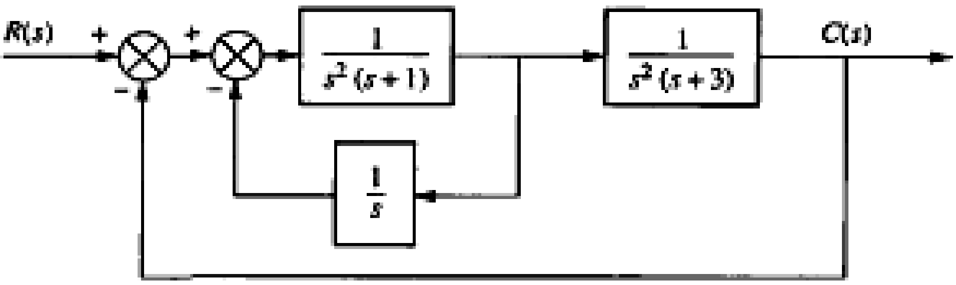
<u>Examples for steady-state error:</u>

1. For the system shown in Figure below, find the following:
   a. The closed-loop transfer function
   b. The system type
   c. The steady-state error for an input of *5u(t)*
   d. The steady-state error for an input of *5tu(t)*



<u>Sol.</u>

a. For the inner loop:

$$G_1(s)=\frac{\frac{1}{s^2(s+1)}}{1+\frac{1}{s^3(s+1)}}=\frac{s}{s^4+s^3+1}$$

$$G_e(s)=\frac{1}{s^2(s+3)}\,G_1(s)=\frac{1}{s(s^5+4s^4+3s^3+s+3)}$$

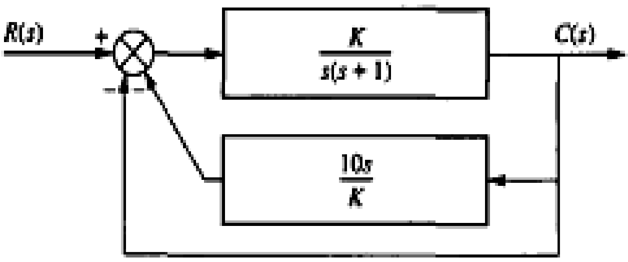
$$T(s)=\frac{G_e(s)}{1+G_e(s)}=\frac{1}{s^6+4s^5+3s^4+s^2+3s+1}$$

b. From $G_e(s)$, system is Type 1.

c. Since system is Type 1, $e_{ss}=0$

d. ; From $G_e(s)$, $K_v=\lim_{s\to 0} sG_e(s)=\frac{1}{3}$. Therefore, $e_{ss}=\frac{5}{K_v}=15$.

2. Given the system of fig. below, design the value of k so that for input of 100tu(t), there will be a 0.01 error in steady state



Sol.

Find the equivalent G(s) for a unity feedback system. $G(s) = \dfrac{\dfrac{K}{s(s+1)}}{1+\dfrac{10}{s+1}} = \dfrac{K}{s(s+11)}$. Thus, $e(\infty) = $

$\dfrac{100}{K_V} = \dfrac{100}{K/11} = 0.01$; from which K = 110,000.

3. For unity feed –back system which have the open loop system is:

$$G(s) = \frac{450(s+8)(s+12)(s+15)}{s(s+38)(s^2+2s+28)}$$

Find the steady-state error for the inputs, 25u(t), 37tu(t), $47t^2u(t)$.

Sol.

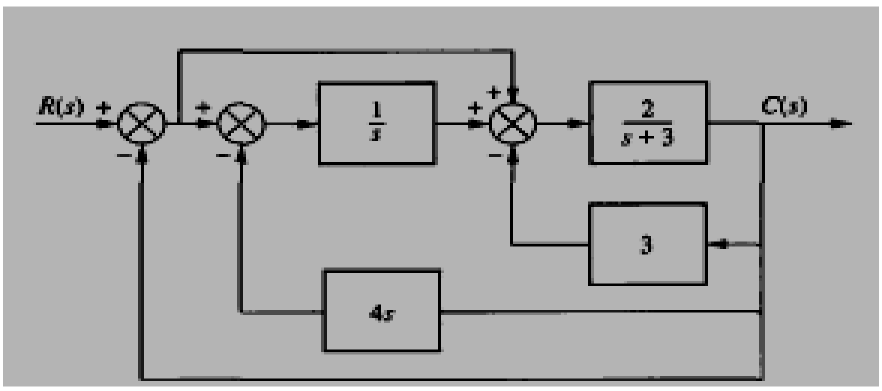
$$e(\infty) = \lim_{s\to 0} sE(s) = \lim_{s\to 0} \frac{sR(s)}{1+G(s)}$$

