Friis Transmission Equation and Radar Range Equation

8.1 Friis Transmission Equation

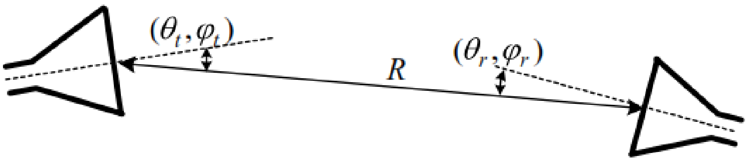
Friis transmission equation is essential in the analysis and design of wireless communication systems. It relates the power fed to the transmitting antenna and the power received by the receiving antenna when the two antennas are separated by a sufficiently large distance ($R >> 2D_{max}^2/\lambda$), i.e., they are in each other's far zones.

A transmitting antenna produces power density $W_t(\theta_t, \varphi_t)$ in the direction (θ_t, φ_t). This power density depends on the transmitting antenna gain in the given direction $G(\theta_t, \varphi_t)$, on the power of the transmitter P_t fed to it, and on the distance R between the antenna and the observation point as

$$W_t(\theta_t, \varphi_t) = \frac{P_t}{4\pi R^2} \cdot G_t(\theta_t, \varphi_t) = \frac{P_t}{4\pi R^2} \cdot e_t \cdot D_t(\theta_t, \varphi_t)$$

Here, e_t denotes the radiation efficiency of the transmitting antenna and D_t is its directivity. The power P_r at the terminals of the receiving antenna can be expressed via its effective area A_{er} and W_t:

$$P_r = A_{er} \cdot W_t$$



To include polarization and heat losses in the receiving antenna, we add the radiation efficiency of the receiving antenna e_r and the PLF:

$$P_r = e_r \cdot PLF \cdot A_{er} \cdot W_t = A_{er} \cdot W_t \cdot e_r \cdot |\hat{\boldsymbol{\rho}}_t \cdot \hat{\boldsymbol{\rho}}_r|^2$$

$$\Rightarrow P_r = \underbrace{D_r(\theta_r, \varphi_r) \cdot \frac{\lambda^2}{4\pi}}_{A_{er}} \cdot W_t \cdot e_r \cdot |\hat{\boldsymbol{\rho}}_t \cdot \hat{\boldsymbol{\rho}}_r|^2$$

Here, D_r is the directivity of the receiving antenna. The signal is incident upon the receiving antenna from a direction (θ_r, φ_r), which is defined in the coordinate system of the receiving antenna:

$$\Rightarrow P_r = D_r(\theta_r, \varphi_r) \cdot \frac{\lambda^2}{4\pi} \cdot \underbrace{\frac{P_t}{4\pi R^2} \cdot e_t \cdot D_t(\theta_t, \varphi_t)}_{W_t} \cdot e_r \cdot |\hat{\boldsymbol{\rho}}_t \cdot \hat{\boldsymbol{\rho}}_r|^2$$

The ratio of the received to the transmitted power is obtained as:

$$\frac{P_r}{P_t} = e_t \cdot e_r \cdot |\hat{\boldsymbol{\rho}}_t \cdot \hat{\boldsymbol{\rho}}_r|^2 \cdot \left(\frac{\lambda}{4\pi R}\right)^2 \cdot D_t(\theta_t, \varphi_t) \cdot D_r(\theta_r, \varphi_r)$$

If the impedance-mismatch loss factor is included in both the receiving and the transmitting antenna systems, the above ratio becomes:

$$\frac{P_r}{P_t} = (1 - |\Gamma_t|^2)(1 - |\Gamma_r|^2) e_t e_r |\hat{\boldsymbol{\rho}}_t \cdot \hat{\boldsymbol{\rho}}_r|^2 \left(\frac{\lambda}{4\pi R}\right)^2 D_t(\theta_t, \varphi_t) D_r(\theta_r, \varphi_r)$$

The above equations are variations of Friis' transmission equation, which is well known in the theory of EM wave propagation and is widely used in the design of wireless systems as well as the estimation of antenna radiation efficiency (when the antenna gain is known).

