Polarization and Related Antenna Parameters

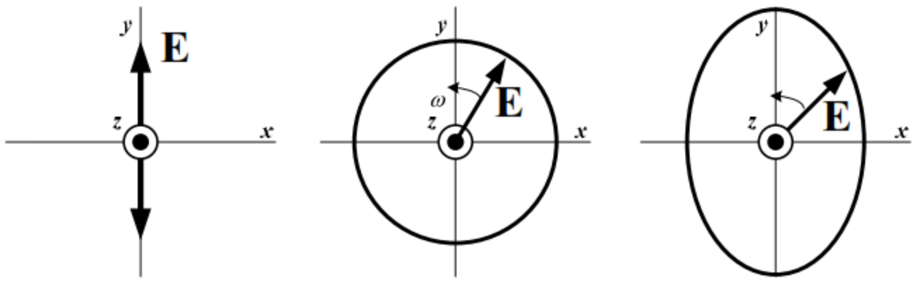
7.1 Polarization of EM fields

The polarization is the locus traced by the extremity of the time-varying field vector at a fixed observation point.

The polarization of the EM field describes the orientation of its vectors at a given point and how it varies with time. In other words, it describes the way the direction and magnitude of the field vectors (usually E) change in time. Polarization is associated with TEM time-harmonic waves where the H vector relates to the E vector simply by $H = E/\eta\,\hat{a}_r$.

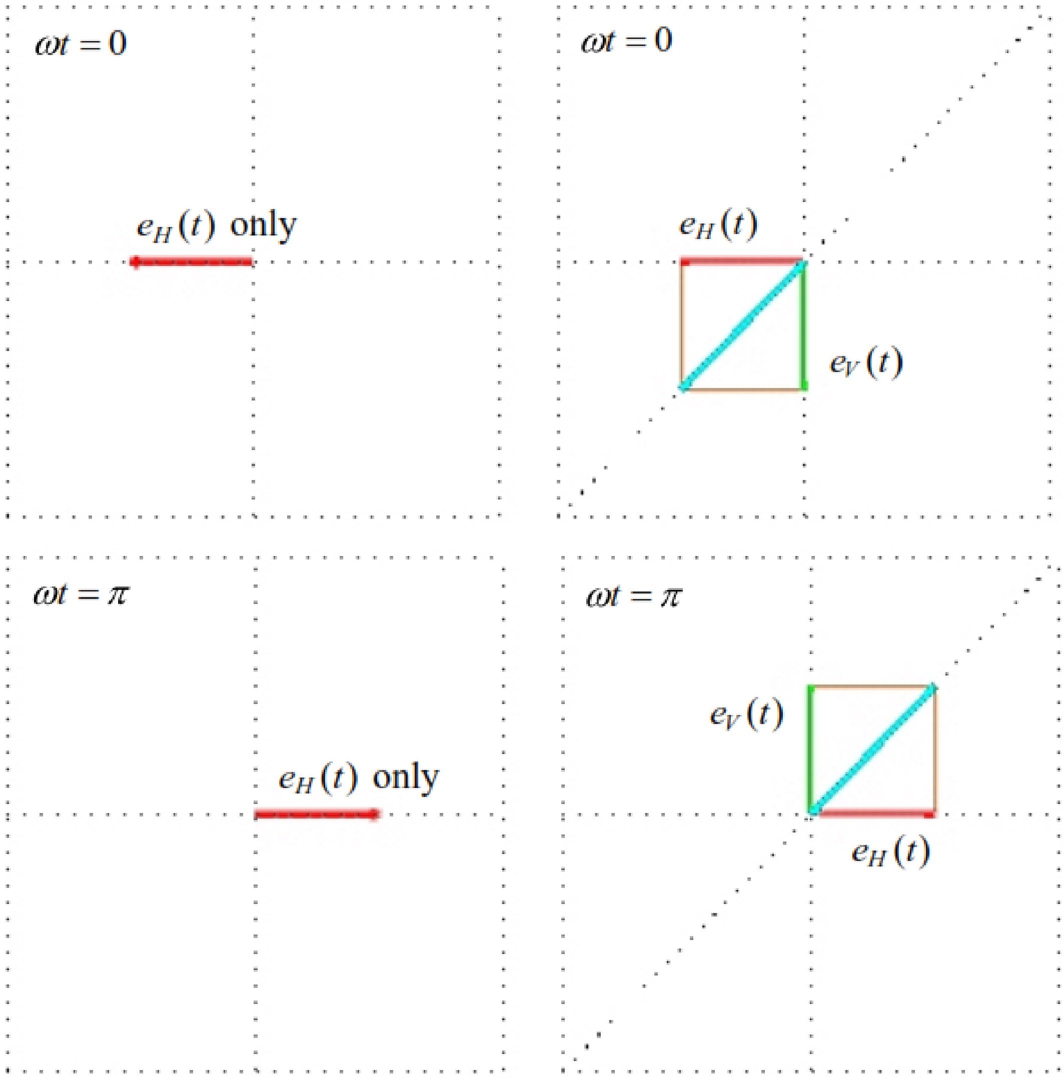
In antenna theory, we are concerned with the polarization of the field in the plane orthogonal to the direction of propagation—this is the plane defined by the vectors of the far field.

