Trip Distribution

After all available and relevant information on the number of trips departing or arriving in each zone has been collected, the next step in transport modeling is to distribute these trips over origin-destination (OD) cells. This can be done either at disaggregated or at aggregate level. In disaggregated trip distribution, destination choice proportions are simulated using information on individual characteristics. In aggregate trip distribution individual characteristics are not considered. In this course we will only discuss aggregate trip distribution models.

The point of departure to the aggregate trip distribution step are the margins of the origin destination (OD) table computed in the trip generation step. This implies estimates are available for either:

- the number of trip departures,
- the number of trip arrivals,
- the number of trip departures and the number of trip arrivals,
- none.

Which case is true depends on the purpose of the study and the availability of data and leads to separate trip distribution models. Table 1 schematizes the trip distribution problem for the case where both the number of trip departures and the number of trip arrivals are known:
the objective is to forecast an OD-table based on estimates of future productions and attractions and measurements of current OD-flows, or measurements of the generalized cost of each trip.

Table 1: Doubly constrained trip distribution

Origins	Destinations				$\sum_j T_{ij}$
	1	2	...	n	
1					P1
2		?			P2
...					...
m					Pm
$\sum_i T_{ij}$	A1	A2	...	An	

Trip Matrix

Generations	Attractions							
	1	2	3	…	j	…	J	$\sum_j T_{ij}$
1	T_{11}	T_{12}	T_{13}	…	T_{1j}	…	T_{1J}	O_1
2	T_{21}	T_{22}	T_{23}	…	T_{2j}	…	T_{2J}	O_2
3	T_{31}	T_{32}	T_{33}	…	T_{3j}	…	T_{3J}	O_3
:	:	:	:		:		:	:
i	T_{i1}	T_{i2}	T_{i3}	…	T_{ij}	…	T_{iJ}	O_i
:	:	:	:		:		:	:
I	T_{I1}	T_{I2}	T_{I3}	…	T_{Ij}	…	T_{IJ}	O_I
$\sum_i T_{ij}$	D_1	D_2	D_3	…	D_j	…	D_J	$\sum_i \sum_j T_{ij} = T$

Two basic categories of aggregate trip distribution methods predominate in urban transportation planning:

- The first basic category of aggregate trip distribution methods is based on the gravity model. For gravity models, typical inputs include one or more flow matrices, an impedance matrix reflecting the distance, time, or cost of travel between zones, and estimates of future levels of productions and attractions. The gravity model explicitly relates flows between zones to interzonal impedance to travel.

  The gravity model was originally motivated by the observation that flows decrease as a function of the distance separating zones, just as the gravitational pull between two objects decreases as a function of the distance between the objects. As implemented for planning models, the Newtonian analogy has been replaced with the hypothesis that the trips between zones i and j are a function of trips originating in zone i and the relative attractiveness and/or accessibility of zone j with respect to all zones.

  Modern derivations of the gravity model show that it can be understood as the most likely spatial arrangement of trips given limited information available on zonal origin totals, zonal destination totals, and various supporting assumptions or constraints about mean trip lengths (Ortúzar en Willumsen, 1994).

  Many different measures of impedance can be used, such as travel distance, travel time, or travel cost. There are also several potential impedance functions used to describe the relative attractiveness of each zone. Popular choices are the exponential functions typically used in entropy models. As an alternative to impedance functions, one can use a friction factor lookup table (essentially a discrete impedance function) that relates the impedance between zones to the attractiveness between zones.

Prior to applying a gravity model, one has to calibrate the impedance function. Typically, calibration entails an iterative process that computes coefficients such that the gravity model replicates the trip length frequency distribution and matches base year productions, or attractions, or both .

- The second basic category of aggregate trip distribution are the growth factor methods. These involve scaling an existing matrix (called base matrix) by applying multiplicative factors (often derived from predicted productions and/or attractions) to matrix cells.

Practical Issues

Some of the classical growth factor methods do not take into account any information about the transportation network, and thus cannot reflect impacts of changes in the network. This may be reasonable for very short-term forecasts, but it is invalid for medium to long-term forecasts for which the network has changed, or to forecast scenarios that include changes in the network. Since most transportation planning involves analysis of transportation networks, gravity models or more sophisticated destination choice models should be used.
In aggregate analysis, the choice of the impedance function should be based upon the mathematical properties of the function and the data distributions to be modeled. In practice, the selection of the functional form of the model should be based on the shape of the measured trip length distribution; consequently, examination of empirical trip length distributions is an important input into the decision process. Both smooth impedance functions and discrete functions (i.e. friction factors) can be used, as well as hybrid functions combining functions or utilizing smoothed discrete values .

Distribution models should be estimated and applied for several trip purposes. The rationale for this is that both the alternatives and individuals' willingness to travel differ greatly by purpose.

Derivation of the gravity model

The gravity model in its general form states that the number of trips between an origin and destination zone is proportional to the following three factors:

- a factor for the origin zone (the production ability)
- a factor for the destination zone (the attraction ability)
- a factor depending on the travel costs between origin and destination zone

Mathematically this is summarized as follows:

$$T_{ij} = \mu Q_i X_j F_{ij}$$

with:
T_{ij} = number of trips from zone i to zone j
Q_i = production ability of zone i
X_j = attraction ability of zone j

F_{ij} = accessibility of j from i (depends on travel costs c_{ij})
μ = measure of average trip intensity in area.

This model will be referred to as the general trip generation model; it will be made more specific for cases where extra information is available, such as the number of arrivals or departures. It can be seen that above model is in line with intuitively clear symmetry assumptions: if two possible destination zones have similar attraction abilities and are equally accessible from an origin zone, there is no reason to expect that more trips will be made from that origin zone to the first destination zone than to the second destination zone.
According to utility theory, decision makers aim at maximizing their perceived net utilities. Utility is derived from activities. To maximize utility, in general multiple types of activities are needed during a day, e.g. working and living.
Individual utility U_{ijp} of making a trip from origin i to destination j for a specific homogeneous travel purpose (e.g. home to work) is:

$$U_{ijp} = U_i + U_j - f(c_{ij}) + \varepsilon_{ijp}$$

U_i = average utility of origin bound activity in i
U_j = average utility of destination bound activity in j
$f(c_{ij})$ = utility value of travel resistance (cost) between i and j
ε_{ijp} = individual error term, accounting for misperceptions, taste variation and non modeled attributes.

