Problem 7.1:

Find the z transform of the following sequences:

(a) $x1(n) = (0.1)^{n-3}u(n)$

(b) $x2(n) = (0.1)^{n}u(n - 3)$

(c) $x3(n) = \cos(0.5\pi n)u(n)$

(d) $x4(n) = -na^{n}u(-n - 1)$

(e) $x5(n) = (\cos(\pi n))\, u(n)$

(f) $x6(n) = (\sin(0.5\pi n))^2\, u(n)$

SOL

A) $x1(n) = (0.1)^{n-3}u(n) = (0.1)^{-3}(0.1)^{n}u(n) = 1000(0.1)^{n}u(n)$

$\mathbb{Z}[1000(0.1)^{n}u(n)] = 1000 * \frac{z}{z-0.1}$ from table

B) $x2(n) = (0.1)^{n}u(n - 3) = (0.1)^{3}(0.1)^{n-3}u(n-3) = 0.001(0.1)^{n-3}u(n-3)$

But $\mathbb{Z}[(0.1)^{n}u(n)] = \frac{z}{z-0.1}$ from table

$\mathbb{Z}[\mathbf{0.001(0.1)^{n-3}u(n-3)}] = \mathbf{0.001} * \{\frac{z}{z-0.1}\mathbf{z^{-3} + 10z^{-2} + 100z^{-1} + 1000}\}$

from delay property

C) $x3(n) = \cos(0.5\pi n)u(n)$

$\mathbb{Z}[\cos(0.5\pi n)u(n)] = \frac{z(z-\cos(0.5\pi))}{z^2-2z\cos(0.5\pi)-1}$ from table

But $\cos(0.5\pi) = 0$

$\mathbb{Z}[\cos(0.5\pi n)u(n)] = \frac{z^2}{z^2-1}$

D) $x4(n) = -na^n u(-n-1)$
At first let $k=-n$
$x4(k)= ka^{-k}u(k-1)$

Then first let $m=k-1$
$x4(m)=(m+1)a^{-m-1}u(m)= (1/a)(m+1)(a^{-1})^m u(m)= (1/a)\{(m(a^{-1})^m + (a^{-1})^m \}u(m)$
$\mathbb{Z}[(1/a)\ (a^{-1})^m\ u(m)]=\frac{1}{a}\frac{z}{z-1/a} = \frac{z}{az-1}$ from table
$\mathbb{Z}[(1/a)m\ (a^{-1})^m\ u(m)]=\frac{1}{a}\frac{-1/a}{(z-1/a)^2} = \frac{-1}{(az-1)^2}$ from table
$\mathbb{Z}[(1/a)\{(m(a^{-1})^m + (a^{-1})^m \}u(m)]=\frac{-1}{(az-1)^2} + \frac{z}{az-1} = \frac{z(1-a)+1}{(az-1)^2}$
$\mathbb{Z}\ [ka^{-k}u(k-1)]=z^{-1}\frac{z(1-a)+1}{(az-1)^2} = \frac{(1-a)+z^{-1}}{(az-1)^2}$ from delay property
$\mathbb{Z}[-na^n u(-n-1)]=\frac{z+(1-a)}{(az^{-1}-1)^2}$ from invers property

E) $x3(n) = \cos(\pi n)u(n)$
$\mathbb{Z}[\cos(\pi n)u(n)]=\frac{z(z-\cos(\pi))}{z^2-2z\cos(\pi)-1}$ from table
But $\cos(\pi) = -1$
$\mathbb{Z}[\cos(\pi n)u(n)]=\frac{z(z+1)}{z^2+2z-1}$

F) $x6(n) = (\sin(0.5\pi n))^2\ u(n)$

At first we simplify the square function
$(\sin(0.5\pi n))^2 = 0.5(1-\cos(2*0.5\pi n)) = 0.5(1-\cos(\pi n))$
$\mathbb{Z}[0.5(1-\cos(\pi n))u(n)]=0.5 * \{\frac{z}{z-1} + \frac{z(z-\cos(\pi))}{z^2-2z\cos(\pi)-1}\}$ from table
But $\cos(\pi) = -1$
$\mathbb{Z}[(\sin(0.5\pi n))^2\ u(n)]=\frac{z/2}{z-1} + \frac{z(z+1)/2}{z^2+2z-1}$

Problem 7.2:

Find the inverse z transform of $H(z) = 0.6z/[(z + 0.1)(z - 0.5)^3]$.