According to the shape of the trace, three types of polarization exist for harmonic fields: ***linear, circular*** and ***elliptical***. Any polarization can be represented by two orthogonal linear polarizations, (E_x, E_y), or (E_H, E_V), whose fields are out of phase by an angle of δ_L.



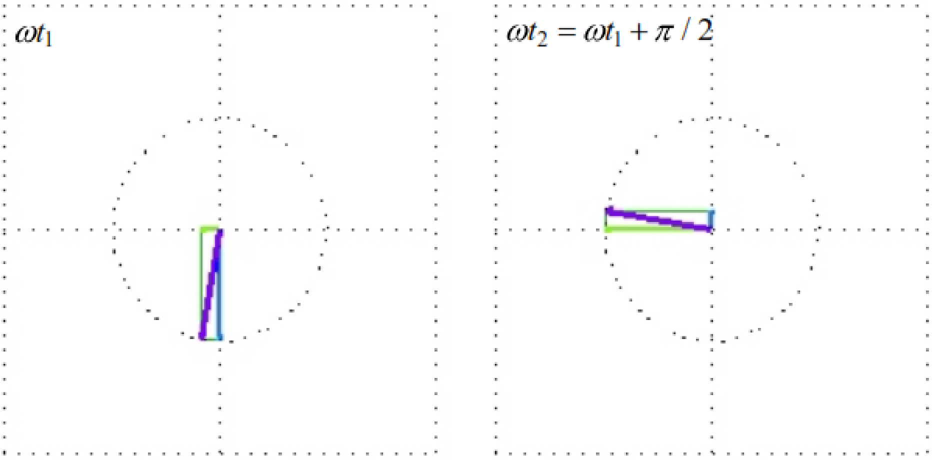
(a) linear polarization (b) circular polarization (c) elliptical polarization

❖ If δ_L=0 or $n\pi$, then linear polarization results.



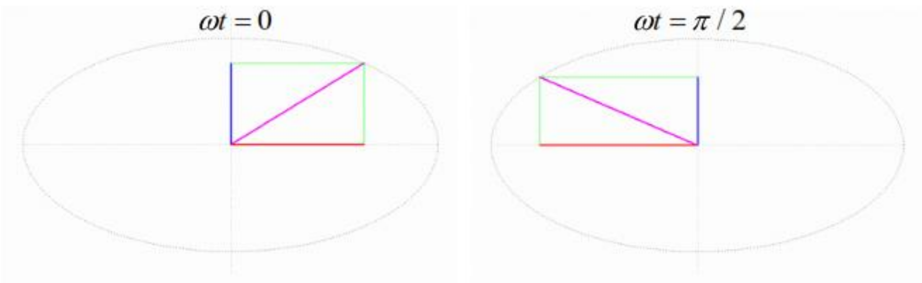
Animation: Linear Polarization, δ_L=0, $E_x = E_y$

❖ If $\delta_L = \pi / 2$ (90°) and $|E_x| = |E_y|$, then circular polarization results.



Animation: Clockwise Circular Rotation

❖ In the most general case, elliptical polarization is defined.



Animation: Counter-clockwise Elliptical Rotation

It is also true that any type of polarization can be represented by a right-hand circular and a left-hand circular polarizations (E_L, E_R). Next, we review the above statements and definitions, and introduce the new concept of polarization vector.