Define:

$$V_{ij} = U_i + U_j - f(c_{ij})$$

Now we can write:

$$U_{ijp} = V_{ij} + \varepsilon_{ijp}$$

If we assume that the error term ε_{ijp} is Gumbel distributed with scale parameter b (logit assumptions), then for each decision maker the probability that he will opt for
a trip from zone i to zone j equals:

$$p_{ij} = \frac{\exp(bv_{ij})}{\sum_{ij} \exp(bv_{ij})} = \frac{1}{k} \exp(bU_i) \cdot \exp\left(bU_j\right) \cdot \exp\left(-bf(c_{ij})\right)$$

with

$$k = \sum_{ij} \exp(bv_{ij})$$

P_{ij} = probability that an individual will make a trip from i to j
b = scale parameter in Gumbel distribution
k = a measure for the number of and variability in trip alternatives. The larger k, the more choice opportunities for a traveler.

With P travelers, the expected number of trips between i and j amounts to:

$$T_{ij} = \mu Q_i X_j F_{ij}$$

which is the general trip distribution model with:

Q_i = production potential = $\exp(bU_i)$
X_j = attraction potential = $\exp(bU_j)$
F_{ij} = accessibility of j from = $\exp(-bf(c_{ij}))$
μ = measure of average trip intensity in area = P / k

Taking this general trip distribution model as starting point, we can formulate various derived models depending on additional constraints imposed on the model, especially on the number of trip arrivals and departures in the zones (see Table 1).

Table 1: Types of distribution models according to imposed constraints on arrivals and departures

	Departures unknown	**Departures known**
Arrivals unknown	Direct Demand	Origin Constraint
Arrivals known	Destination Constraint	Doubly Constraint

1. Direct demand model

In this case, no additional trip constraints are imposed, so this model equals the general trip distribution model:

$$T_{ij} = \mu Q_i X_j F_{ij}$$

with:
Qi = production potential of i
Xj = attraction potential of j
Fij = accessibility of j from i
μ = measure of average trip intensity in area

The production and attraction potentials may be derived from population, area, number of jobs, etc. Both the numbers of departures (productions) and arrivals (attractions) are unknown. They are determined endogenously. The resulting flows are estimated solely on the potentials of zones i and j and the impedance between them.
Although the direct demand model is easy to implement, a disadvantage of the direct demand model is that it predicts a large number of trips per unit of analysis (e.g. person) for particularly accessible origin zones (zones that have many high attraction potential zones nearby). This is not realistic under all circumstances. For example, the number of home-work trips per person will in general not increase, even if many job opportunities are close-by. For this reason this method is rarely used in practice.

Singly constrained trip distribution model
Origin constrained

In the origin constrained trip distribution model, the number of trip departures Pi are imposed as a set constraints on the general trip distribution model:

$$\sum_j T_{ij} = P_i$$

where Pi is the known number of trips departing from zone i, which is determined exogenously (for example estimated using a trip generation model). Combining this with the general trip distribution model

$$T_{ij} = \mu Q_i X_j F_{ij}$$

we can write:

$$\sum_j T_{ij} = \sum_j (\mu Q_i X_j F_{ij}) = \mu Q_i \sum_j (X_j F_{ij}) = P_i$$

Solving for Q_i yields:

$$Q_i = \frac{p_i a_i}{\mu}$$

where a_i is defined as:

$$a_i = \frac{1}{\sum_j X_j F_j}$$

Subditiuting the above equations gives the origin constrained distribution model:

$$T_{ij} = P_i \frac{X_j F_{ij}}{\sum_j X_j F_{ij}} = a_i P_i X_j F_{ij} \qquad \text{Equation (1)}$$

with:
a_i = balancing factor
P_i = number of trips departing from zone i
X_j = attraction potential of zone j
F_{ij} = accessibility of zone j from zone i

The origin constrained trip distribution model is therefore a proportional model that splits the given trip numbers originating in i over the destinations j in proportion to their relative accessibility and utility opportunity.

Although different definitions of accessibility may be used, the factor $\sum_j (X_j F_{ij})$ is often referred to as the accessibility of zone i. By dividing the total number of departures by the accessibility of a zone, we avoid the phenomenon that causes the total number of departures from an origin zone to increase if it is close to zones with high attraction. Of course, if a destination is highly accessible, i.e. many origin zones with high production abilities are close-by, this will still result in a large number of trip arrivals. This might be an unwanted effect if the attraction ability is based on, e.g., the number of jobs in a zone.

Equation (1) shows that the absolute levels of X_j and F_{ij} are not essential in this proportional model. If we multiply each of these variables by an arbitrary constant factor, this would not affect the model outcomes. This characteristic leaves a lot of freedom in specifying both variables.

A practical example of an origin constrained model is the WOLOCAS model. This model is designed to predict the impact of the development of new residential areas (e.g. VINEX). While the number of trips originating from these areas is estimated using detailed trip generation models, no explicit estimate is made of the number of trip arrivals. Instead, attraction abilities are supplied for different trip purposes based on the number of jobs (homework), the number of facilities (shopping), and the number of inhabitants (other trip purposes).

