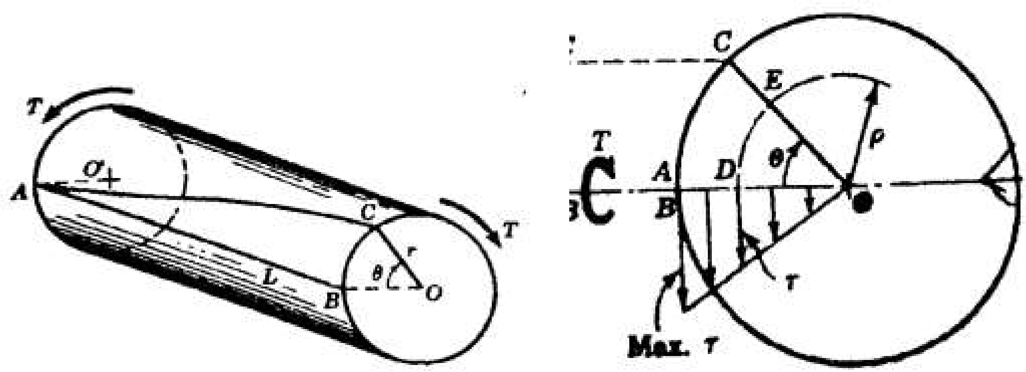
Second Class /Materials Engineering Department

Torsion(twisting)

Derivation of Torsion Formula

In deriving the torsion formulas , we make the following assumptions. The first two apply only to shaft of circular section:

1. Circular section remain circular.
2. Plane section remain plane and don't warp.
3. The projection upon a transverse section of straight radial line in the section remains straight. As show fig.2
4. Shaft is loaded by twisting couples(torque) in planes that are perpendicular to the axis of the shaft.
5. Stresses don't exceed the proportional limit.



(a) (b)

Fig.1

The total shearing deformation δ_s ,as show fig.1 is given by:

$\delta_s = DE = \rho\theta$ -------(a)

where:

DE is the arc length of a circle whose radius ρ

ρ is a circle radius

θ is the twisted angle in radian (not degree)

(((to express θ in degree multiply by $180/\pi = 57.3$ deg/ rad)))

The unit deformation of this fiber(called Shear strain) is

$\gamma = \delta_s / L = \rho\theta / L$ --------(b)

the shearing stress is determined from Hooke's law:

$\tau = G\,\gamma = (G\theta/L)\rho$ -------(c)

where

τ is shear stress

G is modulus of rigidity for shaft materials.

Now , a differential area of section M-N at a radial distance ρ from the axis of the shaft carries the differential load , as show fig.2

$dP = \tau\, dA$.

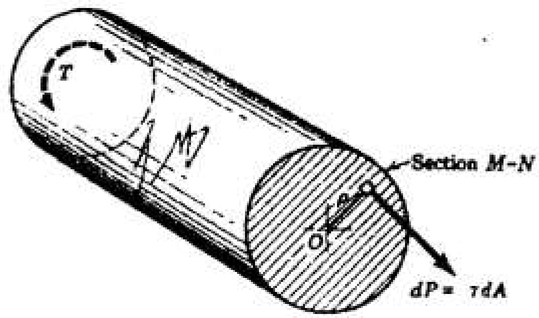


Fig.2

The fact that applied torque T equal the resisting torque T_r , where T_r is the sum of resisting torque developed by all differential loads dP.

$$T = Tr = \int \rho\, dP = \int \rho\, (\tau\, dA)$$

Replacing by its valus from eq. (c) gives

$T = (G\theta/L)\int \rho 2\, dA$

Since $\int \rho 2\, dA = J$

Where

J is the polar moment of intertia of cross section.

$T= G\, \theta\, J/L$

Then

$\theta = TL/GJ$ --------(1)

by replacing the product $G\theta/L$ in eq.(c)by T/J

we obtain

$\tau = T\rho/J$ -------(2)

But, the maximum shearing stress is obtained at surface of shaft then repaced ρ by r radius of shaft.

Max. $\tau = Tr/J$ ------(2a)

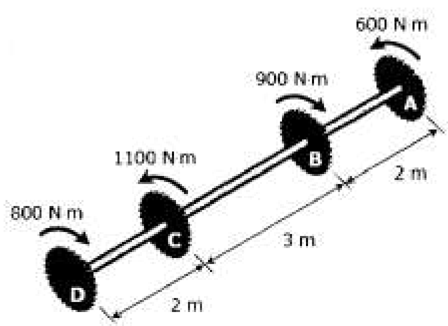
When J value is applied in eq.2a , we obtained:

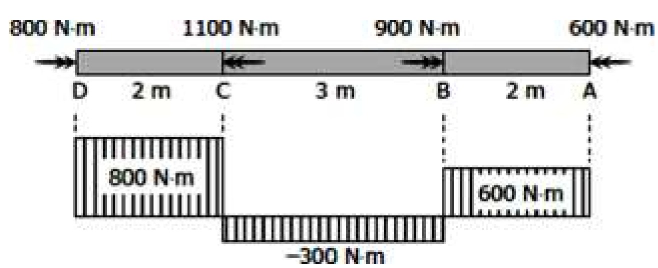
Solid shaft: $$\text{Max. } \tau = \frac{2T}{\pi r^3} = \frac{16T}{\pi d^3}$$

Hollow shaft: $$\text{Max. } \tau = \frac{2TR}{\pi(R^4 - r^4)} = \frac{16TD}{\pi(D^4 - d^4)}$$

Ex/ An aluminum shaft with a constant diameter of 50 mm is loaded by torques applied to gears attached to it as shown in fig below. Using G = 28 GPa, determine the shearing stress for each segment and the relative angle of twist of gear D relative to gear A.

(Note/ review question 210 in the book , you can see the similarity of solution method between two questions)





$$\tau_{max} = \frac{16T}{\pi d^3}$$

$$\tau_{max_{A\text{-}B}} = \frac{16T}{\pi d^3}$$

= 16 x 600 N.m /π (50mm)^3 = 9600 N.m/ π 12500 x 10^{-6}m= ?

$$\tau_{max_{B\text{-}C}} = \frac{16T}{\pi d^3}$$

= 16 x 300N.m/π (50mm)^3 = 4800 N.m/ π 12500 x 10^{-6}m= ?

$$\tau_{max_{C\text{-}D}} = \frac{16T}{\pi d^3}$$

=16 x 800N.m/ π (50mm)^3= 12800 N.m/ π 12500x10^{-6}mm= ?

$$\theta = \frac{TL}{JG}$$

Rotation of D relative to A:

$$\theta_{D/A} = \frac{1}{JG} \Sigma TL$$

$$\theta_{D/A} = \frac{1}{\frac{1}{32}\pi(50^4)(28\,000)} [\,800(2) - 300(3) + 600(2)\,](1000^2)$$

$$\theta_{D/A} = 0.1106 \text{ rad} \;\; \text{X } 57.3 \text{ deg/rad}$$

$$\theta_{D/A} = \text{?} \quad ^\circ$$