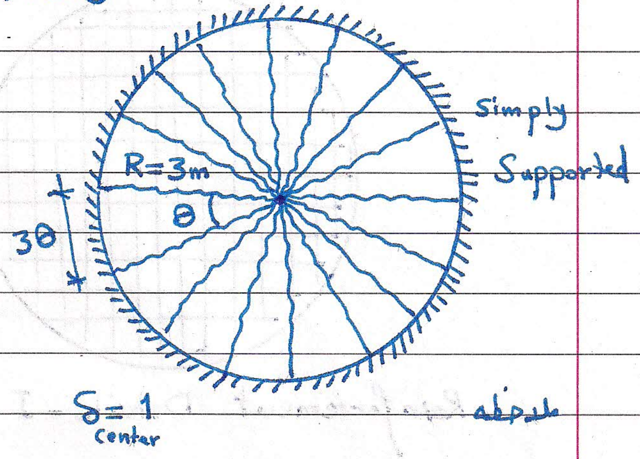
$f'_c = 25\ Mpa$

$f_y = 420\ Mpa$

Solution



$$Area = \frac{1}{2} * R * R\theta = \frac{R^2 * \theta}{2}$$



ملاحظه

$$No.\ of\ Segments = \frac{2\pi}{\theta}$$

كل قطعه يمكن أعتبارها مثلث لأن

(θ) صغيرة جداً

area δ No. segment

$$W_E = w \times \left(\frac{R^2\cancel{\theta}}{\cancel{2}}\right) * \frac{1}{3} * \frac{\cancel{2}\pi}{\cancel{\theta}}$$

$$= \frac{wR^2\pi}{3} = \frac{w(3)^2\pi}{3} = 3\pi w$$

$$W_I = \left(mp * \cancel{R\theta} * \frac{1}{\cancel{R}}\right) * \frac{2\pi}{\cancel{\theta}} = 2\ mp \cdot \pi$$

$$W_E = W_I$$

$$3\pi w = 2mp \cdot \pi \Rightarrow mp = \frac{3}{2} w = \frac{3}{2} * 15 = 22.5\ \frac{kN.m}{m}$$

$$R = \frac{M * 10^6}{0.9\ bd^2} , \quad m = \frac{f_y}{0.85 f'_c} , \quad \rho = \frac{1}{m}\left(1 - \sqrt{1 - \frac{2mR}{f_y}}\right)$$

$$R = \frac{22.5 \times 10^6}{0.9\ (1000) * 112^2} = 1.993 , \quad m = \frac{420}{0.85 \times 25} = 19.76$$

$$\rho = \frac{1}{19.76}\left(1 - \sqrt{1 - \frac{2 \times 19.76 * 1.993}{420}}\right) \approx 0.0056$$

$\rho_{min} = 0.0033$, $\rho_{max} = 0.0189$, $\rho_{min} < \rho < \rho_{max}$ ok

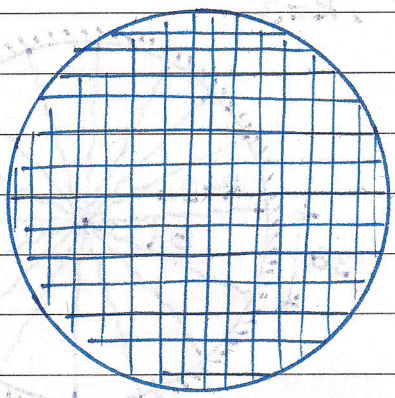
$$As = \rho b d = 0.0056 \times 10^3 \times 112 = 627\ mm^2/m$$

$$spacing = \frac{10^3 * Ab}{As} = \frac{10^3 * \frac{\pi}{4}(12)^2}{627} \approx 180\ mm$$