Constrained to destinations

In analogy with the derivation of the origin constrained trip distribution model, the destination constrained trip distribution may be derived. The internal trip numbers are constrained to exogenously given arrivals. The number of arriving trips A_j in j is known (e.g. by using a separate trip generation model), which implies:

$$\sum_i T_{ij} = A_j \forall j$$

Skipping the derivation (which is analogous to the previous derivation), the model is given below:

$$T_{ij} = A_j \frac{Q_i F_{ij}}{\sum_i (Q_i F_{ij})} = b_j Q_i A_j F_{ij}$$

with:
b_j = balancing factor = $\Sigma_i 1/(Q_i F_{ij})$
Q_i = production potential of i
A_j = number of trips arriving at zone j
F_{ij} = accessibility of j from i

The destination constraint trip distribution model is therefore a proportional model that splits the given trip numbers arriving at j over the origins i in proportion to their relative accessibility and utility opportunity.
The model outcomes are not sensitive to the absolute levels of Q_i and F_{ij} (see equation (1)). Multiplying both variables with an arbitrary constant does not change the results. This characteristic gives considerable freedom in defining these variables.

Doubly constrained trip distribution model

The doubly constrained model arises if both the number of trip departures and the number of trip arrivals are imposed on the general trip distribution model. The derivation of the doubly constrained trip distribution model is as follows. We again start with the general trip distribution model:

$$T_{ij} = \mu Q_i X_j F_{ij}$$

Now we have two sets of constraints in that the numbers of arrivals and departures in the zones are exogenously given. Thus, the number of arriving trips Aj at j is known and the number of departing trips Pi from i is known (again, e.g., using separate models). This yields

$$\sum_j T_{ij} = P_i$$

and

$$\sum_i T_{ij} = A_j$$

Hence,

$$\sum_j T_{ij} = \sum_j \left(\mu Q_i X_j F_{ij}\right) = \mu Q_i \sum_j \left(X_j F_{ij}\right) = P_i$$

and

$$\sum_i T_{ij} = \sum_i \left(\mu Q_i X_j F_{ij}\right) = \mu X_j \sum_i \left(Q_i F_{ij}\right) = A_i$$

Solving for Q_i and X_j

$$Q_i = \frac{P_i a_i}{\mu}$$

$$X_j = \frac{A_j b_j}{\mu}$$

where a_i and b_j are balancing factors for the trip constraints, defined by:

$$a_i = \frac{1}{\sum_j X_j F_{ij}}$$

and

$$b_j = \frac{1}{\sum_i Q_i F_{ij}}$$

Hence, the doubly constrained trip distribution model now is:

$$T_{ij} = \frac{1}{\mu} a_i b_j P_i A_j F_{ij}$$

The parameter μ may be included in the estimated values for a_i and b_j resulting in:

$$T_{ij} = a_i b_j P_i A_j F_{ij} \qquad \text{equation 2}$$

with:
a_i = balancing parameter
b_j = balancing parameter
P_i = number of trips departing at zone i
A_j = number of trips arriving at zone j
F_{ij} = accessibility of zone j from i

Whereas the trip distribution can be computed directly with the non-constrained and the singly constrained trip distribution models (provided sufficient input data are available), this is not the case with the doubly constrained trip distribution model. If all input data are available (i.e. the number of departures, Pi, the number of arrivals, A_j, and the values of the distribution function, F_{ij}), previous equations define the coefficients a_i and b_j in model Equation (2) in an implicit manner. To determine these coefficients, an iterative procedure may be used. The following example illustrates such a procedure.

Example 2 [trip distribution using a doubly constrained model]
Consider a study area consisting of two zones. The following data on population and labor are available:

zone	inhabitants	jobs
1	1000	300
2	800	200

It should be emphasized that in this case the number of inhabitants was determined more accurately than the number of jobs. From national data it follows that the number of work-related trips is on average 0.25 per person per day. The number of work-related trips arriving in a zone is 0.8 for each job. The travel resistance may be assumed to be equal for all OD-pairs in this example.
Questions:
(a) Formulate the doubly constrained distribution model and define its variables.
(b) Compute the trip distribution using the doubly constrained trip distribution model.

Answers:

(a) $T_{ij} = a_i b_j P_i\ A_j\ F_{ij}$
For the definition of the variables, see above.

(b) If the travel resistance is equal for all OD-pairs, distribution function values equal to 1 may be used; i.e. $F_{ij} = 1$. The trip generation is obtained as follows:

$$\begin{pmatrix} P_1 \\ P_2 \end{pmatrix} = 0.25 \begin{pmatrix} 1000 \\ 800 \end{pmatrix} = \begin{pmatrix} 250 \\ 200 \end{pmatrix} \text{ and } \begin{pmatrix} A_1 \\ A_2 \end{pmatrix} = 0.8 \begin{pmatrix} 300 \\ 200 \end{pmatrix} = \begin{pmatrix} 240 \\ 160 \end{pmatrix}$$

Now the values of the balancing factors a_i and b_j need to be computed. This is done in an iterative way. As an initializing step we fill a tableau with the known values of P_i, A_j, and F_{ij}:

from zone	to zone 1	to zone 2	total	P_i	factor
1	1	1	2	250	
2	1	1	2	200	
total	2	2	4	450	
A_j	240	160	400		
factor					

The total number of trip departures (450) does not match the total number of arrivals (400). The first step is therefore to balance them. This is done by multiplying the trip arrivals with a factor 450/400 (because the number of departures is more accurately known):

from zone	to zone 1	to zone 2	total	P_i	factor
1	1	1	2	250	125
2	1	1	2	200	100
total	2	2	4	450	
A_j	270	180	450		
factor					

The following step is factor each row in order to match the row totals (departures):

from zone	to zone 1	to zone 2	total	P_i	factor
1	125	125	250	250	
2	100	100	200	200	
total	225	225	450	450	
A_j	270	180	450		
factor	270/225	180/225			

And each column in order to match the column totals (arrivals):

from zone	to zone 1	2	total	P_i	factor
1	150	100	250	250	1
2	120	80	200	200	1
total	270	180	450	450	
A_j	270	180	450		
factor	1	1			

In this case there is no use in performing more iterations as this would not alter the solution anymore. The table above gives the requested trip distribution.