For the case of impedance-matched and polarization-matched transmitting and receiving antennas, Friis equation reduces to

$$\frac{P_r}{P_t} = e_t e_r \left(\frac{\lambda}{4\pi R}\right)^2 D_t(\theta_t, \varphi_t) D_r(\theta_r, \varphi_r)$$

The factor $(\lambda/4\pi R)^2$ is called the ***free-space loss factor***. It reflects the decrease in the power density due to the spherical spread of the EM wave. Notice that the free-space loss factor is smaller for shorter wavelengths (i.e., for higher frequencies). This is not a propagation effect but is rather due to the increased effective apertures of the antennas for shorter wavelengths.

8.2 Maximum Range of a Wireless Link

Friis transmission equation is frequently used to calculate the ***maximum range*** at which a wireless link can operate. For that, we need to know the nominal power of the transmitter P_t, all the parameters of the transmitting and receiving antenna systems (such as polarization, gain, losses, impedance mismatch), and the minimum power at which the receiver can operate reliably $P_{r\,\min}$. Then,

$$R_{max}^2 = (1 - |\Gamma_t|^2)(1 - |\Gamma_r|^2)e_t e_r |\hat{\boldsymbol{\rho}}_t \cdot \hat{\boldsymbol{\rho}}_r|^2 \left(\frac{\lambda}{4\pi}\right)^2 \left(\frac{P_t}{P_{r\,\min}}\right) D_t(\theta_t, \varphi_t) D_r(\theta_r, \varphi_r)$$

$$R_{max} = \sqrt{(1 - |\Gamma_t|^2)(1 - |\Gamma_r|^2)e_t e_r |\hat{\boldsymbol{\rho}}_t \cdot \hat{\boldsymbol{\rho}}_r|^2 \left(\frac{\lambda}{4\pi}\right)^2 \left(\frac{P_t}{P_{r\,\min}}\right) D_t(\theta_t, \varphi_t) D_r(\theta_r, \varphi_r)}$$

8.3 Radar Cross-Section (RCS) or Echo Area

The RCS is a far-field characteristic of radar targets which create an echo far field by scattering (reflecting) the radar EM wave. The RCS of a target σ is the equivalent area capturing that amount of power, which, when scattered isotropically, produces at the receiver power density equal to that scattered by the target itself:

$$\sigma = \lim_{R\to\infty}\left[4\pi R^2 \frac{W_s}{W_i}\right] = \lim_{R\to\infty}\left[4\pi R^2 \frac{|E_s|^2}{|E_i|^2}\right], m^2$$

Here,

R: is the distance from the target, m;

W_i: is the incident power density, W/m^2;

$W_{s:}$ is the scattered power density at the receiver, W/m^2.

We note that in general the RCS has little in common with any of the cross-sections of the actual scatterer. However, it is representative of the reflection properties of the target. It depends very much on the angle of incidence, on the angle of observation, on the shape of the scatterer, on the EM properties of the matter that it is built of, and on the wavelength. The RCS of targets is similar to the concept of effective aperture of antennas.

Large RCSs result from large metal content in the structure of the object (e.g., trucks and jumbo jet airliners have large RCS, $\sigma > 100$m). The RCS increases also due to sharp metallic or dielectric edges and corners. The reduction of RCS is desired for stealth military aircraft meant to be invisible to radars. This is achieved by careful shaping and coating (with special materials) of the outer surface of the airplane. The materials are mostly designed to absorb electromagnetic waves at the radar frequencies (usually X band).

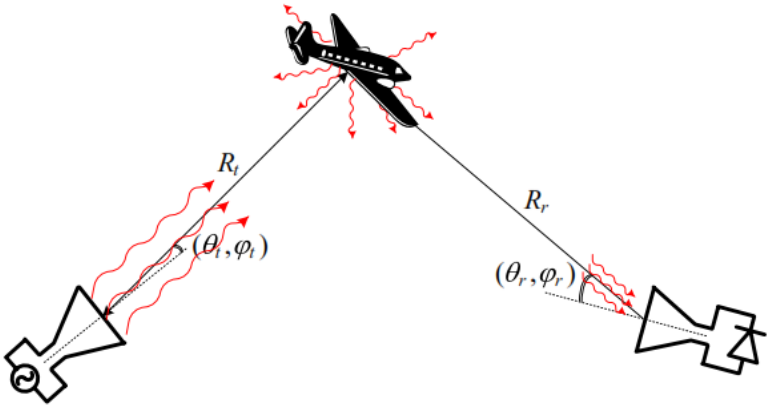
Layered structures can also cancel the backscatter in a particular bandwidth. Shaping aims mostly at directing the backscattered wave at a direction different from the direction of incidence. Thus, in the case of a monostatic radar system, the scattered wave is directed away from the receiver. The stealth aircraft has RCS smaller than 10^{-4} m^2, which makes it comparable or smaller that the RCS of a penny.