7.2 Field Polarization in Terms of Two Orthogonal Linearly Polarized Components

The polarization of any field can be represented by a set of two orthogonal linearly polarized fields. Assume that locally a far-field wave propagates along the z-axis. The far-zone field vectors have only transverse components. Then, the set of two orthogonal linearly polarized fields along the x-axis and along the y-axis, is sufficient to represent any TEM_z field. We use this arrangement to introduce the concept of polarization vector. The field (time-dependent or phasor vector) is decomposed into two orthogonal components:

That mean the electric field of a wave travelling in the z-direction (out of the page). And in general has x and y component and:

$$E_x = E_1 \sin(\omega t - \beta z)$$

$$E_y = E_2 \sin(\omega t - \beta z + \delta_L)$$

At a fixed position (assume z =0), above equations can be written as

$$E_x = E_1 \sin(\omega t)$$

$$E_y = E_2 \sin(\omega t + \delta_L)$$

$$E = E_1 \sin(\omega t)\ \hat{a}_x + E_2 \sin(\omega t + \delta_L)\ \hat{a}_y$$

Or

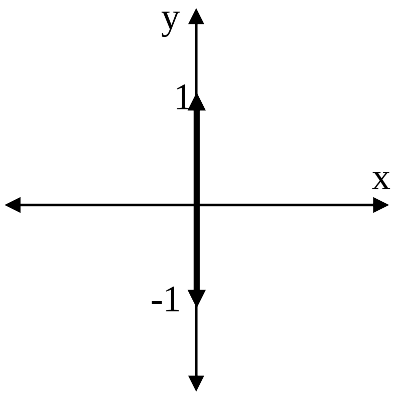
$$E = E_1\ \hat{a}_x + E_2 e^{j\delta_L}\ \hat{a}_y$$

where, δ_L is the relative phase between x and y component of electric field vector

Case 1:

$E_1 = 0, E_2 = 1, \delta_L = 0$

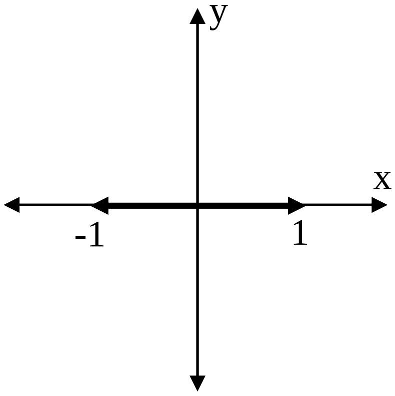
wt	E	wt	E
0	0	180	0
30	0.5	210	-0.5
45	0.707	240	-0.866
60	0.886	270	-1
90	1	300	-0.866
120	0.886	330	-0.5
150	0.5	360	0



This wave is said to be **Vertical Linearly Polarized** (in y-direction) as a function of time and position.

Case 2:

$E_1 = 1, E_2 = 0, \delta_L = 0$

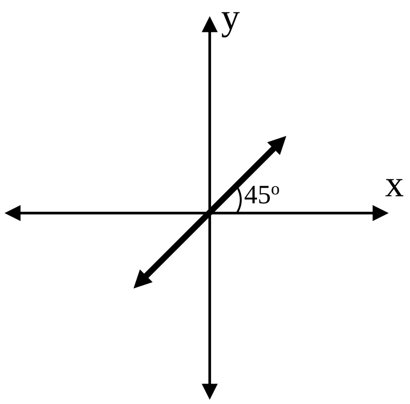


This wave is said to be **Horizontal Linearly Polarized** (in x-direction) as a function of time and position.

Case 3:-

$E_1 = 1, E_2 = 1, \delta_L = 0$

wt	E_x	E_y	$E_T = \sqrt{E_x^2 + E_y^2}$
0	0	0	0
30	0.5	0.5	0.707
45	0.707	0.707	1
60	0.886	0.886	1.224
90	1	1	1.414
120	0.886	0.886	1.224
150	0.5	0.5	0.707
180	0	0	0
210	-0.5	-0.5	0.707
240	-0.866	-0.866	1.224
270	-1	-1	1.414
300	-0.866	-0.866	1.224
330	-0.5	-0.5	0.707
360	0	0	0



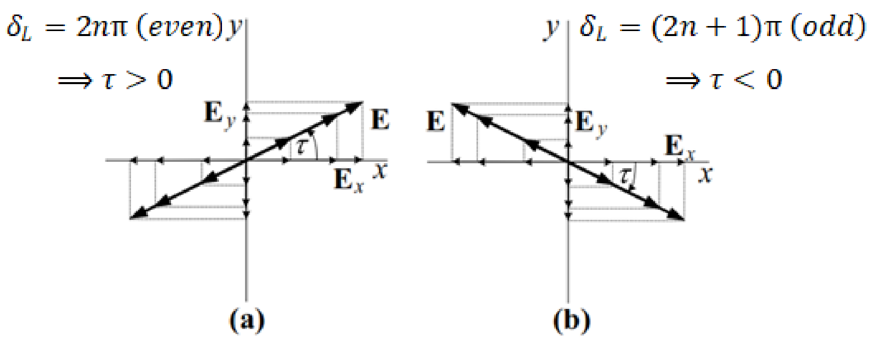
This wave is said to be **Linearly Polarized in the plane at angle 45°**

