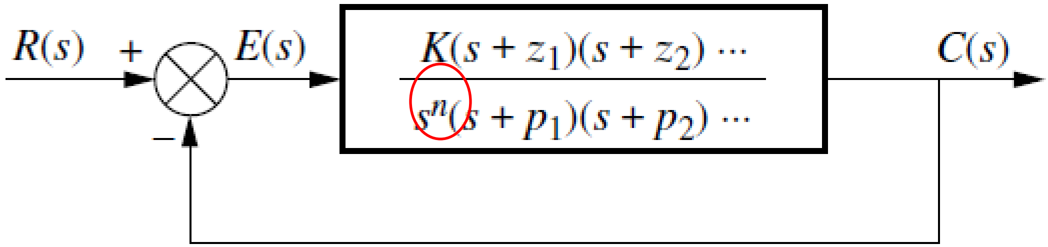
Order of System

- The order of control system is defined as the highest power of s present in denominator of closed loop transfer function G(s) of unity feedback system.

$$\frac{b_m s^m + b_{m-1} s^{m-1} + \cdots + b_0}{(a_n s^n + a_{n-1} s^{n-1} + \cdots + a_0)}$$

System Type



We define system type to be the value of n in the denominator, namely, the number of pure integrations in the forward path.

Example1: Determine order and type of given system

$$TF = G(s) = \frac{s\,(s+2)}{s^4 + 7s^3 + 10s^2 + 5s + 5}$$

Answer: The highest power of equation in denominator of given transfer function is '4'.

Hence the order of given system is

"Forth Order system".

"Type 0"

Example 2 : Determine order and type of given system

Solution:

$$G(s) = \frac{(s+5)(s+2)}{s(s+3)(s+4)}$$

To obtain highest power of denominator, Simplify denominator polynomial.

$$s(s+3)(s+4) = 0$$

$$s(s^2+7s+12) = 0$$

$$s^3+7s^2+12s=0$$

"Third Order system".

"Type 1"

Example 3 : Determine order of and type given

$$G(s) = \frac{K(s+5)}{s^3(7s^2+12s+5)}$$

Solution: To obtain highest power of denominator, Simplify denominator polynomial.

$$s^3(7s^2+12s+5)=0$$

$$7s^5+12s^4+5s^3=0$$

"Fifth Order system".

"Type 3".

What is Time Response Behavior?

- Since time is used as an independent variable in most control system, it is usually of interest to evaluate the output response with respect to time, or simply, the time response.

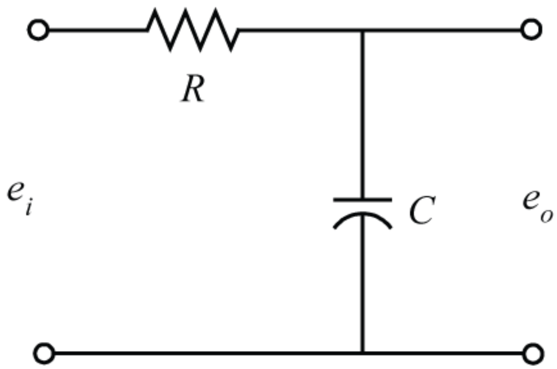
Time response of first-order systems

- First order systems are described by first order differential equations.

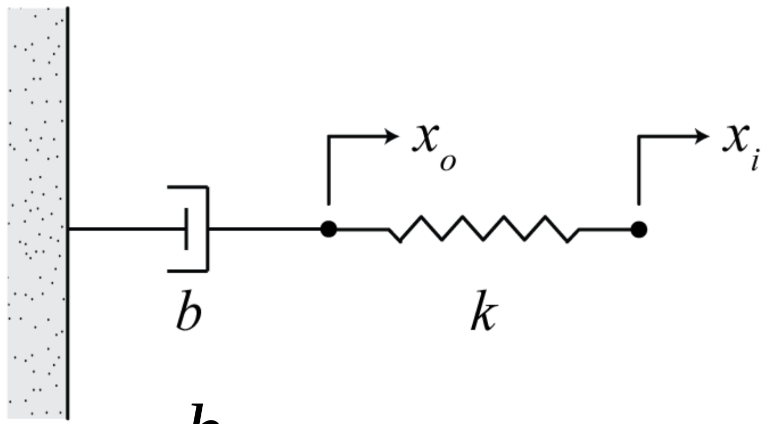
Why learn about first order systems?