<u>Distribution functions</u>

As can be deduced from Example 1, knowledge of distribution functions is essential when applying trip distribution models. The distribution function F (also referred to as deterrence function) represents the relative willingness to make a trip as a function of the generalized travel costs c_{ij}. In general such a function will be a monotonously decreasing function of travel costs.

Usually different distribution functions are used, depending on the trip purpose and the attributes of the trip maker. The difference can be attributed to the fact that different categories of a population (students, housewife, employees) value travel resistance in a different way, due to differences in monetary and time budgets. A requirement to apply these different distribution functions of course is that the trip production and attraction can be estimated by category. In the most trip distribution models distinguish between different trip purposes (e.g. work, business and others), while some also distinguish between different categories of travelers (e.g. car available, and no car available).

A typical example of the shape of distribution functions for various travel modes is shown in Figure 1. All functions decrease monotonously. The distribution function for the bicycle exceeds the others for small travel distances, but decreases to approximately zero at 10 km.
The public transit curve is smallest initially, but barely decreases. At a distance of approximately 50 kilometres the public transit curve exceeds the car curve.

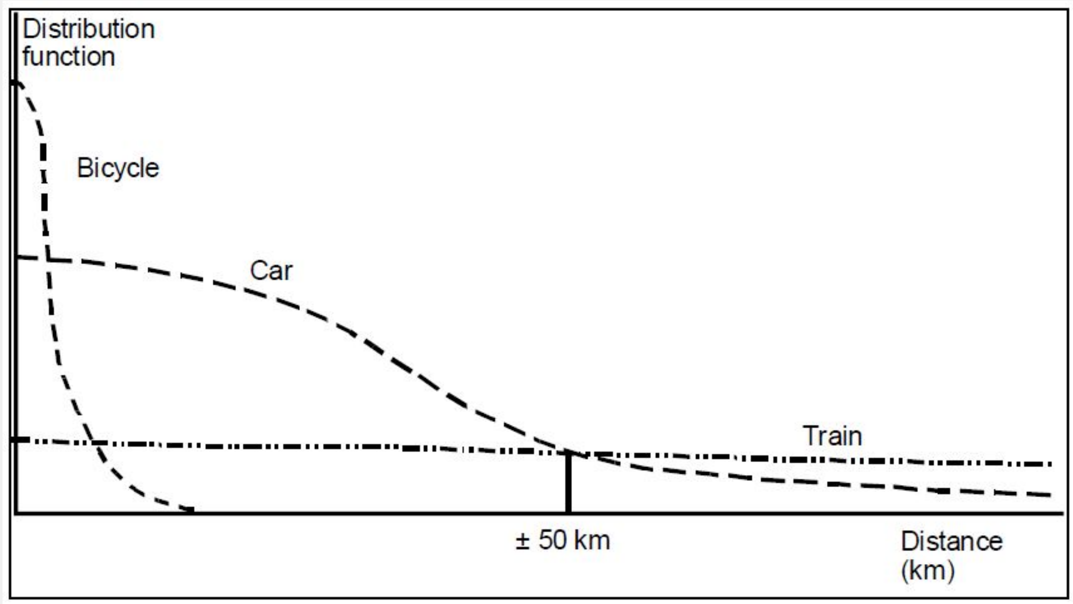

Figure 1: Example of distribution function for various modes

Mathematical requirements

Distribution functions are usually estimated on the basis of empirical data. In this procedure the following theoretical requirements may be imposed:

1. The function should be decreasing with generalized travel time; a larger travel time should lead to diminished willingness to make a trip:

$$F(C_{ij}) \geq F\left(c_{ij} + \Delta_c\right) if\ \Delta_c > 0$$

2. The expression $\int_0^\infty F(c)cdc$ is finite.

This requirement implies that a limited number of trips originate from each zone, even if the study area is not bounded. If this requirement is not met, the number of trips depends on the boundaries of the study area, preventing model parameters from being transferable to other studies. Power functions (see next section) with an exponent less than or equal to 2 do not meet this requirement.

3. The fraction $\frac{F(ac_{ij})}{Fc_{ij}}$ depends on the value of c_{ij}.

This requirement expresses that if the travel costs decrease with a constant factor, this has an impact on the trip distribution.

4. Fixed absolute changes should have a diminishing relative impact on the willingness to

make a trip:

$$\frac{F(C_{ij}+\Delta C)}{F(C_{ij})} > \frac{F(C_{ij}+A+\Delta C)}{F(C_{ij}+A)} \, if \; A > 0$$

<u>Continuous distribution functions</u>

Over time, different mathematical forms of distribution function have been proposed. The following overview is by no means complete, but is quite representative for practice. Note that all distribution functions are written with an index *m* (mode choice). If only one mode is considered, this index may be omitted.

<u>Power:</u>

$$F_{ijm}\left(C_{ijm}\right) = C_{ijm}^{-\alpha_m}$$

This function is believed to be relatively accurate for large travel distances or costs, and less so for small distances. It is rarely used in practice. If α_m=2, the Newtonian model is obtained. Although the name may suggest differently, in transport planning the name 'gravity model' is used for trip generation models using a wide range of distribution functions, including the Newtonian.

<u>Exponential:</u>

$$F_{ijm}\left(C_{ijm}\right) = \alpha \exp\left(\beta_m C_{ijm}\right)$$

A counterintuitive property of this function is that a fixed absolute increase in travel time results in a fixed relative decrease in the (modeled) willingness to make a trip. Therefore the function is not believed to be accurate when the range of traveled distances in the study area exceeds 15 km.
Nevertheless, this function appears quite often in theoretical research due to its nice mathematical properties. Most derivations of the trip distribution model in the literature result in a model with an exponential distribution function.