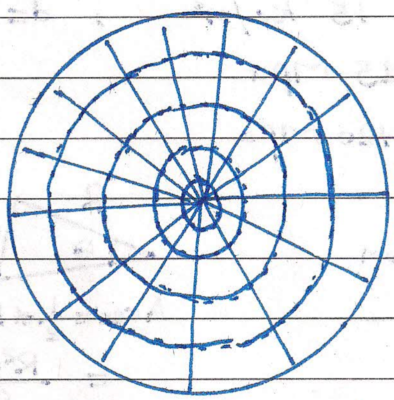
YL

11

use ϕ 12 @ 175 mm



OR



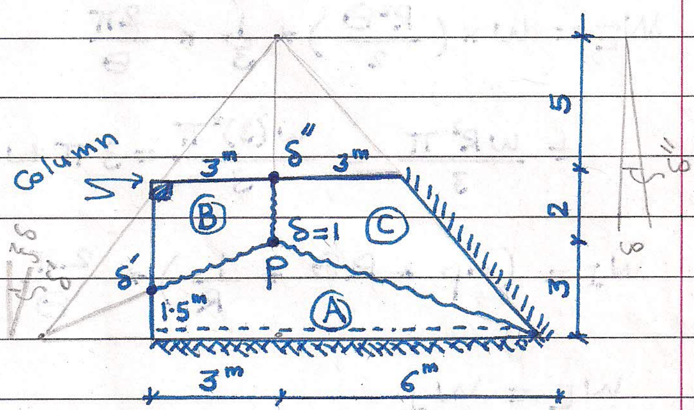
Reinforcement Details - I

Reinforcement Details - II

Example - 13

Find (P)

Solution



$m^- = m^+ = m$

$\frac{\delta}{3} = \frac{\delta'}{1.5} \Rightarrow \delta' = 0.5$

$\frac{\delta}{7} = \frac{\delta''}{5} \Rightarrow \delta'' = \frac{5}{7} \simeq 0.714$

$W_E = P * 1 = P$

$W_{I_{(A)}} = m^- * 9 * \frac{1}{3} + m^+ * 9 * \frac{1}{3} = 6m$

$W_{I_{(B)}} = m * 3 * \frac{\delta'}{(5-1.5)} + m * (5-1.5) * \frac{\delta''}{3}$

$= m * 3 * \frac{0.5}{3.5} + m * 3.5 * \frac{0.714}{3} = 1.262\,m$

$W_{I_{(C)}} = m * 6 * \frac{\delta}{7} + m * 5 * \frac{\delta''}{3} = m * 6 * \frac{1}{7} + m * 5 * \frac{0.714}{3}$