8.4 Radar Range Equation

The radar range equation (RRE) gives the ratio of the transmitted power (fed to the transmitting antenna) to the received power, after it has been scattered (re-radiated) by a target of cross-section σ.

In the general radar scattering problem, there is a transmitting and a receiving antenna, and they may be located at different positions as it is shown in the figure below. This is called ***bistatic scattering***.

Often, one antenna is used to transmit an EM pulse and to receive the echo from the target. This case is referred to as ***monostatic scattering*** or ***backscattering***. Bear in mind that the RCS of a target may considerably differ as the location of the transmitting and receiving antennas change.



Assume the power density of the transmitted wave at the target location is:

$$W_t = \frac{P_t}{4\pi R_t^2}\cdot G_t(\theta_t,\varphi_t) = \frac{P_t}{4\pi R_t^2}\cdot e_t\cdot D_t(\theta_t,\varphi_t),\quad W/m^2$$

The target is represented by its RCS σ, which is used to calculate the captured $P_c = \sigma\cdot W_t$ (W), which when scattered isotropically gives the power density at the receiving antenna that is actually due to the target. The density of the re-radiated (scattered) power at the receiving antenna is

$$W_r = \frac{P_c}{4\pi R_r^2} = \frac{\sigma\cdot W_t}{4\pi R_r^2} = \sigma\cdot e_t\cdot\frac{P_t\cdot D_t(\theta_t,\varphi_t)}{(4\pi R_t R_r)^2}$$

The power transferred to the receiver is

$$P_r = e_r \cdot A_{er} \cdot W_r = e_r \cdot \left(\frac{\lambda^2}{4\pi}\right) D_r(\theta_r, \varphi_r) \cdot \sigma \cdot e_t \cdot \frac{P_t \cdot D_t(\theta_t, \varphi_t)}{(4\pi R_t R_r)^2}$$

Re-arranging and including impedance mismatch losses as well as polarization losses, yields the complete radar range equation:

$$\frac{P_r}{P_t} = e_t e_r (1 - |\Gamma_t|^2)(1 - |\Gamma_r|^2) |\hat{\boldsymbol{\rho}}_t \cdot \hat{\boldsymbol{\rho}}_r|^2 \sigma \left(\frac{\lambda}{4\pi R_t R_r}\right)^2 \frac{D_t(\theta_t, \varphi_t) D_r(\theta_r, \varphi_r)}{4\pi}$$

For polarization matched loss-free antennas aligned for maximum directional radiation and reception

$$\frac{P_r}{P_t} = \sigma \left(\frac{\lambda}{4\pi R_t R_r}\right)^2 \frac{D_t(\theta_t, \varphi_t) D_r(\theta_r, \varphi_r)}{4\pi}$$

The radar range equation is often used to calculate the ***maximum range of a radar system***. As in the case of Friis transmission equation, we need to know all parameters of both the transmitting and the receiving antennas, as well as the minimum received power at which the receiver operates reliably. Then,

$$(R_t R_r)^2_{max} = e_t e_r (1 - |\Gamma_t|^2)(1 - |\Gamma_r|^2) |\hat{\boldsymbol{\rho}}_t \cdot \hat{\boldsymbol{\rho}}_r|^2 \left(\frac{P_t}{P_{r\,\min}}\right) \sigma \left(\frac{\lambda}{4\pi}\right)^2 \frac{D_t(\theta_t, \varphi_t) D_r(\theta_r, \varphi_r)}{4\pi}$$

Example:

ATS-6 Satellite, 20GHz transmitter on board has 2w of power into 37dB gain, 45.7cm diameter parabolic dish antenna. The 1.22 diameter parabolic ground station antenna at the Virginia earth terminal has 45.8dB gain. The distance from the satellite to the earth station is 36, 941.031km. Find P_r.