<u>Top-exponential (Tanner):</u>

$$F_{ijm}\left(C_{ijm}\right) = \alpha_m C_{ijm}^{\gamma_m} \exp(\beta_m C_{ijm})$$

<u>Lognormal:</u>

$$F_{ijm}\left(C_{ijm}\right) = \alpha_m \exp(\beta_m ln^2 C_{ijm} + 1))$$

<u>Top-lognormal:</u>

$$F_{ijm}\left(C_{ijm}\right) = \alpha_m \mathrm{C}_{\mathrm{ijm}}^{\gamma_\mathrm{m}} \exp(\beta_m ln^2 (C_{ijm} + 1))$$

<u>Log-logistic:</u>

$$F_{ijm}\left(C_{ijm}\right) = \frac{MAX^m}{1 + \exp\left[\beta_m + \gamma_m \log(C_{ijm})\right]}$$

The top-exponential and top-lognormal distribution functions are used in practice as an alternative for the exponential and lognormal functions. They ignore the requirement that a distribution function should be monotonously decreasing (see Figure 2).

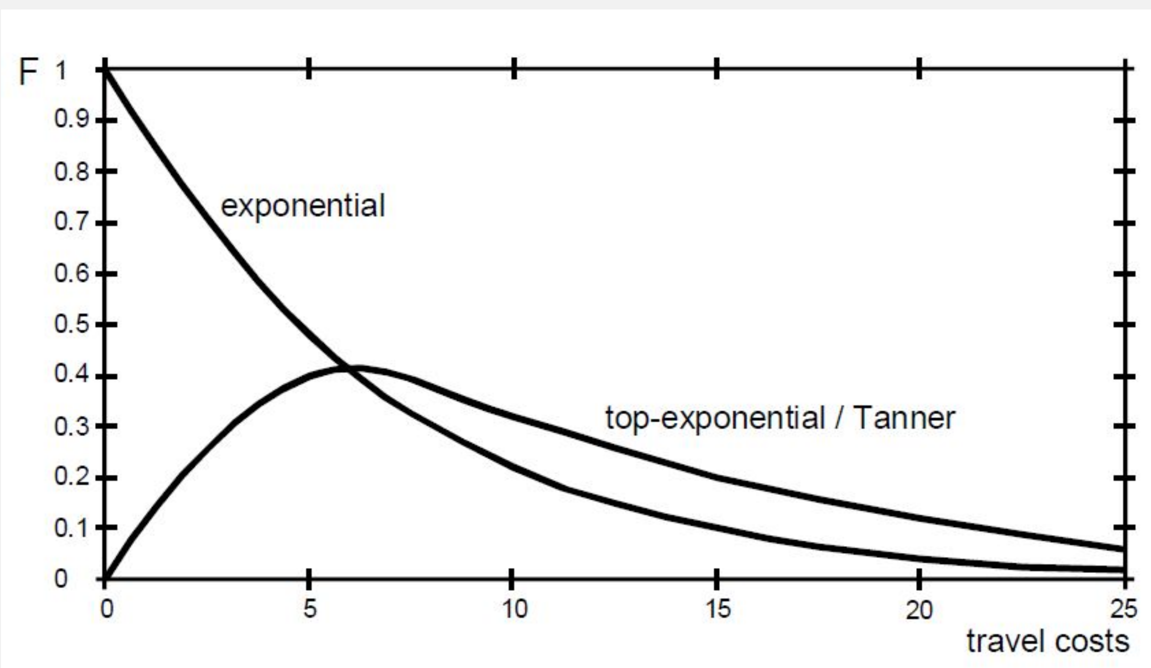

Figure 2: *Exponential distribution functions (parameters a = 1, β = -0.15), and top-exponential distribution function (parameters a = 0.25, β = -0.15, γ = 0.75).*

Discrete distribution functions

As an alternative to the continuous distribution, a discrete or piecewise constant distribution function may be used. The mathematical form of this function is:

$$F_{ijm}\left(C_{ijm}\right) = \sum_{k=1}^{K} F_{ijm}^{k}\, d^{k}(C_{ijm})$$

with

$$F_{ijm}^{k} \geq 0 \; and \;\; d^{k}(C_{ijm}) \in \{0,1\}$$

where k is the cost bin, K is the number of cost bins (e.g. 10), and F_{ijm}^{k} is the value of the distribution function for cost bin k. The function $d^{k}\,(C_{ijm})$ is the membership function which is 1 if C_{ijm} lies in cost bin k and zero otherwise.
This function defines a fixed distribution function value for each cost-bin. A property of this approach is that no assumptions on the shape of the distribution function are imposed. In Figure 3 an example of a discrete distribution function is shown.

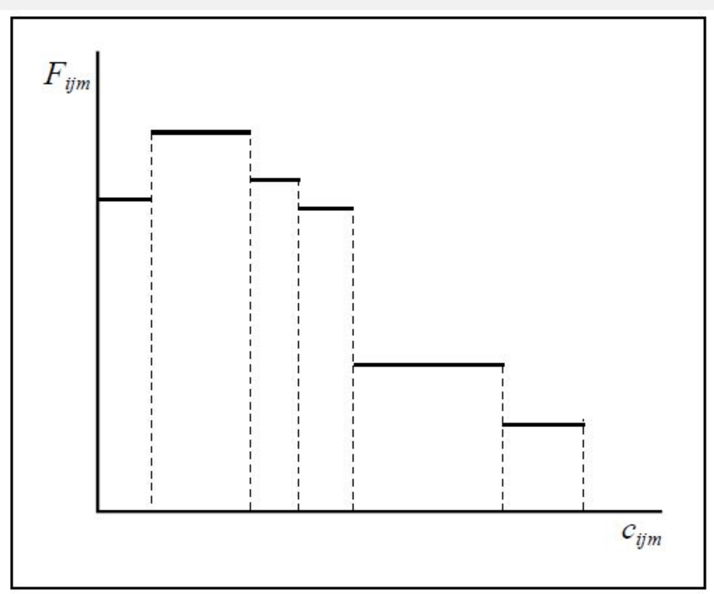

Figure 3: Example discrete distribution function.

