4.5. Plastic Moment of Resistance

A simply supported steel beam of a doubly symmetrical cross section is braced to prevent lateral torsional buckling while its flanges and web are compact to prevent local buckling. The beam is subjected at its middle to load (P) produces maximum bending moment of $(P.L/4)$ and equals compressive and tensile stresses. So, flexural stresses produced in the beam are illustrated in Figure 4.5 and generally given by:

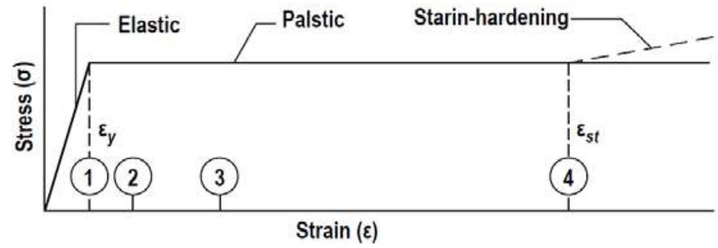
$$f = M/S$$

where:

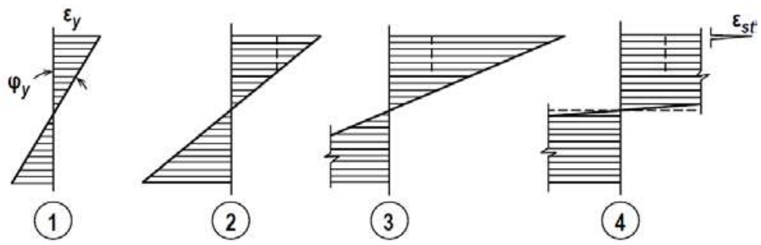
f: flexural stress.

M: bending moment.

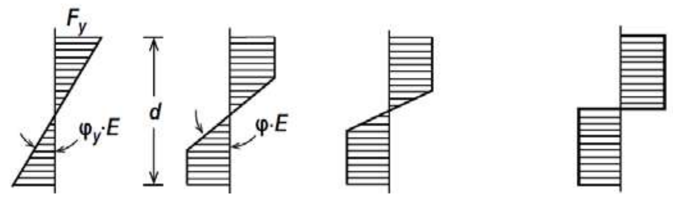
S: section modulus of the beam.



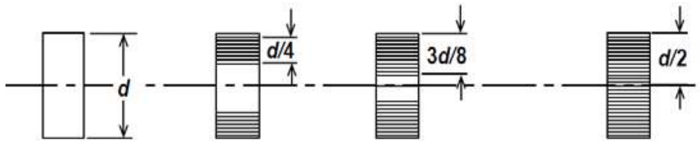
a. Stress-Strain Curve of Structural steel



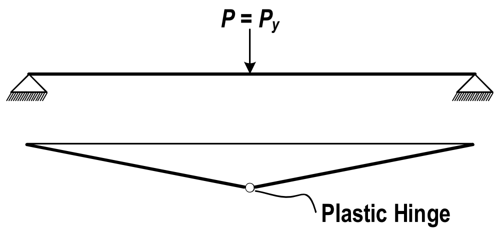
b. Strain Distributions



c. Stress Distributions



d. Yield Distributions



e. Plastic Hinge Development

Figure 4.5: Flexural Stress Development in Steel Beam

By gradual increasing of (P), the flexural stresses (f) in the beam increase linearly. At a stage of load increasing, the maximum stress in the extreme fibers of the beam firstly reach the yield stress (F_y) while stresses in the rest fibers still less than and the stress distribution remains linear, as in Figure 4.5c. At this stage, the corresponding moment is given by:

$$M_y = F_y.S$$

where

M_y: yield moment.

F_y: yield stress of steel

As the load (P) is increased further, the behavior of steel changes from elastic to plastic but the stresses in the extreme fibers remain constant at (F_y) level. The plastic range portion of the curve is indicated by ①-④ as shown in Figure 4.5a. Within the plastic portion, the stresses in more and more interior fibers increase and approach (F_y) level.

At the instant when the flexural stresses in all the beam fibers reach (F_y) a plastic hinge will form at the point of maximum bending moment; the applied load becomes (P_y) as shown in Figure 4.5a and the referred to as collapse load while the corresponding moment is given by:

$$M_P = F_y.Z$$

where

M_P: plastic moment.

Z: plastic section modulus of the beam.

Finally, the nominal flexural strength of the section can be designed as equal to the plastic moment ($M_n = M_P$) in a procedure called the plastic design method or capacity design method; while called the plastic analysis method in case of structural analysis. However, this method is only applicable for steel structures because it takes advantage of structural steel ductility; it referred to as inelastic design method in AISC specifications from 2005 and permitted for steels with yield strength not greater than 65 ksi.