$= 2.05\,m$

$W_{I_{Total}} = 6m + 1.262m + 2.05m = 9.312\,m$

$W_E = W_I \Rightarrow P = 9.312\,m$ Answer

Example - 14 -

$W_E = P * 1 = P$

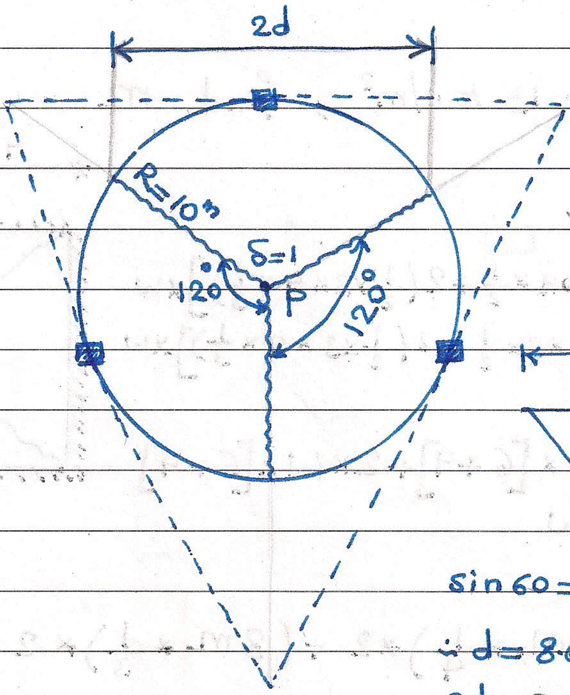
$W_I = 3(m * 17.32)\frac{1}{10}$

$= 5.196\, m$

$W_E = W_I$

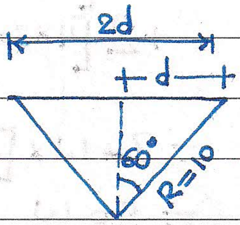
$\therefore P = 5.196\, m$

Answer



Y.L بتاع مسقط

على محور الدوران



$\sin 60 = \frac{d}{R} = \frac{d}{10}$

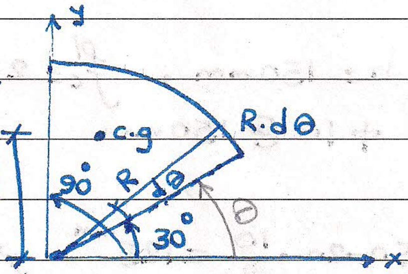
$\therefore d = 8.66\, mm$

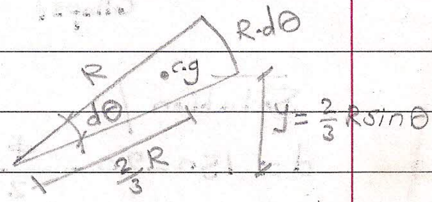
$2d = 2 * 8.66 = 17.32^{m}$

ملاحظة/

اذا كان الحمل UDL فيجب ايجاد مركز القطعه من القانون

$\bar{y} = \left(\frac{\int y \cdot dA}{\int dA}\right) * 2$





$dA = \frac{1}{2} * R * R \cdot d\theta$

$y = \left(\frac{2}{3}R\right) \cdot \sin\theta$

$\therefore \bar{y} = \left(\frac{\int_{30}^{90} \left(\frac{2}{3}R\right) \cdot \sin\theta * \cancel{\frac{1}{2}} * R * \cancel{R} \cdot d\theta}{\int_{30}^{90} \cancel{\frac{1}{2}} * R * \cancel{R} \cdot d\theta}\right) * 2$

$= 2\,\frac{\frac{2}{3}R \int_{30}^{90} \sin\theta \cdot d\theta}{\int_{30}^{90} d\theta} = \frac{4}{3}R\,\frac{\left[-\cos\theta\right]_{30}^{90}}{(90-30) * \frac{2\pi}{180}}$

↑ Deg → Rad

$= \frac{4R}{3\left(\frac{2}{3}\pi\right)}\left[-\left(\cos 90 - \cos 30\right)\right]$

$\bar{y} = 0.5513\, R$

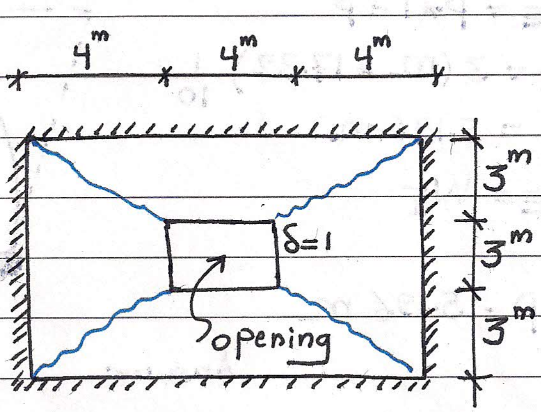
YL

13

Example-15-

If $w = 15\ kN/m^2$, find m

Solution



$$W_E = 2\left[3\times4\times\tfrac{1}{2} + 2\left(\tfrac{1}{2}\times3\times4\times\tfrac{1}{3}\right)\right]\times w + 2\left[4\times3\times\tfrac{1}{2} + 2\left(\tfrac{1}{2}\times3\times4\times\tfrac{1}{3}\right)\right]\times w$$

$$= 2 * w \times [6+4] + 2 * w \times [6+4]$$
$$= 40\,w$$

$$W_I = \left(6m \times \tfrac{1}{4}\right) * 2 + \left(8m \times \tfrac{1}{3}\right) * 2 = \frac{25}{3}\,m$$

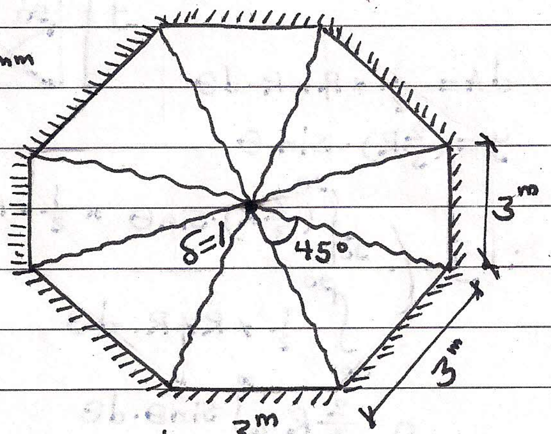
$$W_E = W_I \Rightarrow 40\,w = \frac{25}{3}\,m \Rightarrow m = \frac{40\times3}{25} * 15 = 72\ kN.m/m$$