- First-order systems are the simplest systems, and they make a good place to begin a study of system dynamics.
- First-order system concepts form the foundation for understanding more complex systems.

Some example systems



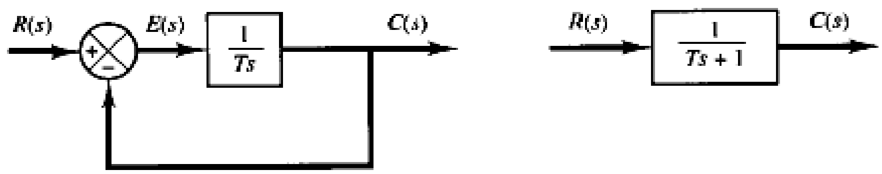
$$RC\dot{e}_o + e_o = e_i$$



$$\frac{b}{k}\dot{x}_o + x_o = x_i$$

Time Response of First Order Control Systems

When the maximum power of s in the denominator of a transfer function is one, the transfer function represents a first order control system. Commonly, the first order control system can be represented as



(a) (b)

$$\frac{C(s)}{R(s)} = \frac{1}{sT+1}$$

Time Response for Step Function

Now a unit step input is given to the system, then let us analyze the expression of output

$$\frac{C(s)}{R(s)} = \frac{1}{sT+1}$$

substituting $R(s) = 1/s$ into Equation

$$C(s) = \frac{1}{Ts + 1}\frac{1}{s}$$

Expanding $C(s)$ into partial fractions gives

$$C(s) = \frac{1}{s} - \frac{T}{Ts + 1} = \frac{1}{s} - \frac{1}{s + (1/T)}$$

Taking the inverse Laplace transform

$$c(t) = 1 - e^{-t/T}, \qquad \text{for } t \geq 0$$

It is seen from the error equation that if the time approaching to infinity, the output signal reaches exponentially to the steady-state value of one unit. As the output is approaching towards input exponentially, the steady-state error is zero when time approaches to infinity.

$$Error\ e(t) = r(t) - c(t)$$

$$\therefore Steady\ State\ Error$$

$$= \lim_{t\to\infty}\left[1 - \left(1 - e^{-t/T}\right)\right] = \lim_{t\to\infty} e^{-t/T} = 0$$

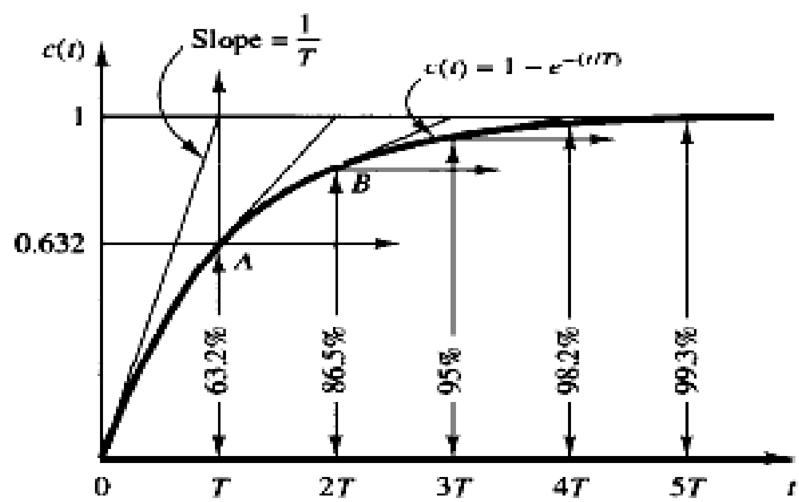
Let us put t = T in the output equation and then we get,

$$c(T) = 1 - e^{-T/T} = 1 - e^{-1} = 1 - 0.368 = 0.632$$

This T is defined as the time constant of the response and the time constant of a response signal is that time for which the signal reaches to its 63.2% of its final value.

