Input, Output and Characteristics of Control System

1- Introduction

The main part in any control system is the plant or process which is needed to be controlled. Actually a plant is a black box that transforms an input to an output. It is usually not difficult to recognize the plant itself, but identifying what to use as control input and output. Therefore we can define the input and output of the control system as in the following:

a) **Input or control (process) input** is a quantity that we can adjust directly. By adjusting the input, we hope to see the effect on the output in a favorable way.

b) **Output or process output** is the quantity we want to control: we want the output to track (follow) the reference value (the set point).

2- Standard Test Signals of control systems

After mathematical modelling of the required system such as the design of either electrical, mechanical, or electromechanical systems or any other engineering system, the next step would be to obtain and analyze its response according to specific input . Now question is the input can be a time varying function or it may be a *random signal.* Thus we need some standard test signals of control systems which strain the system very severely.

These standard input signals are

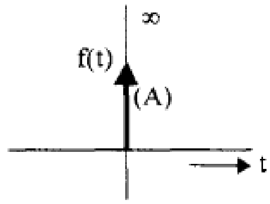
- impulse input ,
- step input ,
- ramp input
- Parabolic input.

By using above standard test signals of control systems, analysis and design of control systems are carried out, defining certain performance measures for the system.

It will be described briefly the standard input signals of control system as in the following:-

a) Impulse Signal

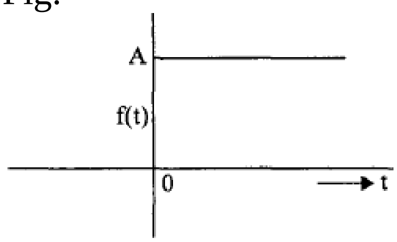
An impulse signal is denoted by f (t) = A (t). If A = 1 it is called a unit impulse function.
An impulse signal is shown in Fig



The impulse function is zero for all *t* not equal to 0 and it is infinity at *t* = 0.

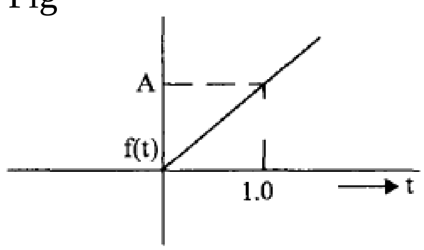
b) Step Signal

It is zero for *t* < 0 and suddenly rises to a value A at *t* = 0 and remains at this value for *t* > 0: It is denoted by *f(t)* = *Au (t)*. If A = 1, it is called a *unit step function.*
In bellow a step signal is shown in Fig.



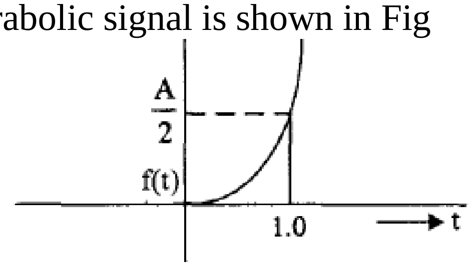
c) Ramp signal

It is zero for *t* < 0 and uniformly increases with a slope equal to A. It is denoted by *f (t)* = *At.*
If the slope is unity, then it is called a *unit ramp signal.*
In bellow a ramp signal is shown in Fig



d) Parabolic signal

A parabolic signal is denoted by $f(t) = A\frac{t^2}{2}$. If A is equal to unity then it is known as a *unit parabolic signal.* In bellow a parabolic signal is shown in Fig



3- Time Domain Specifications of Control System

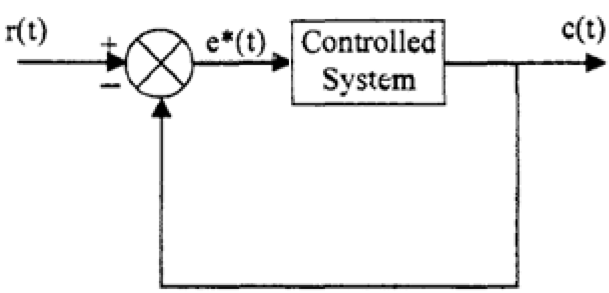
We can analyze the response of the control systems in both the time domain and the frequency domain. Let us now discuss about the time response analysis of control systems.

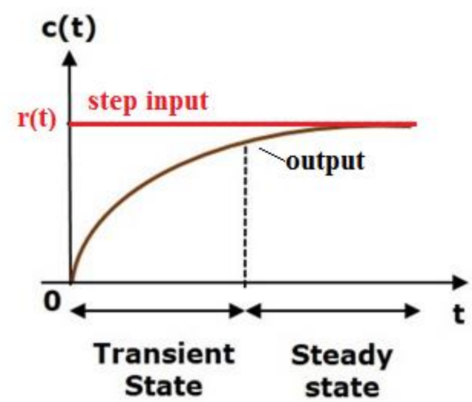
What is Time Response?