Primarily, this method was developed for indeterminate structures such as continuous beams and frames not for determinate structures such as simple beams. Because when a plastic hinge is developed, the simple beam will be unstable due to presence of three hinges; two at the supports and one at the span.

4.6. Shape Factor for Indeterminate Members

According to the following expression:

$$M_P/M_y = Z/S$$

The ratio (Z/S) is defined as the shape factor which is a dimensionless parameter always > 1.0 so that (M_P) is always greater than (M_y). The shape factor depends on the cross section of members and its value equal to:

- Wide-flange sections: 1.1
- Rectangular sections: 1.5
- Solid circular sections: 1.7

The determination of the shape factor value for any section is depending on (S) value about the elastic neutral axis of the section and (Z) value about the plastic neutral axis of the same section.

$$S = I/c$$

where:
I: elastic moment of inertia of beam.
c: centroid of required area about neutral axis.

The (Z) value can be derived from Figure 4.6 below:

$$\sum F = 0 \rightarrow F_y.A_1 - F_y.A_2 = 0$$

$$\therefore A_1 = A_2 = A/2$$

$$M_P = F.\text{arm} = F_y.A/2(\text{y}_1 + \text{y}_2)$$

$$\therefore Z = 0.5A(\text{y}_1 + \text{y}_2)$$

where:
A: total area of the section.
y_1: centroid of above area about plastic neutral axis.
y_2: centroid of bottom area about plastic neutral axis.

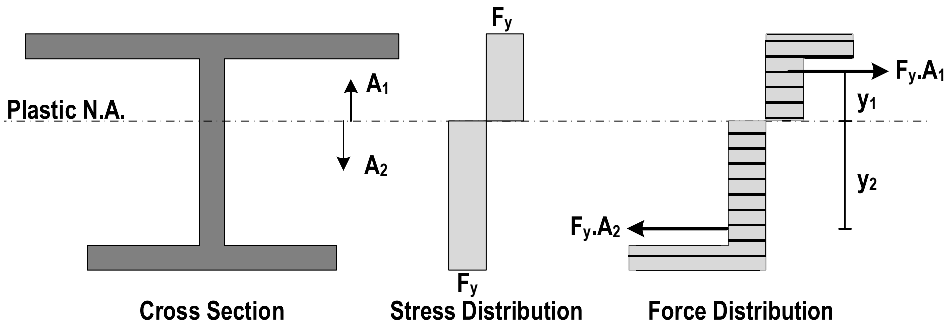


Figure 4.6: Equilibrium About Plastic Neutral Axis

Example 4.3

Determine the elastic section modulus (S), plastic section modulus (Z), yield moment (M_y) and the plastic moment (M_P) of the cross-section shown below. What is the design moment for the beam cross-section? Assume 50 ksi steel.

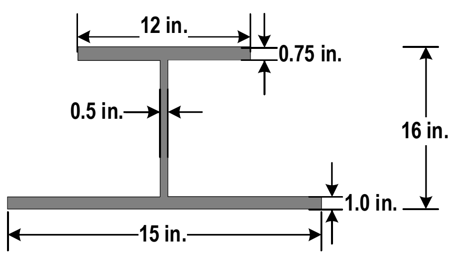


Figure 4.7: Details for Example 4.3

Solution 4.3

- Determine the areas

$$A_{f1} = 12(0.75) = 9 \text{ in}^2$$

$$A_{f2} = 15(1.0) = 15 \text{ in}^2$$

$$A_w = 0.5(16 - 0.75 - 1.0) = 7.125 \text{ in}^2$$

$$A = 9 + 15 + 7.125 = 31.125 \text{ in}^2$$

- Determine the yield moment

$$\bar{y} = [9(16 - 0.75/2) + 7.125(8.125) + 15(0.5)]/31.125 = 6.619 \text{ in}$$
$$I = 12(0.75^3)/12 + 9(9.006^2) + 0.5(14.25^3)/12 + 7.125(1.506^2) + 15(1^3)/12 + 15(6.119^2) = 1430 \text{ in}^4$$
$$S = I/c = 1430/(16 - 6.619) = 152.43 \text{ in}^3$$
$$M_y = F_y.S = 50(152.43) = 7624.5 \text{ kip-in.} = 635.125 \text{ kip-ft.}$$

- Determine the plastic moment

$$15(1.0) + 0.5\left(\bar{y}_p - 1.0\right) = 31.125/2 = 15.5625$$
$$\bar{y}_p = 2.125 \text{ in}^2$$
$$\mathrm{y}_1 = [9(13.5) + 6.5625(6.5625)]/15.5625 = 10.5746 \text{ in}$$
$$\mathrm{y}_2 = [0.5625(05625) + 15(1.625)]/15.5625 = 1.5866 \text{ in}$$
$$Z = 0.5A(\mathrm{y}_1 + \mathrm{y}_2) = 15.5625(10.5746 + 1.5866) = 189.26 \text{ in}^3$$
$$M_p = F_y.Z = 50(189.26) = 9462.93 \text{ kip-in.} = 788.58 \text{ kip-ft}$$