where

$$G(s) = \frac{450(s+12)(s+8)(s+15)}{s(s+38)(s^2+2s+28)}.$$

For step, $e(\infty) = 0$. For 37tu(t), $R(s) = \dfrac{37}{s^2}$. Thus, $e(\infty) = 6.075\text{x}10^{-2}$. For parabolic input, $e(\infty) = \infty$

4. For the system shown in Figure below, find the steady-state error for the inputs, 15u(t), 15tu(t)



Sol.

Reduce the system to an equivalent unity feedback system by first moving 1/s to the left past the summing junction. This move creates a forward path consisting of a parallel pair, $\left(\frac{1}{s}+1\right)$ in cascade with a feedback loop consisting of $G(s)=\frac{2}{s+3}$ and $H(s)=7$. Thus,

$$G_e(s)=\left(\frac{(s+1)}{s}\right)\left(\frac{2/(s+3)}{1+14/(s+3)}\right)=\frac{2(s+1)}{s(s+17)}$$

Hence, the system is Type 1 and the steady-state errors are as follows:

Steady-state error for $15u(t)=0$.

Steady-state error for $15tu(t)=\frac{15}{K_v}=\frac{15}{2/17}=127.5$.

5. Determine the step, ramp, and parabolic error constants of the following unity-feedback control systems. The forward-path transfer functions are given.

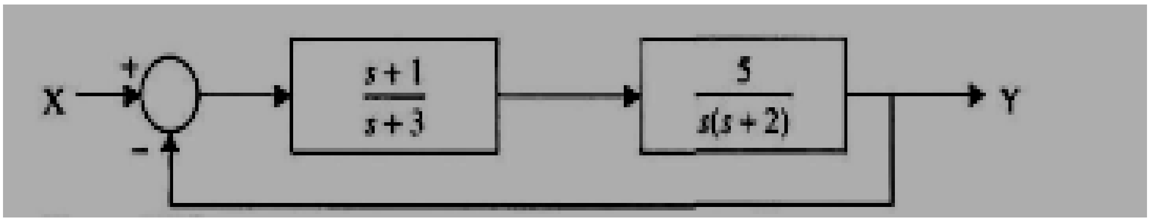
(c) $G(s)=\frac{K}{s(1+0.1s)(1+0.5s)}$ (d) $G(s)=\frac{100}{s^2(s^2+10s+100)}$

Sol.

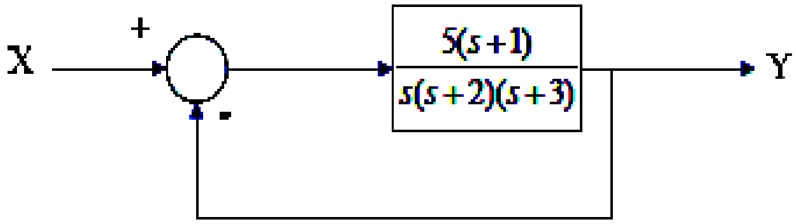
(c) $K_p=\lim_{s\to 0}G(s)=\infty$ $K_v=\lim_{s\to 0}sG(s)=K$ $K_a=\lim_{s\to 0}s^2G(s)=0$

(d) $K_p=\lim_{s\to 0}G(s)=\infty$ $K_v=\lim_{s\to 0}sG(s)=\infty$ $K_a=\lim_{s\to 0}s^2G(s)=1$

6. Find the position, velocity, and acceleration error constants for the system given in Fig. below.



Sol.



$$G(s) = \frac{5(s+1)}{s(s+2)(s+3)}$$

a) Position error: $K_p = \lim_{s \to 0} G(s) = \lim_{s \to 0} \frac{5(s+1)}{s(s+2)(s+3)} = \infty$

b) Velocity error: $K_v = \lim_{s \to 0} sG(s) = \lim_{s \to 0} \frac{5(s+1)}{(s+2)(s+3)} = \frac{5}{6}$

c) Acceleration error: $K_a = \lim_{s \to \infty} s^2 G(s) = \lim_{s \to \infty} \frac{5s(s+1)}{(s+2)(s+3)} = 0$