

Q1/ Use the Limit Comparison Test to determine whether the following series converge or diverge?

$$\sum_{n=1}^{\infty} \frac{1}{2n^3 - n}$$

Solution -

$$a_n = \frac{1}{2n^3 - n}$$

$$\text{Let } b_n = \frac{1}{n^3}$$

$$\lim_{n \rightarrow \infty} \frac{a_n}{b_n} = \lim_{n \rightarrow \infty} \frac{1}{2n^3 - n} \cdot \frac{n^3}{1} = \frac{1}{2}$$

$\therefore \lim_{n \rightarrow \infty} \frac{a_n}{b_n} = \frac{1}{2} > 0$ (as (1) both convergents or both Diverges.

to test $\sum b_n$

$$\sum_{n=1}^{\infty} b_n = \sum_{n=1}^{\infty} \frac{1}{n^3}$$

p-series with $p = 3 > 1$ $\therefore$ Converges

$$\therefore \sum a_n = \sum_{n=1}^{\infty} \frac{1}{2n^3 - n} \text{ Converges.}$$

Q2/ Determine whether the series

$$\sum_{n=1}^{\infty} \frac{n^2 \cdot 2^{n-1}}{5^n}$$

is convergent or divergent.

solution :

we use the root test :

$$\lim_{n \rightarrow \infty} \sqrt[n]{a_n} = \lim_{n \rightarrow \infty} \sqrt[n]{\frac{n^2 \cdot 2^{n-1}}{5^n}}$$

Q3/ Determine whether the series

$$\sum_{n=1}^{\infty} \frac{n^2 \cdot 2^{n-1}}{5^n}$$

is convergent or divergent.

solution:

Let $a_n = \frac{n^2 \cdot 2^{n-1}}{5^n}$ we use the root test:

$$\lim_{n \rightarrow \infty} \sqrt[n]{a_n} = \lim_{n \rightarrow \infty} \sqrt[n]{\frac{n^2 \cdot 2^{n-1}}{5^n}}$$

$$= \lim_{n \rightarrow \infty} \frac{(\sqrt[n]{n})^2 \cdot 2^{\frac{n-1}{n}}}{5^{\frac{n}{n}}} = \frac{(1)^2 \cdot 2^1}{5^1}$$

$$= \frac{2}{5} < 1$$

$\therefore R < 1 \therefore$ the series is convergent.

$$\sqrt[n]{n} = 1$$

Q4/ Test the convergence of the following series:

$$\sum_{n=1}^{\infty} \frac{3+2\sin(n^2)}{n^2}$$

solution: using comparison Test

$$a_n = \frac{3+2\sin(n^2)}{n^2}$$

$$b_n = \frac{3}{n^2}$$

$$a_n \leq b_n$$

To test $\sum b_n$

$$\sum_{n=1}^{\infty} \frac{3}{n^2}$$

$\therefore \sum b_n$ converges

p-series
 $p = 2 > 1$
 $\therefore$ the series is converges.

$$\therefore \sum a_n = \sum_{n=1}^{\infty} \frac{3+2\sin(n^2)}{n^2} \text{ converges}$$

Q5/ Determine whether $\sum_{n=1}^{\infty} n^2 e^{-n^3}$ converges or diverges.

Solution:

using integral test

$$f(x) = x^2 e^{-x^3}$$

$$\int_{n=N}^{\infty} f(x) dx = \int_1^{\infty} x^2 e^{-x^3} dx$$

$$= -\frac{1}{3} \int_1^{\infty} -3x^2 e^{-x^3} dx$$

$$= -\frac{1}{3} \left[e^{-x^3} \right]_1^{\infty} = -\frac{1}{3} \left[\frac{1}{e^{x^3}} \right]_1^{\infty}$$

$$= -\frac{1}{3} \left[\frac{1}{e^{\infty}} - \frac{1}{e^1} \right] = \frac{1}{3e} \neq \infty$$

$$\sum_{n=1}^{\infty} n^2 e^{-n^3}$$

Converges.