- Check the shape factor and determine the design moment

$$M_p/M_y = 788.58/635.125 = 1.24 < 1.5 \qquad \text{OK!}$$
$$\text{Design Moment} = \phi_b M_p = 0.9(788.58) = 709.72 \text{ kip-ft}$$

4.7. Built-Up Sections

Built-up sections consist of two or more components connected together when single standard shape does not accommodate the structural or architectural requirements. AISC Manual Table 1-19 provides the section properties of W- and S-shapes with cap channels. The properties of other built-up sections must be determined manually.

Example 4.4

Determine the plastic moment of resistance for the cover plated W10x88 section shown below. Ignore the effect of the web fillets.

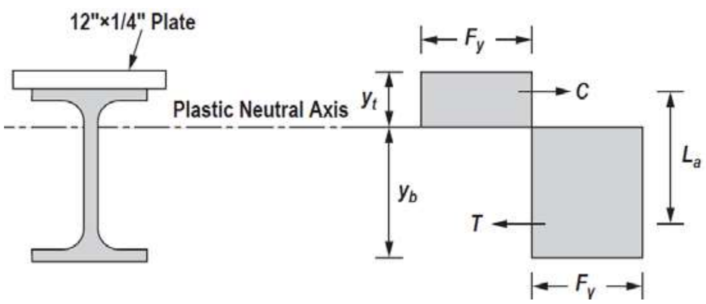


Figure 4.8: Details for Example 4.4

Solution 4.4

- Determine the areas

Area of W-shape

Table 1-1	Section	A	d	t_w	b_f	t_f
page 1-26	W10×88	26.0	10.8	0.605	10.3	0.990

$A_W = 26.0 \text{ in}^2$

$A_f = 10.3(0.99) = 10.2 \text{ in}^2$

$A_w = 0.605[10.8 - 2(0.99)] = 5.34 \text{ in}^2$

The area of plate

$A_P = 12(0.25) = 3.0 \text{ in}^2$

The total area of the built-up section

$A_{comp} = A_W + A_P = 26 + 3 = 29 \text{ in}^2$

- Determine the plastic moment

The areas in tension and compression after the formation of a plastic hinge

$A_C = A_T = A/2 = 29/2 = 14.5 \text{ in}^2$

The plastic neutral axis above the base

$A_T = A_f + t_w(y_b - t_f)$

$14.5 = 10.2 + 0.605(y_b - 0.99)$

$\therefore y_b = 8.1 \text{ in}$

$y_t = 11.05 - 8.1 = 2.95 \text{ in}$

The centroids of plastic areas about PNA

$y_1 = [0.605(1.71^2)/2 + 10.2(2.205) + 3(2.825)]/14.5 = 2.2 \text{ in}$

$y_2 = [0.605(7.11^2)/2 + 10.2(7.605)]/14.5 = 6.4 \text{ in}$

The lever arm

$L_a = y_1 + y_2 = 2.2 + 6.4 = 8.6 \text{ in}$

The plastic section modulus

$Z = A_T . L_a = 14.5(8.6) = 124.7 \text{ in}^3$

The plastic moment of resistance

$M_p = F_y . Z = 50(124.7)/12 = 519.58 \text{ kip-ft}$

Assignment

A simply supported beam spanning 30 ft with concentrated loads at the third points of the span. Each load consists of a dead load component of $w_D = 5$ kips, which includes an allowance for the weight of the beam, and a live load component of $w_L = 15$ kips. The beam is continuously braced on its compression flange and has a yield stress of $F_y = 50$ ksi. Find the lightest W18 section with adequate flexural capacity. Then, determine the flexural capacity of your selected section according to plastic design method.

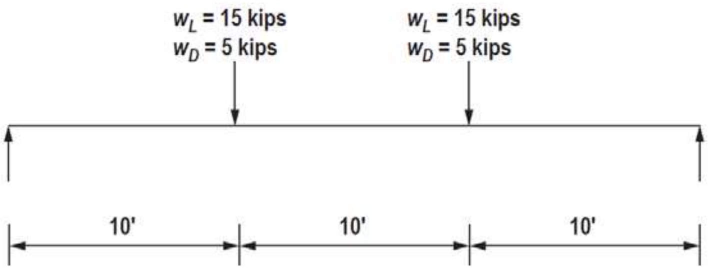


Figure 4.9: Details for Assignment