If the output of control system for an input varies with respect to time, then it is called the **time response** of the control system. The time response consists of two parts.

- Transient response
- Steady state response

The response of control system in time domain is shown in the following figure.





In this control system there is both the transient and the steady states as indicated in the figure. The responses corresponding to these states are known as transient and steady state responses.

Mathematically, we can write the time response of the output control signal *c (t)* as

$$c\ (t) = c_{tr}\ (t) + c_{ss}\ (t)$$

Where,

- $c_{tr}(t)$ is the transient response
- $c_{ss}(t)$ is the steady state response

Example

Find the transient and steady state terms of the time response of the control system

$$c\,(t)=10+5e^{-t}$$

Sol:-

Here,

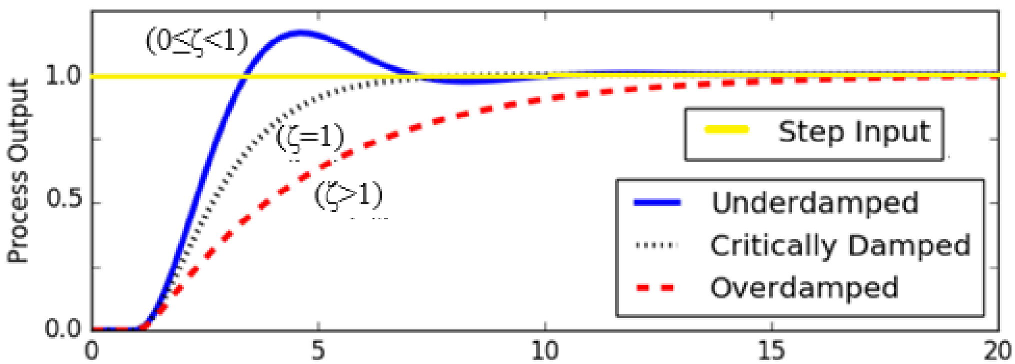
$$\lim_{t\to\infty} c_{tr}(t) = 0$$

The second term ($5e^{-t}$) will be zero as (**t**) denotes infinity. So, this is the **transient term**.
And the first term 10 remains even as (***t***) approaches infinity. So, this is the **steady state term**.

4- Over damped, underdamped and Critical damped in control system

Practically, a second-order linear system is a common description of many dynamic processes. The response depends on whether it is an over damped, critically damped, or underdamped second order system depending on the value of damping factor (ζ) due to the response of the second order system to a step input, hence there is three cases of control system response as shown in figure.

1- Over damped (ζ>1)
2- critically damped (ζ=1)
3- Under damped (0≤ζ<1)



To understand over damped, under damped and Critical damped in control system, Let we take the closed loop transfer function in generic form and analysis that to find out different condition

$$T\,(s) = \frac{\omega_n^{\,2}}{s^2 + 2\delta\omega_n s + \omega_n^{\,2}}$$

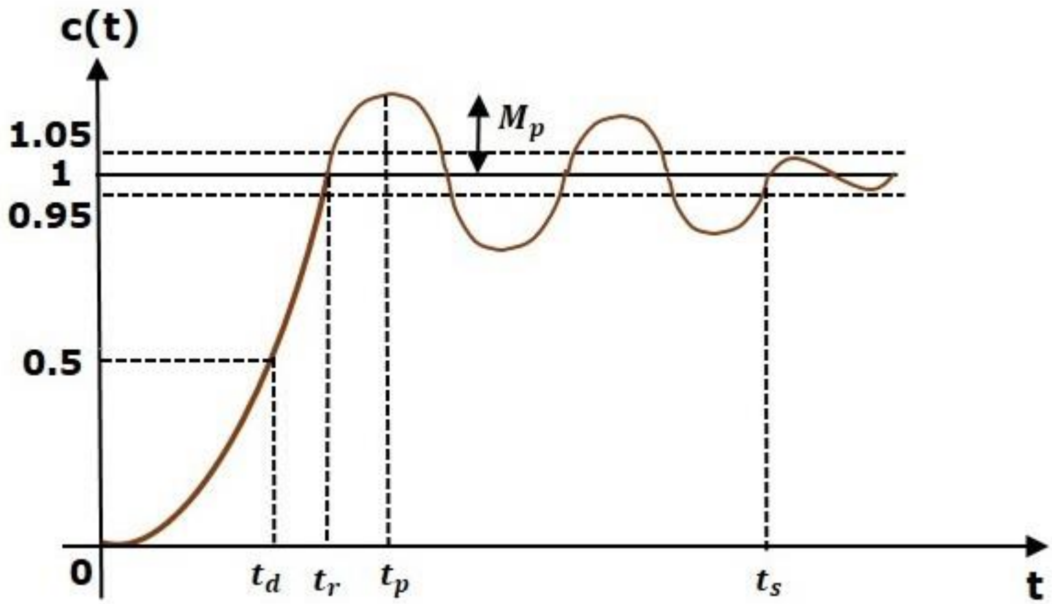
So in this standard case the denominator polynomial of T(s), is