Summary:

Linear polarization: $\delta_L = n\pi$, $n = 0,1,2, \ldots$

$$E = E_1 \sin(\omega t)\ \hat{a}_x + E_2 \sin(\omega t \pm n\pi)\ \hat{a}_y$$

$$E = E_1\ \hat{a}_x \pm E_2\ \hat{a}_y$$



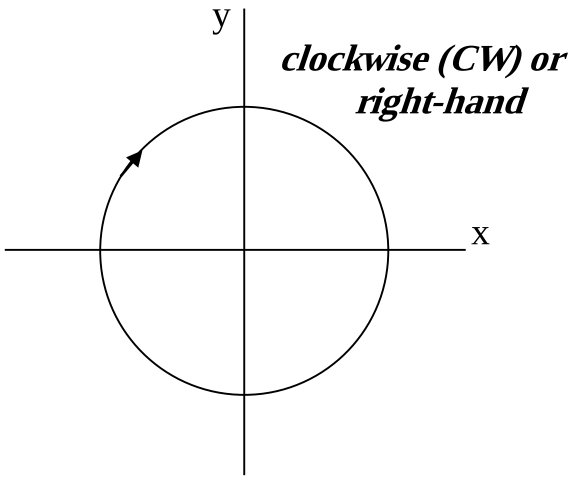
$$\tau = \pm \tan^{-1}\left(\frac{E_2}{E_1}\right)$$

where τ is tilt angle

Case 4:-

$E_1 = 1, E_2 = 1, \delta_L = 90^o$

wt	E_x	E_y	$E_T = \sqrt{E_x^2 + E_y^2}$
0	0	1	1
30	0.5	0.866	1
45	0.707	0.707	1
60	0.886	0.57	1
90	1	0	1
120	0.886	-0.5	1
135	0.707	-0.707	1
150	0.5	-0.866	1
180	0	-1	1
210	-0.5	-0.866	1
240	-0.866	-0.5	1
270	-1	0	1
300	-0.866	0.5	1
330	-0.5	00.866	1
360	0	1	1



This type of polarization is called circular polarization, in this type of polarization, the electric field E rotate as a function of time in a circular form. This type of polarization is occurred only when $E_1 = E_2$ and $\delta_L = 90^o$. circular polarization is a special type of the elliptical polarization. If $\delta_L = 90^o$ the electric field will rotate Clock wise (right hand) , while when $\delta_L = -90^o$ the electric field will rotate counter-Clock wise (left hand).

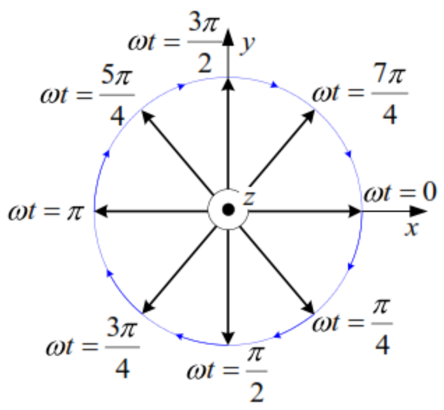
Summary:

Circular polarization: $E_1 = E_2 = E_m,\ \delta_L = \pm\left(\frac{\pi}{2} + 2n\pi\right), n = 0,1,2, \dots$

$$E = E_1 \sin(\omega t)\ \hat{a}_x + E_2 \sin(\omega t \pm \frac{\pi}{2} + 2n\pi)\ \hat{a}_y$$

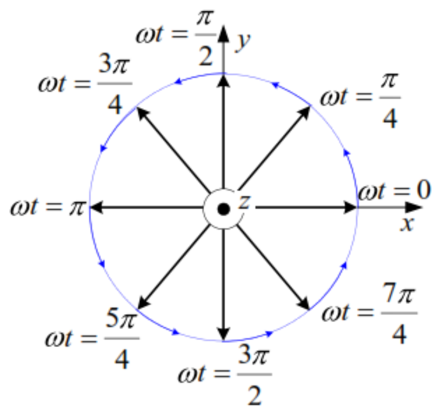
$$E = E_1\, \hat{a}_x \pm jE_2\, \hat{a}_y = E_m(\,\hat{a}_x \pm j\,\hat{a}_y)$$

$$\mathbf{E} = E_m(\hat{\mathbf{x}} + j\hat{\mathbf{y}})$$
$$\delta_L = +\frac{\pi}{2} + 2n\pi$$