Example-16-

Find $w_{Collapse}$, $h = 150\,mm$, $f'_c = 25\ Mpa$, $f_y = 350\ Mpa$

$\phi 10 @ 150\,mm$

Solution /



$$d = 150 - 20 - \frac{\phi}{2} = 150 - 20 - \frac{10}{2} = 125^{mm}$$

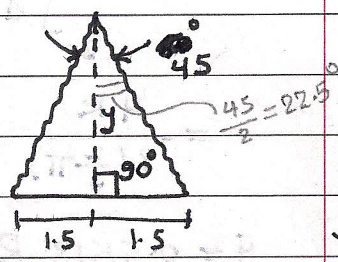
$$W_E = 8w\left(\tfrac{1}{2}\times3\times3.62\times\tfrac{1}{3}\right)$$
$$= 14.48\,w$$

$$W_I = 8\left(mp^+ \times 3 \times \frac{1}{3.62}\right) = 6.63\,mp^+$$

$$mp^+ = \rho b d^2 f_y \left(1 - 0.59\rho\frac{f_y}{f'_c}\right)$$

$$\rho = \frac{As}{bd}, \quad As = \frac{Ab * 1000}{S}$$

$$\frac{360}{8} = 45^\circ$$



$As = 524\ mm^2/m$

$$\rho = \frac{524}{10^3\times125} = 0.00419$$

$$\tan 22.5 = \frac{1.5}{y}$$
$$\therefore y = 3.62\ m$$

$mp^+ \approx 22\ kN.m/m$

YL

$\therefore W_I = 6.63 \times 22 = 145.86$

$W_E = W_I \Rightarrow w = \dfrac{145.86}{14.48} \approx 10 \ kN/m^2$

Answer

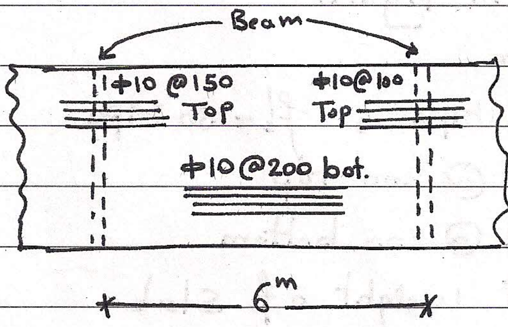
Example _ 17 _

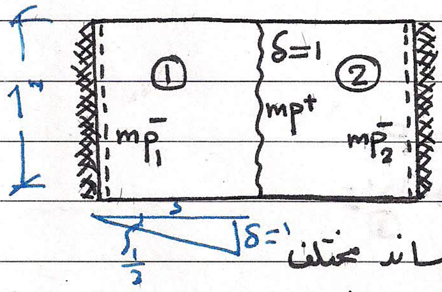
Find $W_{collopse}$ if :-

$h = 160 \ mm$

$b = 1 \ m$

$f_y = 420 \ Mpa$, $f'_c = 30 \ Mpa$





ملاحظه

∵ التسليح عند الساند مختلف

∴ يجب حساب الشغل الداخلي لكل قطعه على حدا.

$W_E = 2\,(3 \times 1) * \frac{1}{2}\, w = 3w$

$W_I = \underset{①}{W_I} + \underset{②}{W_I}$

$= m\bar{p}_1 \times 1 \times \frac{1}{3} + m\overset{+}{p} \times 1 \times \frac{1}{3}$

$+ m\bar{p}_2 \times 1 \times \frac{1}{3} + m^{+}p \times 1 \times \frac{1}{3}$

$W_I = \dfrac{m\bar{p}_1}{3} + \dfrac{m\bar{p}_2}{3} + \dfrac{2}{3}\, mp^{+}$

$d = h - c - \dfrac{\phi}{2}$

$= 160 - 20 - \dfrac{10}{2} = 135^{mm}$

$mp = \rho\, b d^2 f_y \left(1 - 0.59\, \rho\, f_y / f'_c\right)$, $\rho = \dfrac{A_s}{bd}$, $A_s = \dfrac{1000 \times A_b}{S}$