Now if we put t = 4T in the above output response equation, then we get,

$$c(T) = 1 - e^{-4T/T} = 1 - e^{-4} = 1 - 0.018 = 0.982$$



When actual value of response, reaches to the 98% of the desired value, then the signal is said to be reached to its steady-state condition. This required time for reaching the signal to 98% of its desired value is known as setting time and naturally <u>setting time is four times of the time constant of the response</u>. The condition of response before setting time is known as transient condition and condition of the response after setting time is known as steady-state condition. From this explanation it is clear that if the time constant of the system is smaller, the response of the system reaches to its steady-state condition faster.

<u>Time Response for Ramp Function</u>

Now a unit ramp input is given to the system, then let us analyze the expression of output

unit-ramp function is $1/s^2$, we obtain the output of the system

$$\frac{C(s)}{R(s)} = \frac{1}{sT+1}$$

$$C(s) = \frac{1}{Ts+1}\frac{1}{s^2}$$

Expanding $C(s)$ into partial fractions gives

$$C(s) = \frac{1}{s^2} - \frac{T}{s} + \frac{T^2}{Ts+1}$$

Taking the inverse Laplace transform

$$c(t) = t - T + Te^{-t/T}, \qquad \text{for } t \geq 0$$

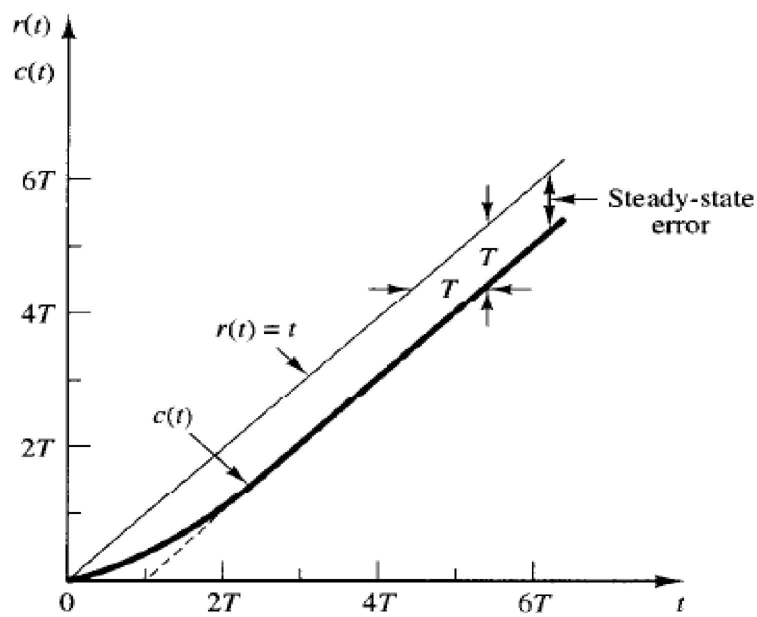
$$Error\ e(t) = r(t) - c(t)$$

$$\therefore Steady\ State\ Error$$

$$= \lim_{t\to\infty} \left[t - \left(t - T + Te^{-t/T}\right)\right] = \lim_{t\to\infty} \left(T - Te^{-t/T}\right)$$

$$= \lim_{t\to\infty} T\left(1 - e^{-t/T}\right) = T$$

In this case during steady-state condition, the output signal lags behind input signal by a time equal to the time constant of the system. If the time constant of the system is smaller, the positional error of the response becomes lesser.