If $+\hat{\mathbf{z}}$ is the direction of propagation: ***clockwise (CW)*** or ***right-hand*** polarization

$$\mathbf{E} = E_m(\hat{\mathbf{x}} - j\hat{\mathbf{y}})$$
$$\delta_L = -\frac{\pi}{2} - 2n\pi$$

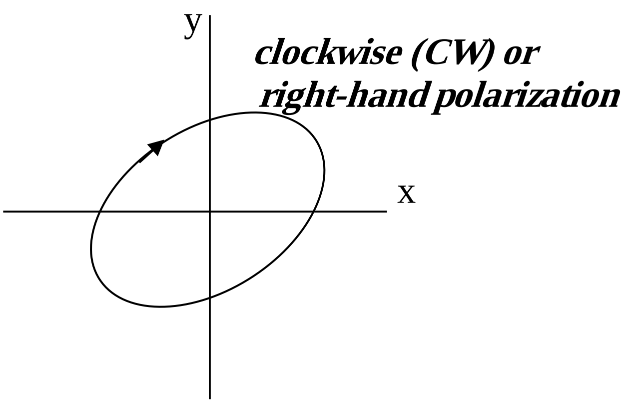


If $+\hat{\mathbf{z}}$ is the direction of propagation: ***counterclockwise (CCW)*** or ***left-hand*** polarization

Case 5:-

$$E_1 = 1, E_2 = 1, \delta_L = 45^o$$

wt	E_x	E_y	$E_T = \sqrt{E_x^2 + E_y^2}$
0	0	0.707	0.707
30	0.5	0.966	1.087
45	0.707	1	1.224
60	0.886	0.966	1.31
90	1	0.707	1.224
120	0.886	0.258	0.923
135	0.707	0	0.707
150	0.5	-0.258	0.562
180	0	0.707	0.707
210	-0.5	-0.966	1.087
240	-0.866	-0.966	1.31
270	-1	-0.707	1.224
300	-0.866	-0.258	0.923
330	-0.5	0.258	0.563
360	0	0.707	0.707

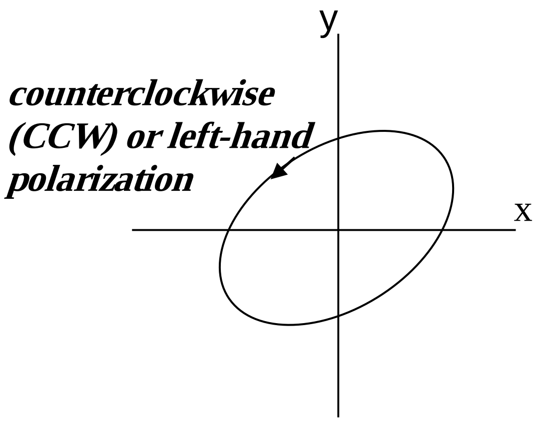


In this more general situation the wave is said to be ***clockwise (CW) or right-hand elliptically polarization.*** At E rotates as a function of time, the tip of the vector describing an ellipse.

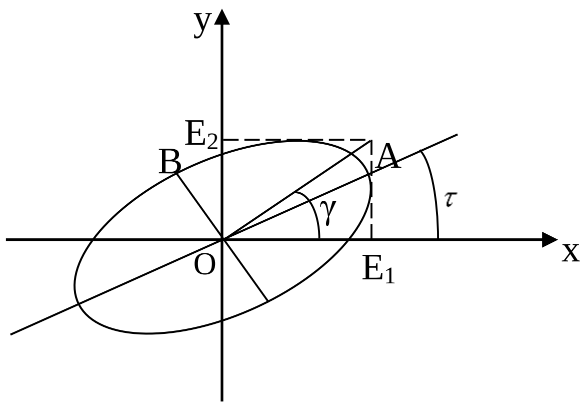
Case 6:-

$$E_1 = 1, E_2 = 1, \delta_L = -45^o$$

wt	E_x	E_y	$E_T = \sqrt{E_x^2 + E_y^2}$
0	0	-0.707	0.707
30	0.5	-0.258	0.563
45	0.707	0	0.707
60	0.886	0.966	1.31
90	1	0.707	1.306
120	0.886	0.966	1.31
135	0.707	1	1.306
150	0.5	0.966	1.087
180	0	0.707	0.707
210	-0.5	0.2588	0.563
240	-0.866	-0.258	0.923
270	-1	-0.707	1.224
300	-0.866	-0.966	1.31
330	-0.5	-0.966	1.086
360	0	-0.707	0.707