SOL

$$\mathrm{H(z)} = \frac{0.6z}{(z+0.1)(z-0.5)^3}$$

At first use the partial fraction to find A, B, C and D where

$$\frac{\mathrm{H(z)}}{z} = \frac{0.6}{(z+0.1)(z-0.5)^3} = \frac{A}{(z+0.1)} + \frac{B}{(z-0.5)} + \frac{C}{(z-0.5)^2} + \frac{D}{(z-0.5)^3}$$

A=-2.778, B=-5.556, C=-1.667, D=1

$$H(z) = \frac{2.778}{(z+0.1)} - \frac{5.556}{(z-0.5)} - \frac{1.667}{(z-0.5)^2} + \frac{1}{(z-0.5)^3}$$

$$h(n) = 2.778(-0.1)^n - 5.556(0.5)^n - 1.667 * 2n(0.5)^n + 0.5n(n - 1)(0.5)^n$$

$$\boldsymbol{h(n) = \{2.778(-0.1)^n - 5.556(0.5)^n - 3.834n(0.5)^n + 0.5n^2(0.5)^n\}u(n)}$$

Problem 8.1:

Find the total response y(n) of the discrete-time system described by the following difference equation

$$y(n) - 0.3y(n-1) + 0.02y(n-2) = x(n) - 0.1x(n-1)$$

where y(−1) = 0, y(−2) = 0, and $x(n) = (-0.2)^n u(n)$.

SOL

$$y(n) - 0.3y(n-1) + 0.02y(n-1) = x(n) - 0.1x(n-1)$$

$$Y(z) - 0.3\{z^{-1}Y(z)\} + 0.02\{z^{-2}Y(z)\} = X(z) - 0.1\{z^{-1}X(z)\}$$

$$Y(z)\{1 - 0.3z^{-1} + 0.02z^{-2}\} = X(z)\{1 - 0.1z^{-1}\}$$

$$Y(z) = \frac{1-0.1z^{-1}}{1-0.3z^{-1}+0.02z^{-2}}X(z)$$

$$X(z) = \frac{1}{1+0.2z^{-1}}$$

$$Y(z) = \frac{1-0.1z^{-1}}{1-0.3z^{-1}+0.02z^{-2}} * \frac{1}{1+0.2z^{-1}}$$

$$Y(z) = \frac{1-0.1z^{-1}}{(1-0.1z^{-1})(1-0.2z^{-1})(1+0.2z^{-1})} = \frac{1}{(1-0.2z^{-1})(1+0.2z^{-1})} = \frac{z^2}{(z-0.2)(z+0.2)}$$