Q6/ Test the convergence of the following series :

$$\sum_{n=2}^{\infty} \frac{1}{n[\ln(n)+1]}$$

solution :

using integral test

$$f(x) = \frac{1}{x[\ln(x)+1]}$$

$$\int_{n=N}^{\infty} f(x) dx = \int_{n=2}^{\infty} \frac{1}{x[\ln(x)+1]} dx$$

$$= \int_2^{\infty} \frac{\frac{1}{x}}{[\ln(x)+1]} dx$$

$$= \ln[\ln(x)+1] \Big|_2^{\infty} = \infty - \infty = \infty$$

so by integral test

$$\sum_{n=2}^{\infty} \frac{1}{n[\ln(x)+1]} \text{ Diverges}$$

Q7/ Test the convergence of the following series :

$$\sum_{n=1}^{\infty} \frac{4^n}{3^{n+2}}$$

solution :

let $a_n = \frac{4^n}{3^{n+2}}$ we use the root test :

$$\lim_{n \rightarrow \infty} \sqrt[n]{a_n} = \lim_{n \rightarrow \infty} \frac{[4^n]^{\frac{1}{n}}}{[3^{n+2}]^{\frac{1}{n}}}$$

$$= \lim_{n \rightarrow \infty} \frac{4}{3^{\frac{n+2}{n}}} = \lim_{n \rightarrow \infty} \frac{4}{3^{\frac{1+2/n}{1}}}$$

$$= \frac{4}{3^{\frac{1+0}{1}}} = \frac{4}{3} > 1 \quad \text{the series is Diverges}$$

Q8/ Test the convergence of the following series :

$$\sum_{n=1}^{\infty} \frac{\sqrt{n^4+1}}{n^5-3}$$

solution : using the L.C.T

$$a_n = \frac{\sqrt{n^4+1}}{n^5-3}$$

$$\text{let } b_n = \frac{\sqrt{n^4}}{n^5} = \frac{n^2}{n^5} = \frac{1}{n^3}$$

$$\lim_{n \rightarrow \infty} \frac{a_n}{b_n} = \lim_{n \rightarrow \infty} \frac{\sqrt{n^4+1}}{n^5-3} \cdot \frac{n^5}{\sqrt{n^4}}$$

$$= \lim_{n \rightarrow \infty} \frac{n^5}{n^5-3} \cdot \sqrt{\frac{n^4+1}{n^4}}$$

$$= \lim_{n \rightarrow \infty} \frac{1}{1 - \frac{3}{n^5}} \cdot \sqrt{\frac{1 + \frac{1}{n^4}}{1}}$$

$$= \frac{1}{1 - \frac{3}{\infty}} \cdot \sqrt{\frac{1 + \frac{1}{\infty}}{1}} = 1 \cdot 1 = 1 > 0$$

Case (1) both
converges or both
Diverges

To test $\sum b_n$

$$\sum_{n=1}^{\infty} b_n = \sum_{n=1}^{\infty} \frac{1}{n^3}$$

$$\therefore \sum_{n=1}^{\infty} a_n = \sum_{n=1}^{\infty} \frac{\sqrt{n^4+1}}{n^5-3}$$

p-series with
 $p=3 > 1 \therefore$ Converges
Converges.

Q9 Test the convergence of the following series :

$$\sum_{n=1}^{\infty} \frac{1}{\sqrt{1+4n^2}}$$

solution : using comparison test

$$a_n = \frac{1}{\sqrt{1+4n^2}} \quad , \quad b_n = \frac{1}{\sqrt{n^2}} = \frac{1}{n}$$

$$a_n \leq b_n$$

To test b_n , $\sum_{n=1}^{\infty} \frac{1}{n}$

افضل الشرط
لذلك نستخدم L.C.T

p-series with $p=1$ diverges so we need to test by L.C.T

$$\lim_{n \rightarrow \infty} \frac{a_n}{b_n} = \lim_{n \rightarrow \infty} \frac{1}{\sqrt{1+4n^2}} \cdot \frac{\sqrt{n^2}}{1}$$

$$= \lim_{n \rightarrow \infty} \sqrt{\frac{n^2}{1+4n^2}} = \lim_{n \rightarrow \infty} \sqrt{\frac{1}{\frac{1}{n^2} + 4}}$$

$$= \sqrt{\frac{1}{\frac{1}{\infty} + 4}} = \sqrt{\frac{1}{4}} > 0 \quad \text{Cas(1)}$$

$$\therefore \sum a_n = \sum \frac{1}{\sqrt{1+4n^2}} \text{ is Diverges}$$

both converges
or both diverges