2. Growth factor models

As an alternative to trip distribution models, growth factor models may be used. In this approach a base year matrix is needed ('basismatrix'). Each cell of this matrix is multiplied by a growth factor. Growth factors may be computed in a number of ways, e.g. as the output of an economic model, a trend model, etc. However, in these course notes, we only discuss methods of computing growth factors based on trip generation modeling.
The base year matrix contains an estimate of the trips being made in the base year. Theoretically it is possible to directly observe a base year matrix using a travel survey. However, if the study area is decomposed into many zones, and the travel survey represents only a part of all travel, directly observing a base year matrix would result in a matrix mainly consisting of zero cells. This can be seen from the following example:

Consider a town with 100.000 inhabitants that produce 15.000 trips during a two hour peak period. If this town is divided in 50 zones, there are more than 2.000 OD cells, and the average number of trips per OD-cell is 15.000/2.000 = 8. If 10% of the population is surveyed (10% is a large number for an survey) on average 0.8 trips per cell are reported in the observed matrix. This means that at least 20% of all observed cells must be zero (note the distinction between observed zero and unobserved cell).
Applying a growth factor model to a base year matrix mainly consisting of zeros results in a predicted matrix mainly consisting of zeros. A better approach is to impose extra constraints on the base year matrix in order to supply a realistic value for the cells in which no trips are observed, for example by requiring that the base year matrix complies with the gravity model, i.e.:

$$T_{ij} = Q_i X_j f(c_{ij})$$

Where:
T_{ij}^0 : base year matrix

X_i, Q_j : parameters in gravity model, calibrated in such a way that the expression observed
$\left\| T_{ij}^0 \; T_{ij}^{observed} \right\|$minimized

$f(c_{ij})$:value of distribution function for OD-pair i-j

Note that imposing such a structure can also 'spoil' an observed matrix, for example by causing bias. When larger zones are used, determining an appropriate value for the travel costs, cij, that is representative for all trips between i and j becomes impossible. This might introduce inaccuracies in the approach described above.

Another source of information that can be used to estimate base year matrices is traffic counts. Dedicated estimation techniques are available to adapt a prior base year matrix to a set of traffic counts.

The growth factor methodology is pointed out in Figure 3. A trip distribution model is used to compute a current and future trip matrix. Combining these matrices results in a growth factor that is then applied to the base year matrix, resulting in a predicted trip matrix.

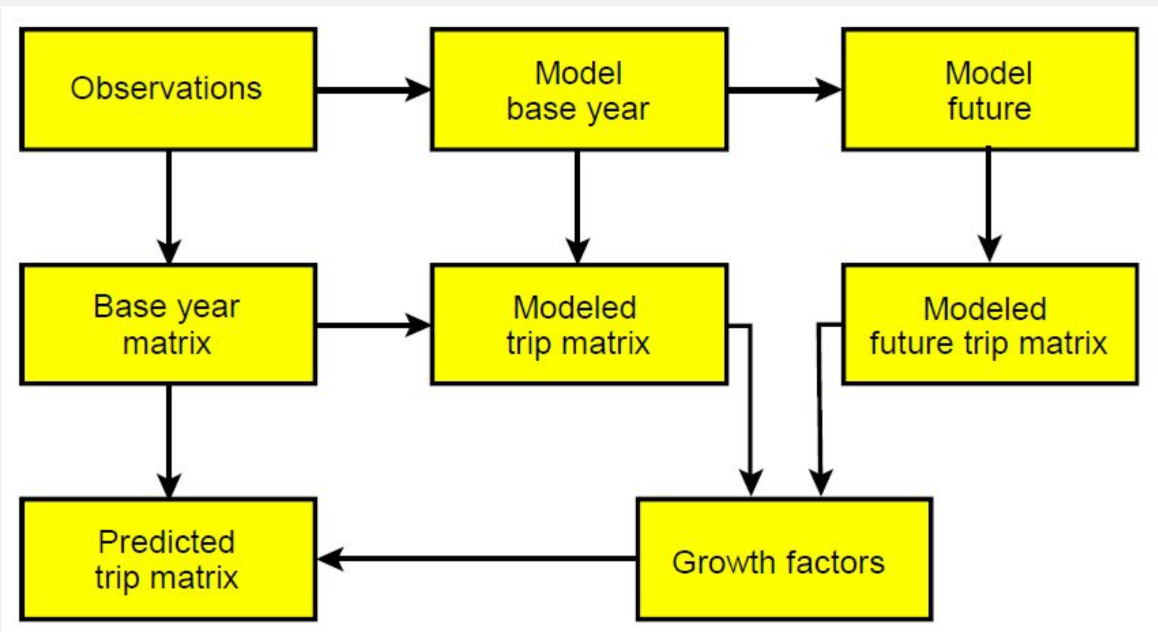

Figure 3: Predicting future trip matrix using growth factor methodology.

The base year matrix is the best possible estimate of current origin-destination trip flows in the study area. In the growth factor methodology the base year matrix is the starting point for making predictions of future states. It is considered a better base for the future than using a trip distribution model on its own.

Reasons for this are among others:

- models cannot capture peculiarities in trip making that often can be found in study areas. In contrast, such peculiarities can be included in a base matrix to a large extent because it is based on observations.
- in planning practice it is necessary that parties involved in planning all agree on the fundamentals for planning. In this respect, a base matrix is a better tool to gain confidence in the fundamentals than a model because it is more understandable, it is verifiable, etc.

So, a widely accepted approach to prediction is to take a base matrix and adapt this using growth factors derived from models.