In this case the wave is said to be ***counterclockwise (CCW) or left-hand elliptically polarization***.



$$\gamma = tan^{-1}\frac{E_2}{E_1}$$

The parameters of the polarization ellipse are given below.

a) Major axis ($2 \times OA$)

$$OA = \sqrt{\frac{1}{2}\left[E_1^2 + E_2^2 + \sqrt{E_1^4 + E_2^4 + 2E_1^2E_2^2\cos(2\delta_L)}\right]}$$

b) Minor axis ($2 \times OB$)

$$OB = \sqrt{\frac{1}{2}\left[E_1^2 + E_2^2 - \sqrt{E_1^4 + E_2^4 + 2E_1^2E_2^2\cos(2\delta_L)}\right]}$$

c) Tilt angle

$$\tau = \frac{1}{2}tan^{-1}\left[\frac{2E_1E_2}{E_1^2 - E_2^2}cos\,\delta_L\right]$$

d) axial ratio

$$AR = \frac{OA}{OB} = \frac{major \quad axis}{minor \quad axis} \qquad 1 \le AR \le \infty$$

Notes:

The linear and circular polarizations as special cases of the elliptical polarization:

- if $\delta_L = \pm\left(\frac{\pi}{2} + 2n\pi\right)$ and $E_1 = E_2$, then OA=OB=E_1=E_2 ; the ellipse becomes a circle.
- if $\delta_L = n\pi$, then OB=0 and $\tau = \pm\tan^{-1}E_2/E_1$; the ellipse collapses into line.

7.3 Polarization vector

The ***polarization vector*** is the normalized phasor of the electric field vector.

$$\hat{\boldsymbol{\rho}}_L = \frac{E}{E_m} = \frac{E_1}{E_m}\hat{a}_x + \frac{E_2}{E_m}e^{j\delta_L}\,\hat{a}_y$$

$$E_m = \sqrt{E_1^2 + E_2^2}$$

The polarization vector takes the following specific forms:

- Linear polarization

$$\hat{\boldsymbol{\rho}}_L = \frac{E_1}{E_m}\hat{a}_x + \frac{E_2}{E_m}\hat{a}_y$$

$$E_m = \sqrt{E_1^2 + E_2^2}$$

Where E_1 and E_2 are real numbers.

- Circular polarization

$$\hat{\boldsymbol{\rho}}_L = \frac{1}{\sqrt{2}}(\hat{a}_x \pm j\hat{a}_y)$$

$$E_m = \sqrt{2}\cdot E_1 = \sqrt{2}\cdot E_2$$

7.4 Antenna polarization

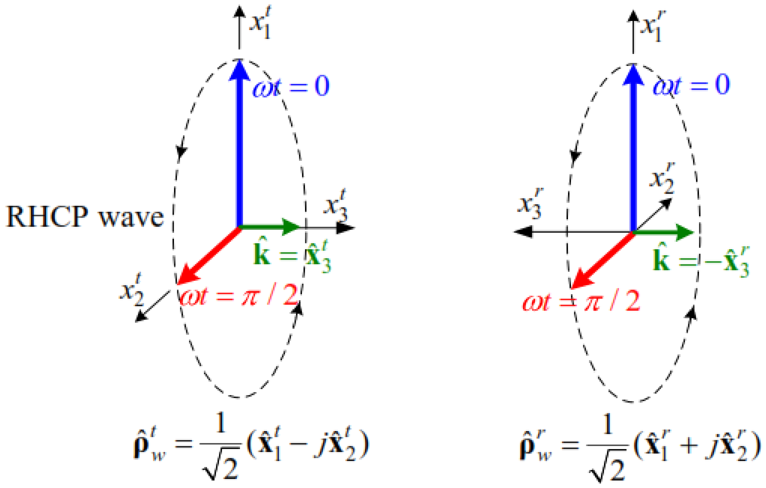
The ***polarization of a transmitting antenna*** is the polarization of its radiated wave in the far zone. ***The polarization of a receiving antenna*** is the polarization of a plane wave, incident from a given direction, and having given power flux density, which results in maximum available power at the antenna terminals.

The antenna polarization is defined by the polarization vector of the radiated (transmitted) wave. Notice that the polarization vector of a wave in the coordinate system of the ***transmitting antenna*** is represented by its ***complex conjugate*** in the coordinate system of the ***receiving antenna***.

$$\hat{\boldsymbol{\rho}}_w^r = (\hat{\boldsymbol{\rho}}_w^t)^*$$

This is illustrated in the figure below with a right-hand CP wave. Let the coordinate triplet $(x_1^t, x_2^t\,, x_3^t)$ represent the coordinate system of the transmitting antenna while $(x_1^r, x_2^r\,, x_3^r)$ represents that of the receiving antenna.