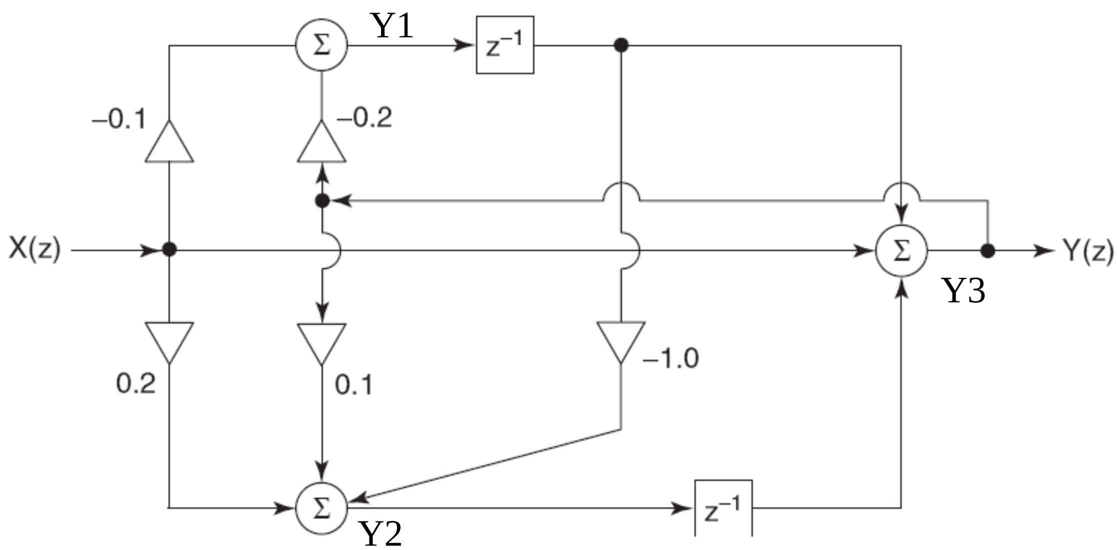
$$\frac{Y(z)}{z} = \frac{z}{(z-0.2)(z+0.2)} = \frac{A}{(z-0.2)} + \frac{B}{(z+0.2)} = \frac{0.5}{(z-0.2)} + \frac{0.5}{(z+0.2)}$$

$$Y(z) = \frac{0.5z}{(z-0.2)} + \frac{0.5z}{(z+0.2)}$$

$$y(n) = \{0.5(0.2)^n + 0.5(-0.2)^n\}u(n)$$

Problem 8.2:

Derive the transfer function of the digital filter shown in Figure and find the samples h(0), h(1), and h(2).



SOL

$y1(n) = -0.2y3(n) - 0.1x(n)$

$\rightarrow Y1(z) = -0.2Y3(z) - 0.1X(z)$ (1)

$y2(n) = 0.1y3(n) - 0.1y1(n-1) + 0.2x(n)$

$\rightarrow Y2(z) = 0.1Y3(z) - 0.1z^{-1}Y1(z) + 0.2X(z)$ (2)

$y3(n) = y1(n-1) + y2(n-1) + x(n)$

$\rightarrow Y3(z) = z^{-1}Y1(z) + z^{-1}Y2(z) + X(z)$ (3)

Substitute (2) in (3)

$Y3(z) = z^{-1}Y1(z) + z^{-1}\{0.1Y3(z) - 0.1z^{-1}Y1(z) + 0.2X(z)\} + X(z)$

$Y3(z)(1 - 0.1z^{-1}) = (z^{-1} - 0.1z^{-2})Y1(z) + (1 + 0.2z^{-1})X(z)$

Substitute (1)

$$Y3(z)\{(1-0.1z^{-1})+0.2(z^{-1}-0.1z^{-2})\}$$

$$=\{-0.1(z^{-1}-0.1z^{-2})+(1+0.2z^{-1})\}X(z)$$

$$Y3(z)\{1+0.1z^{-1}-0.02z^{-2}\}=\{1+0.1z^{-1}-0.1z^{-2}\}X(z)$$

$$H(z)=\frac{Y3(z)}{X(z)}=\frac{1+0.1z^{-1}-0.1z^{-2}}{1+0.1z^{-1}-0.02z^{-2}}=\mathbf{1}-\frac{0.08z^{-2}}{1+0.1z^{-1}-0.02z^{-2}}$$

$$H(z)=\mathbf{1}-\frac{0.08z^{-2}}{1+0.1z^{-1}-0.02z^{-2}}=\mathbf{1}-\frac{0.08}{z^2+0.1z-0.02}=\mathbf{1}-\frac{0.08z}{z(z-0.1)(z+0.2)}$$