The methodology pointed out in Figure 3 also has a number of disadvantages of which the most important ones are:

- if new building sites are developed (e.g. VINEX) the resulting changes in trip distribution are difficult to capture in a growth factor model. This is because the travel behavior of the present inhabitants of these building sites is not representative for the future, especially when an agriculturally oriented environment changes to an urban environment.
- the base year matrix is influenced to a great extent by historical travel patterns. These patterns might fade away in a few decades time. This is particularly true if new cities have arisen as a result of suburbanization: initially the travel of suburbs is oriented toward the nearby town, but in time such strong historic ties vanish, and a more balanced trip making pattern arises. Of course, planners have to take account of these phenomena. Simply applying growth factors in this case would not lead to the desired result.

Computation of growth factors

Throughout this section we use the following notation:

T_{ij}^{0} base matrix (known current OD-table)
Tij future OD-table to be predicted τ growth factor
$\widehat{T_{ij}^{0}}$ model outcome of trip quantity (current situation)
$\hat{T}_{ij}$ model outcome of trip quantity (future situation)

We can distinguish various levels of updating a base matrix, ranging from simple to complex. In a first class of approaches, the growth factors do not reflect changes in the network and consider only changes in socio-economic conditions in the study area. The other approaches do reflect changes in interzonal accessibility.

A. Network independent base matrix updating.
A.1 General growth factor τ

$$T_{ij} = \tau T_{ij} \qquad \forall i, j$$

τ may be determined by general factors expressing growth in activities such as demographic growth, economic growth etc.

A.2 Origin or destination specific growth factors τ

$$T_{ij} = \tau_i T_{ij}^0 \qquad or\ T_{ij} = \tau_j T_{ij}^0 \qquad \forall i, j$$

Growth factors τ may be derived from trip end models applied to current and future conditions respectively.

A.3 Two sets of independently and exogenously determined growth factors for origins and destinations.

$$T_{ij} = \tau_j \tau_i T_{ij}^0 \qquad \forall i, j$$

with constraints:

$$\tau_i \sum\nolimits_j T_{ij}^0 = \sum_j T_{ij}\, \forall i$$
$$\tau_j \sum\nolimits_i T_{ij}^0 = \sum_j T_{ij}\, \forall i$$

$$\sum\nolimits_i (\tau_i \sum_j T_{ij}^0) = \sum_j (\tau_j \sum_i T_{ij}^0)$$

This updating problem can be iteratively solved by bi-proportional fitting. Alternatively, growth factors τ may be derived from trip end models applied to current and future conditions successively.

B. Network dependent base matrix updating.
In this case the growth factors reflect changes that are OD-relation specific. These changes can be calculated using trip distribution models applied to current and future conditions.

$$T_{ij} = \tau_{ij} T_{ij}^0 \quad with \quad \tau_{ij} = \hat{T}_{ij} / \hat{T}_{ij}^0$$

It may even be considered to combine the approaches A.3 and B into one joint base year matrix updating.

Derived quantities; network performance

After trip distribution has been computed, various quantities can be derived. These quantities play a key role in judging a transport network. They may be derived in the following ways:

Notation:
B_t = total travel time
B_k = total travel costs
B_l = total traveled distance

From link characteristics

$$B_t = \sum_a q_a t_a$$

$$B_k = \sum_a q_a c_a$$

$$B_l = \sum_a q_a l_a$$

q_a: flow on link *a*
t_a : travel time on link *a*
c_a : travel costs of link *a*
l_a : length of link *a*

From route-characteristics

$$B_t = \sum_i \sum_j \sum_r T_{ij}^r t_{ij}^r$$

$$B_k = \sum_i \sum_j \sum_r T_{ij}^r C_{ij}^r$$

$$B_l = \sum_i \sum_j \sum_r T_{ij}^r l_{ij}^r$$

T_{ij}^{r} = number of trips from i to j via route r
t_{ij}^{r} = travel time from i to j via route r
C_{ij}^{r} = travel costs from i to j via route r
l_{ij}^{r} = length of route r from i to j

In Figure 4 and Figure 5 these computational schemes are illustrated

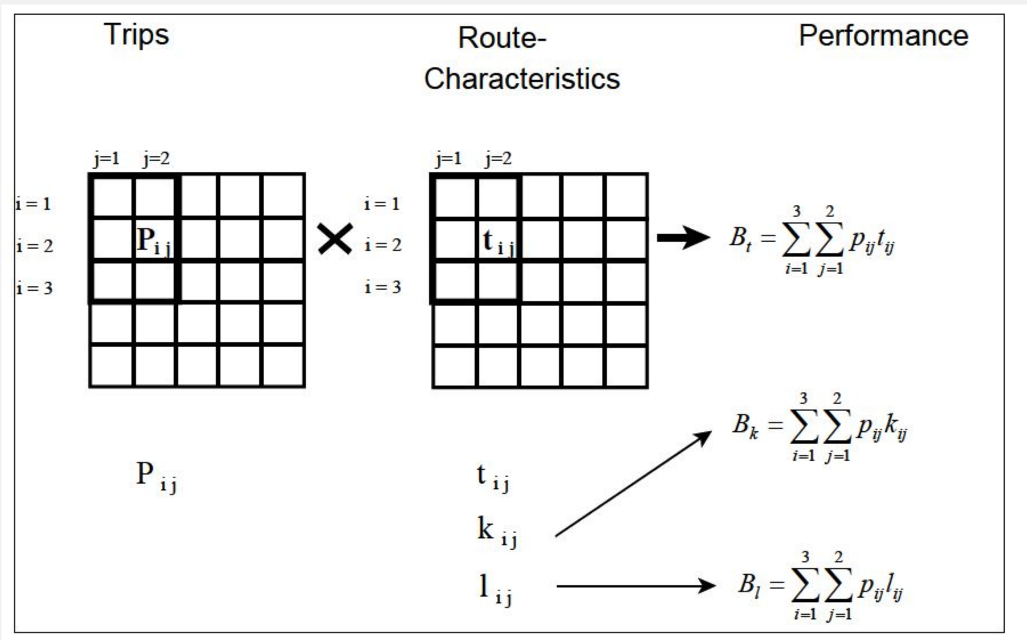

Figure 4: Computing network performance index from trip distribution data.