Since the transmitting and receiving antennas face each other, their coordinate systems are oriented so that $\hat{x}_3^t = -\hat{x}_3^r$. If we align the axes x_1^t and x_1^r, then $x_2^t = -x_2^r$ must hold so that $\hat{x}_3^t = -\hat{x}_3^r$. This changes the sign in the imaginary part of the wave polarization vector.



Bearing in mind the definitions of antenna polarization in transmitting and receiving modes, we conclude that the transmitting-mode polarization vector of an antenna is the conjugate of its receiving-mode polarization vector.

7.5 Polarization Loss Factor and Polarization Efficiency

Generally, the polarization of the receiving antenna is not the same as the polarization of the incident wave. This is called ***polarization mismatch***. The **P**olarization **L**oss **F**actor (PLF) characterizes the loss of EM power because of polarization mismatch:

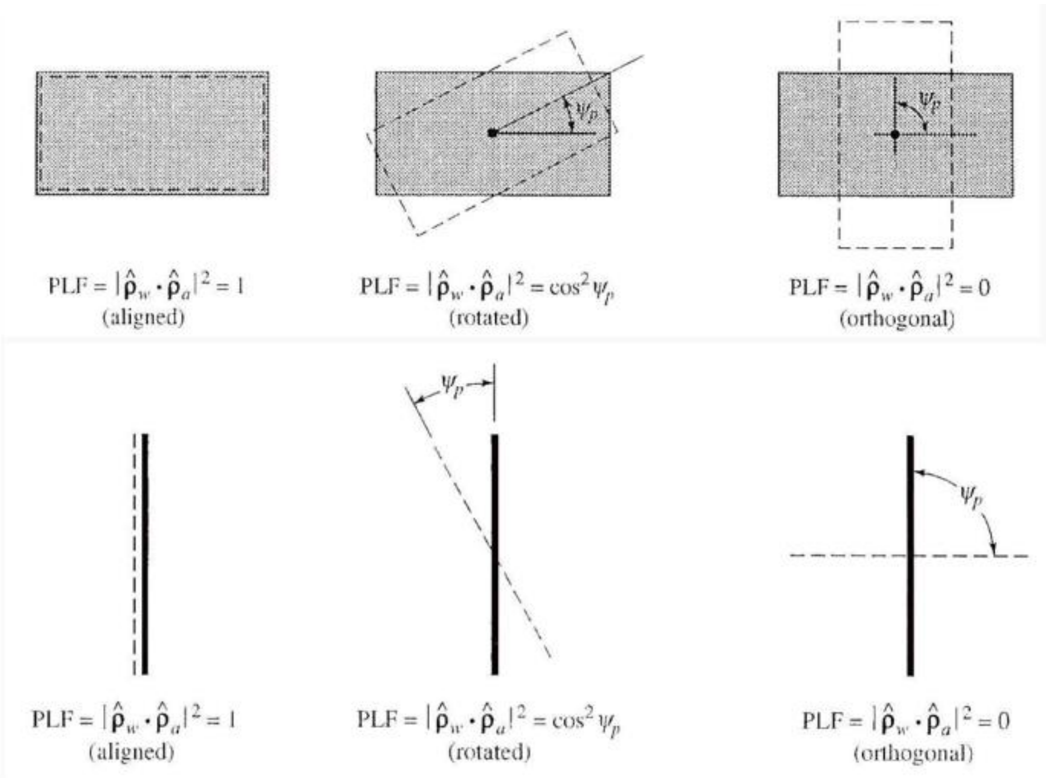
$$\text{PLF} = |\hat{\boldsymbol{\rho}}_i \cdot \hat{\boldsymbol{\rho}}_a|^2 = |\hat{\boldsymbol{\rho}}_w \cdot \hat{\boldsymbol{\rho}}_a|^2$$