$$H(z)=\mathbf{1}+\frac{4z}{z}-\frac{\frac{8}{3}z}{(z-0.1)}+\frac{\frac{8}{3}z}{(z+0.2)}=\mathbf{5}-\frac{\frac{8}{3}z}{(z-0.1)}+\frac{\frac{8}{3}z}{(z+0.2)}$$

$$\mathrm{h(n)}=5\delta(\mathrm{n})-\left\{\frac{8}{3}(0.1)^{\mathrm{n}}+\frac{8}{3}(-0.2)^{\mathrm{n}}\right\}u(n)$$

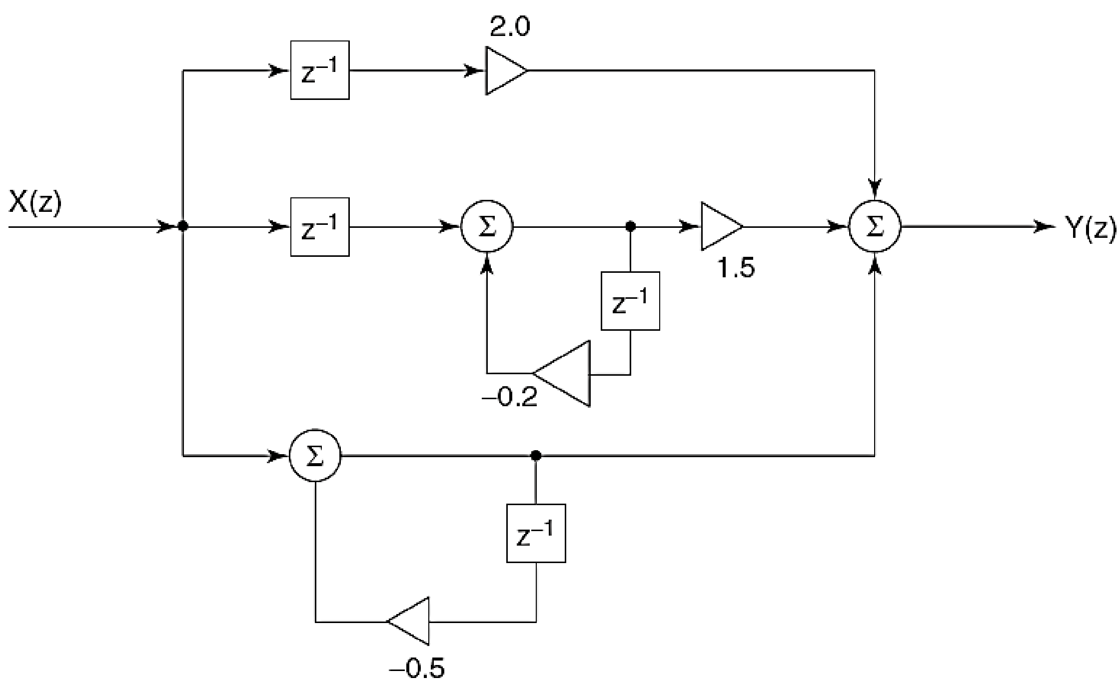
$$\mathrm{h}(0)=5-\frac{8}{3}+\frac{8}{3}=5$$

$$\mathrm{h}(1)=0-\frac{8}{3}(0.1)\quad+\frac{8}{3}(-0.2)\quad=-0.8$$

$$\mathrm{h}(2)=0-\frac{8}{3}(0.1)^2+\frac{8}{3}(-0.2)^2=0.08$$

HW 7: Repeat the problems 7.1, 7.2 and 8.1 and with proposed questions.

HW 8: Repeat the problem 8.2 with for the following figure with any additional path.



HW 8.1: Find the error in the solved problems 7.1, 7.2, 8.1 and 8.2 if found.