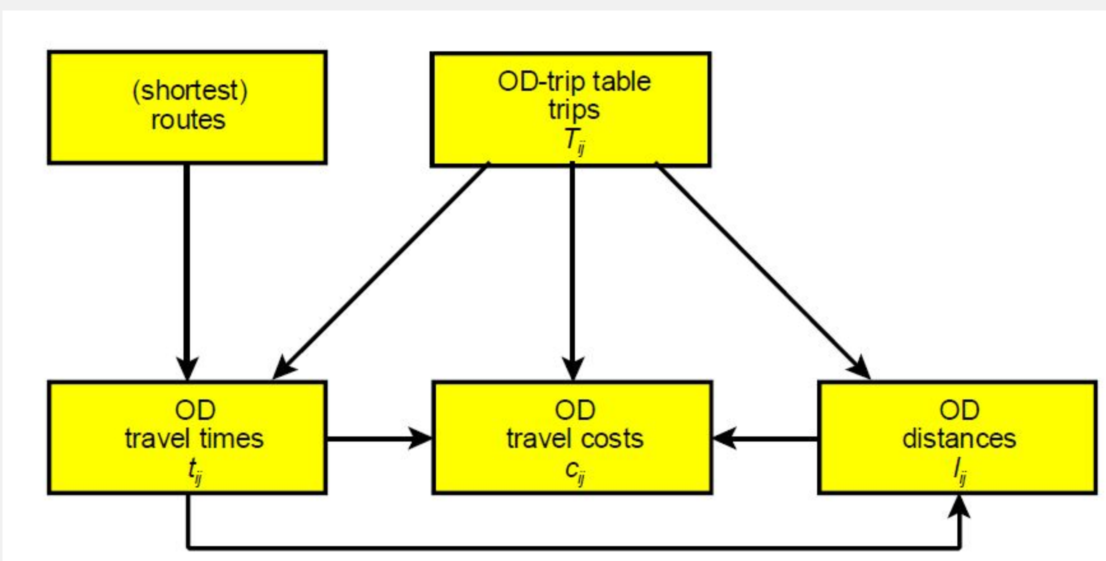

Figure 5: Computing network performance index.

Departure time choice

The *time* of an activity influences the utility that is derived from it, and hence influences the utility of the trip that is needed for this activity. Employees usually have preferred times to start and end their daily job. Starting early or late brings about a certain disutility. This disutility is sometimes accepted if making the trip at the preferred time would bring about an even higher disutility, due to congestion on the roads or discomfort and irregularity in public transit. The phenomenon of travelers avoiding the peak hour is referred to as peak spreading.

Peak spreading is one of the pitfalls in transport planning. Not taking peak spreading into account leads to, among other things, an underestimation of travel demand in the peak hour: building new infrastructure in most cases leads to an inevitable 'back to the peak' effect. A way to introduce departure time choice into the chain of transport models is to value early and late arrivals using the utility scale. If travelers start early to avoid congestion, they trade off the disutility of starting early against the disutility of incurring extra travel time due to congestion. Departure time choice (given travel mode) is usually modeled as a choice between a number of discrete time intervals, assuming that travelers maximize a utility that may be decomposed in the following components:

- utility of the activity (constant)
- disutility of the free flow travel time (constant)
- disutility of the travel time loss due to congestion (time dependent)
- disutility of arriving early (time dependent)
- disutility of arriving late (time dependent)

The way these components are valued differs per from person to person.

Figure 6 illustrates the (hypothetical) disutility function D given the preferred arrival time AT. Arriving late is penalized more than arriving early, and arriving much early is associated with a large disutility per time unit (see the slope in Figure 6). New infrastructure causes changes in congestion levels.

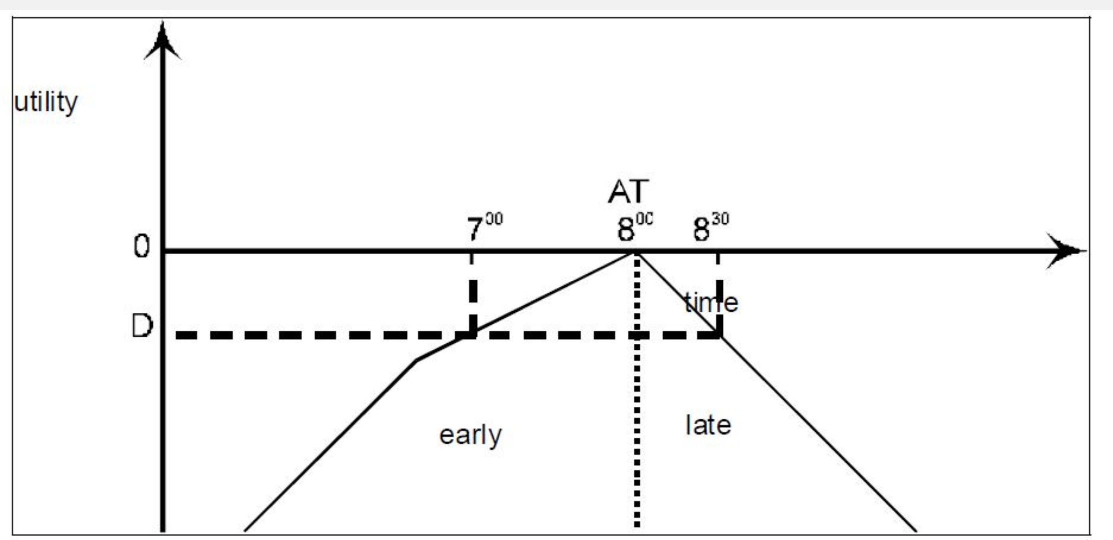

Figure 6: (dis)utility as a function of time (AT: preferred arrival time).