$\bar{A_{s_1}} = \dfrac{78 \times 10^3}{150} = 520 \ mm^2/m \rightarrow \bar{\rho_1} = 0.00385 \rightarrow m\bar{p}_1 = 28.5 \ kN.m/m$

$A_s^{+} = \dfrac{78 \times 10^3}{200} = 390 \ mm^2/m \longrightarrow \rho^{+} = 0.0029 \rightarrow mp^{+} = 21.66 \ kN.m/m$

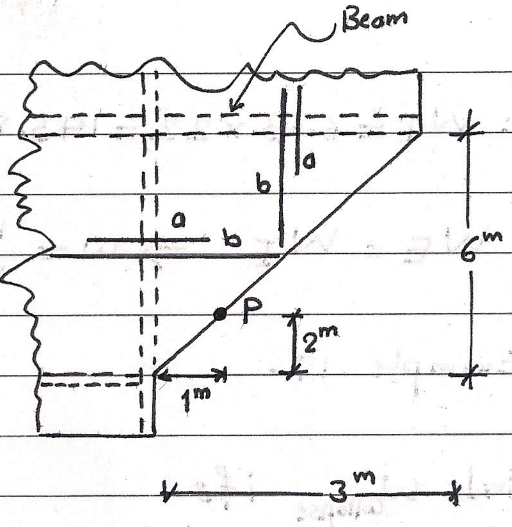
$A\bar{s}_2 = \dfrac{78 \times 10^3}{100} = 780 \ mm^2/m \rightarrow \bar{\rho_2} = 0.0058 \longrightarrow m\bar{p}_2 = 42.27 \ kN.m/m$

$\therefore W_I = \dfrac{28.5}{3} + \dfrac{42.27}{3} + \dfrac{2}{3} \times 21.66 = 38.03$

$W_E = W_I \Rightarrow 3w = 38.03 \Rightarrow w = 12.67 \ kN/m^2$

Answer

Example-18



Find max. Concentrated Force (P) that the triangular slab can sustain if it is reinforced as shown in Figure.

$d = 140\,mm$

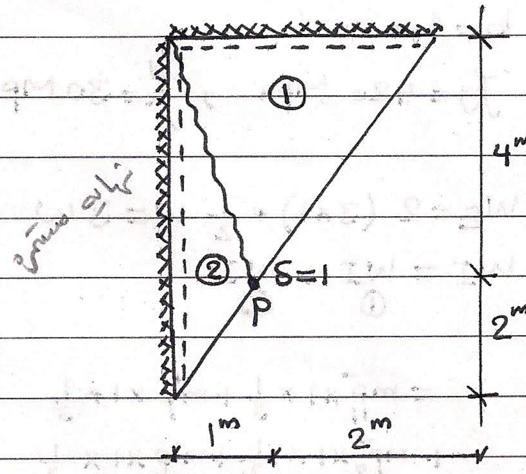
$f_y = 420$ Mpa ; $f'_c = 30$ Mpa

$a = \phi 12$ @ 200 top

$b = \phi 12$ @ 200 bottom

Neglect weight of slab.

Solution :-



$W_E = P \times 1 = P$

$W_I = \underset{①}{W_I} + \underset{②}{W_I}$

$= m_p^- \times 3 \times \frac{1}{4} + m_p^+ \times 1 \times \frac{1}{4}$
$+ m_p^- \times 6 \times \frac{1}{1} + m_p^+ \times 4 \times \frac{1}{1}$

$\because$ Reinforcement (a) same as Reinforcement (b) $\rightarrow m_p^+ = m_p^-$

$\therefore W_I = m_p\left(\frac{3}{4} + \frac{1}{4} + 6 + 4\right) = 11\,m_p$

$m_p = \rho b d^2 f_y (1 - 0.59 \rho\, f_y / f'_c)$

$\rho = \frac{A_s}{bd}$, $A_s = \frac{A_b \times 10^3}{S}$

$A_s = \frac{\frac{\pi}{4}(12)^2 \times 10^3}{200} = 565\ mm^2/m \rightarrow \rho = 0.00404 \rightarrow m_p = 32.1\ KN.m/m$

$\therefore W_I = 32.1 \times 11 = 353.1$

$W_E = W_I \Rightarrow P = 353.1\ KN$

Answer

YL