For the unit ramp input, $R(s) = \frac{1}{s^2}$. Therefore,

$$C(s) = \frac{1}{s^2} \cdot \frac{1}{s\tau + 1}$$

$$= \frac{A}{s^2} + \frac{B}{s} + \frac{C}{s\tau + 1} \text{ (say)}$$

Then[1],

$$A = \left[s^2 C(s)\right]_{s=0} = \left[\frac{1}{s\tau + 1}\right]_{s=0} = 1$$

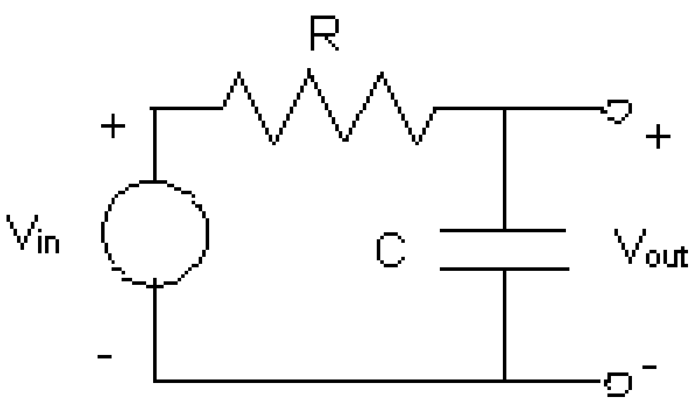
$$B = \left[\frac{d}{ds}\left\{s^2 C(s)\right\}\right]_{s=0} = \left[\frac{d}{ds}\left(\frac{1}{s\tau + 1}\right)\right]_{s=0} = -\left.\frac{\tau}{(s\tau + 1)^2}\right|_{s=0} = -\tau$$

$$C = \left[(s\tau + 1)C(s)\right]_{s=-\frac{1}{\tau}} = \left.\frac{1}{s^2}\right|_{s=-\frac{1}{\tau}} = \tau^2$$

Thus,

$$C(s) = \frac{1}{s^2} - \frac{\tau}{s} + \frac{\tau^2}{s\tau + 1}$$

A Resistor-Capacitor Circuit



Differential equation:

$$v_i(t) = R * i(t) + v_c(t)$$

$$i(t) = C\frac{dv_c(t)}{dt} \;, v_o(t) = v_c(t)$$

$$v_i(t) = R * C\frac{dv_o(t)}{dt} + v_o(t)$$

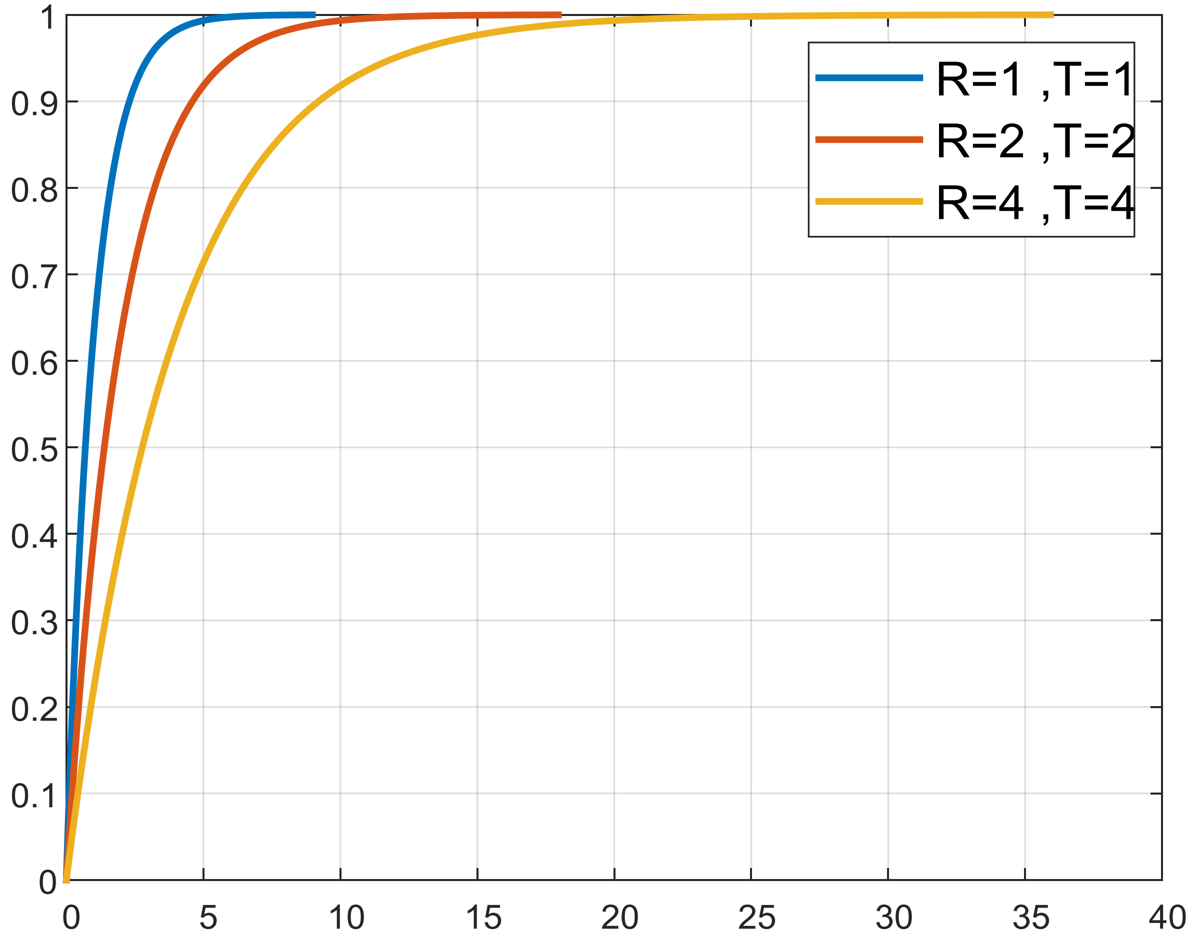
Taking Laplace transform

$$V_i(s) = RCsV_o(s) + V_o(s) = (1 + RCs)V_o(s)$$

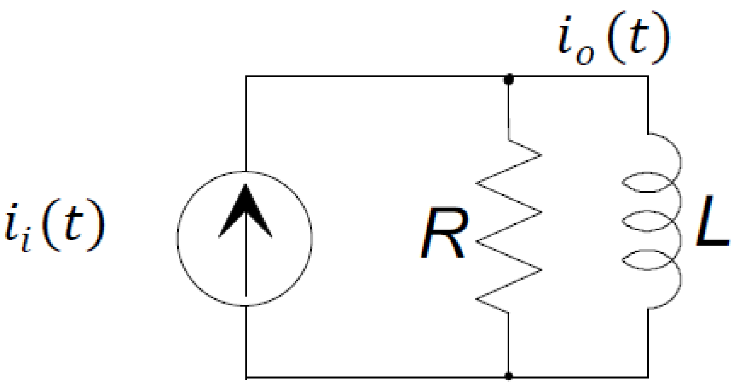
$$\frac{V_o(s)}{V_i(s)} = \frac{1}{1+RCs} \qquad T=RC$$

$$\frac{V_o(s)}{V_i(s)} = \frac{1}{1+RCs}$$

C=1 F ,T=RC



A Resistor Inductance Circuit



Differential equation:

$$i_i(t) = i_R(t) + i_L(t)$$

$$i_R(t) = \frac{v_L(t)}{R} \;, i_o(t) = i_L(t)$$

$$v_L(t) = L\frac{di_L(t)}{dt} = L\frac{di_o(t)}{dt}$$

$$i_i(t) = i_R(t) + i_L(t) = \frac{L}{R}\frac{di_o(t)}{dt} + i_o(t)$$

Taking Laplace transform

$$I_i(s) = \frac{L}{R}sI_o(s) + I_o(s) = (1 + \frac{L}{R}s)I_o(s)$$

$$\frac{I_o(s)}{I_i(s)} = \frac{1}{1+\frac{L}{R}s}$$ T=L/R

$$\frac{I_o(s)}{I_i(s)} = \frac{1}{1+\frac{L}{R}s}$$

L=1 H T=L/R