Solution

$$\lambda = \frac{c}{f} = \frac{3*10^8}{20*10^9} = 0.015m$$

$$D_t = 37dB = 10^{3.7} = 5011.87$$

$$D_r = 45.8dB = 10^{4.58} = 38018.9$$

$$P_r = P_t \frac{D_t D_r \lambda^2}{(4\pi R)^2} = 2\frac{5011.87 * 38018.9 * 0.015^2}{(4\pi * 36941031)^2} = 3.98 * 10^{-10} \quad \text{mW}$$

Example:

In an experiment to determine the radar cross-section of Tomahawk cruise missile, a 1000w, 300MHz signal was transmitted toward the target, and the received power was measured to be 0.1425 mW. The same antenna, whose gain was 75, was used for both transmitting and receiving. the distance between the antenna and missile was 500m. what is the radar cross section of the cruise missile?

Solution:

$$P_t = 1000W, f = 300MHz, P_r = 0.1425mW, G_t = G_r = 75, R = 500m$$

$$\lambda = \frac{c}{f} = \frac{3*10^8}{3*10^6} = 1 \quad m$$

$$\sigma = \frac{P_r (4\pi)^3 R^4}{P_t \lambda^2 G^2} = \frac{(0.1425*10^{-3})(4\pi)^3 (500)^4}{(1000)(1)^2 (75)^2} = 3141.96 \quad m^2$$

Example:

a CW circularly polarized uniform plane wave is traveling in +z direction. Find the polarization loss factor (dimensionless and in dB) assuming the receiving antenna (in its transmitting mode) is

a) CW circularly polarized.

b) CCW circularly polarized.

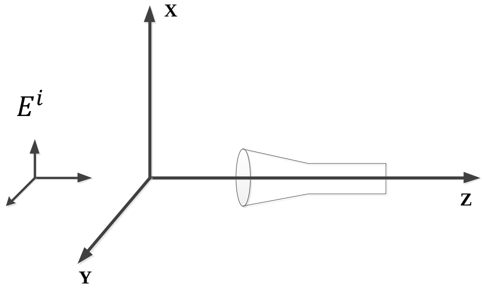
Solution:

Since the incident wave is CW therefore; $E^i = E_m(\,\hat{a}_x + j\,\hat{a}_y)$

Then polarization vector is $\hat{\boldsymbol{\rho}}_i = \frac{1}{\sqrt{2}}(\hat{a}_x + j\hat{a}_y)$

a) $E^a = E_m(\,\hat{a}_x - j\,\hat{a}_y)$ and $\hat{\boldsymbol{\rho}}_a = \frac{1}{\sqrt{2}}(\hat{a}_x - j\hat{a}_y)$

$$\text{PLF} = |\hat{\boldsymbol{\rho}}_i \cdot \hat{\boldsymbol{\rho}}_a|^2 = \left|\frac{1}{\sqrt{2}}(\hat{a}_x + j\hat{a}_y)\cdot\frac{1}{\sqrt{2}}(\hat{a}_x - j\hat{a}_y)\right|^2 = \left(\frac{1-j^2}{2}\right)^2 = 1 = 0\text{d}B$$



b) $E^a = E_m(\,\hat{a}_x + j\,\hat{a}_y)$ and $\hat{\boldsymbol{\rho}}_a = \frac{1}{\sqrt{2}}(\hat{a}_x + j\hat{a}_y)$

$$\text{PLF} = |\hat{\boldsymbol{\rho}}_i \cdot \hat{\boldsymbol{\rho}}_a|^2 = \left|\frac{1}{\sqrt{2}}(\hat{a}_x + j\hat{a}_y)\cdot\frac{1}{\sqrt{2}}(\hat{a}_x + j\hat{a}_y)\right|^2 = \left(\frac{1+j^2}{2}\right)^2 = 0 = -\infty\text{d}B$$