The above definition is based on the representation of the incident field and the antenna polarization by their polarization vectors. If the incident field is

$$E^i = E_m^i \hat{\boldsymbol{\rho}}_i = E_m^i \hat{\boldsymbol{\rho}}_w$$

then the field of the same magnitude that would produce maximum received power at the antenna terminals is

$$E_a = E_m^i \hat{\boldsymbol{\rho}}_a$$

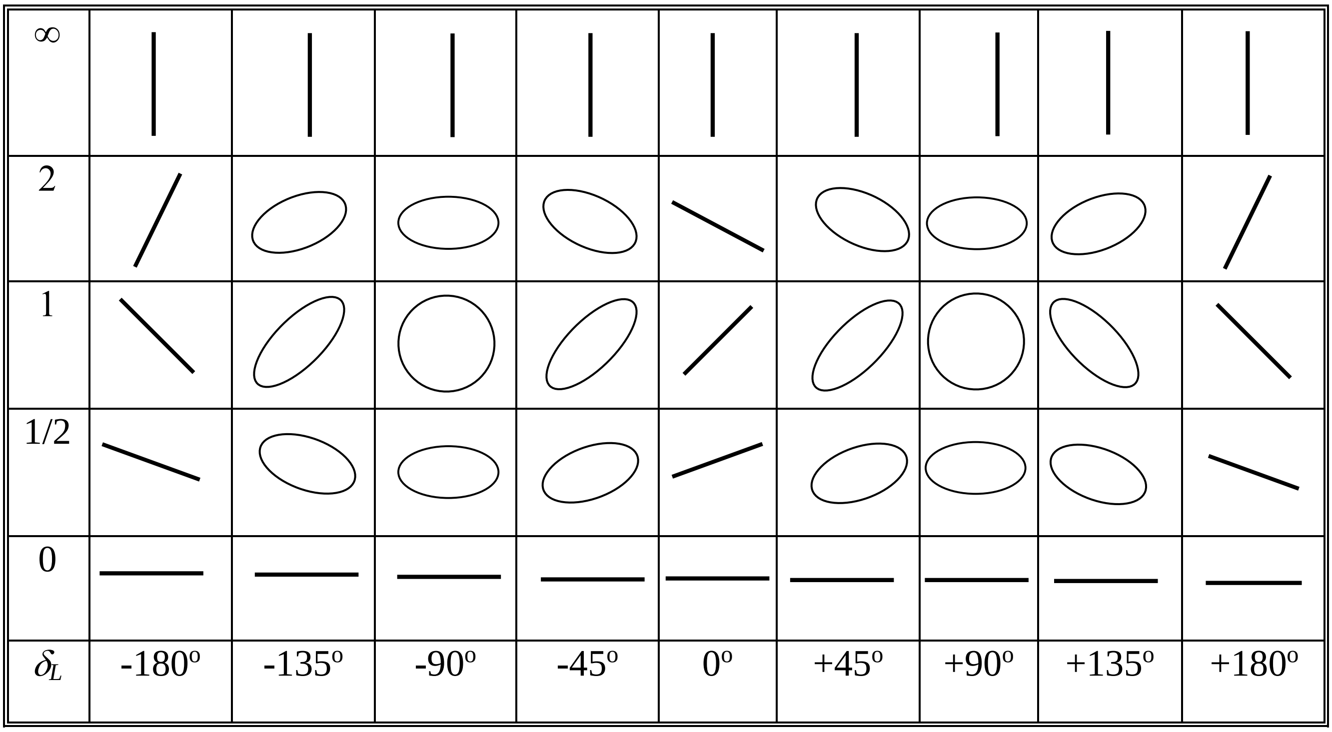


If the antenna is polarization matched, then PLF =1, and there is no polarization power loss. If PLF = 0, then the antenna is incapable of receiving the signal.

$$0 \le \text{PLF} \le 1$$

The ***polarization efficiency*** means the same as the PLF

The polarization of electrical field as a function of E_2/E_1 and phase difference angle δ_L



Example:

A wave radiated by an antenna is traveling in the outward radial direction along the +z axis. Its radiated field in the far zone region is described by its spherical components, and its polarization is right-hand (clockwise) circularly polarized. This radiated field impinging upon a receiving antenna whose polarization is also right-hand (clockwise) circularly polarized and whose polarization unit vector is represented by

$$E_a = E(r,\theta,\phi)\left(\hat{a}_\theta - j\hat{a}_\phi\right)$$

Determine the polarization loss factor (PLF)

Solution: -

$$\hat{\rho}_w = \frac{1}{\sqrt{2}}\left(\hat{a}_\theta + j\hat{a}_\phi\right), \quad \hat{\rho}_a = \frac{1}{\sqrt{2}}\left(\hat{a}_\theta - j\hat{a}_\phi\right)$$

The polarization Loss factor= $PLF = \left|\hat{\rho}_\omega \cdot \hat{\rho}_a\right|^2 = \frac{1}{4}|1+1|^2 = 1 = 0 \quad dB$