$$D(s) = s^2 + 2\delta\omega_n s + \omega_n^2$$

This denominator is known as the *characteristic polynomial* of the system and D(s) = 0 is known as the *characteristic* equation of the system.

5- Time Domain Specifications of control system

Let us discuss the time domain specifications of the second order system. The step response of the second order system for the underdamped case is shown in the following figure.



All the time domain specifications are represented in this figure. The response up to the settling time is known as transient response and the response after the settling time is known as steady state response. The time domain specifications of control system depend on damping factor (ζ)and natural frequency(w_n) and can be defined as in the follows:-

1- Delay Time (t_d)

It is the time required for the response to reach **half of its final value** from the zero instant.

$$t_d = \frac{1+0.7\delta}{\omega_n}$$

2- Rise Time(t_r)

It is the time required for the response to rise from **0% to 100% of its final value**. This is applicable for the **under-damped systems**. (For the over-damped systems, consider the duration from 10% to 90% of the final value).

$$t_r = \frac{\pi - \theta}{\omega_d} \quad , \quad \theta = \cos^{-1}\delta \quad ,$$

damped frequency ω_d as $\omega_d = \omega_n\sqrt{1-\delta^2}$

3- Peak Time (t_p)

It is the time required for the response to reach the **peak value** for the first time.

$$t_p = \frac{\pi}{\omega_d}$$

4- Peak Overshoot (M_p)

Defined as the deviation of the response at peak time from the final value of response. It is also called the **maximum overshoot**.

5- Percentage of peak overshoot (%Mp)

The percentage of peak overshoot will decrease if the damping ratio δ increases.

$$\%M_p = \left(e^{-\left(\frac{\delta\pi}{\sqrt{1-\delta^2}}\right)}\right) \times 100\%$$

6- Settling time (t_s)

It is the time required for the response to reach the steady state and stay within the specified tolerance bands around the final value. In general, the tolerance bands are 2% and 5%.

$$t_s = \frac{4}{\delta\omega_n}$$

Example

Find the time domain specifications of a control system having the closed loop transfer function

$$\frac{4}{s^2+2s+4}$$

When the unit step signal is applied as an input to this control system.

Sol:-

We know that the standard form of the transfer function of the second order closed loop control system as

$$\frac{\omega_n^2}{s^2 + 2\delta\omega_n s + \omega_n^2}$$

By equating these two transfer functions, we will get the following:-

1- The undamped natural frequency ω_n= 2 rad/sec

2- The damping ratio δ =0.5.

3-

$$\omega_d = \omega_n \sqrt{1-\delta^2} \quad \Rightarrow \omega_d = 2\sqrt{1-(0.5)^2} \quad \Rightarrow \omega_d = 1.732\, rad/sec$$

$$\theta = \cos^{-1}\delta \quad \Rightarrow \theta = \cos^{-1}(0.5) = \frac{\pi}{3}\, rad$$

The following table shows the formulae of time domain specifications, substitution of necessary values and the final values.

Formula	Substitution of values in Formula	Final value
$t_d = \frac{1+0.7\delta}{\omega_n}$	$t_d = \frac{1+0.7(0.5)}{2}$	t_d =0.675 sec
$t_r = \frac{\pi-\theta}{\omega_d}$	$t_r = \frac{\pi-(\frac{\pi}{3})}{1.732}$	t_r =1.207 sec
$t_p = \frac{\pi}{\omega_d}$	$t_p = \frac{\pi}{1.732}$	t_p =1.813 sec
$\%M_p = \left(e^{-\left(\frac{\delta\pi}{\sqrt{1-\delta^2}}\right)}\right) \times 100\%$	$\%M_p = \left(e^{-\left(\frac{0.5\pi}{\sqrt{1-(0.5)^2}}\right)}\right) \times 100\%$	$\% M_p$ =16.32%
$t_s = \frac{4}{\delta\omega_n}$	$t_S = \frac{4}{(0.5)(2)}$	t_s =4 sec