5. Design of Steel Beams for Shear and Torsion

5.1. Shear Stress

The applied loading on a beam results in a shear force V on the beam. Provided that the stresses produced in the beam are within the elastic limit, the shear stress produced at a specific level in the cross section of a member is given by the expression:

$$f_v = V.Q/I.t$$

where:

f_v: shear stress in the section.

V: applied shear force on the section.

Q: statical moment of the area, above the level considered, about the neutral axis of the section.

I: moment of inertia of the section.

t: width of section at the level considered.

Figure 5.1 shows the plot of above expression over the height of a W-section. It is clear from the figure that the maximum shear stress occurs at the neutral axis of the section. Most of the shear capacity of the section is provided by the web of the W-section while only a small portion provided by the flanges.

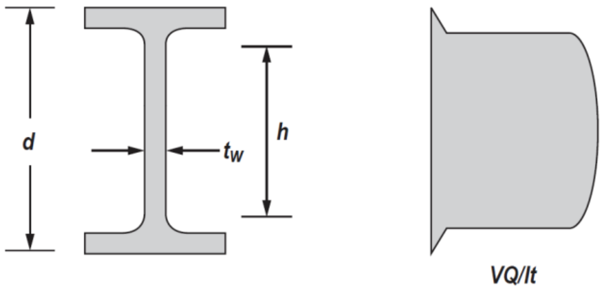

Figure 5.1: Shear distribution in a W-section

It is suitable in design to assume that the applied shear force is resisted by an area equal to the product of the depth of the beam and the thickness of the web. This gives a uniform shear stress over the depth of the beam of

$$f_v = V/A_w$$
$$A_w = d.t_w$$

where:

f_v: shear stress in web.

d: overall depth of the beam.

t_w: web thickness.

This uniform shear stress is approximately 88% of the maximum shear stress.

f_v: shear stress in web.

t_w: web thickness.

Example 5.1

A W16×89 beam, with a yield stress of 50 ksi, is simply supported over a span of 21 ft. The beam supports a uniformly distributed load of 6 kip/ft that includes the self-weight of the beam. The beam is laterally braced at the supports and at the third points of the span. Determine the maximum shear stress and the average shear stress in the beam.

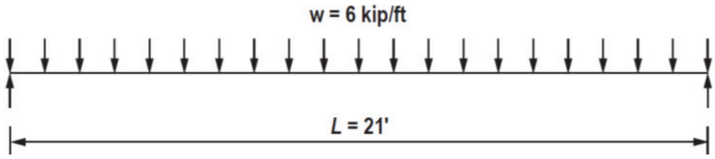

Figure 5.2: Details for Example 5.1

Solution 5.1

Shear force acting on the section

$V = w.L/2$

$= 6(21)/2 = 63$ kip

Properties of W16x89

Table 1-1	Section	d	t_w	I_x
page 1-22	W16×89	16.8	0.525	1300

Properties of WT-shape split from W16x89

Table 1-8	Section	A	d	y
page 1-60	W8x44.5	13.1	8.38	1.7

The statical moment of the area of the W16x89 above the neutral axis

$Q = A(d - y)$

$= 13.1(8.38 - 1.7) = 87.51$ in^3

The shear stress at the neutral axis of the W16x89

$f_v = V.Q/I.t_w$

$= 63(87.51)/[1300(0.525)] = 8.08$ ksi

The average shear stress over the depth of the beam is

$f_v = V/d.t_w$

$= 63/[16.8(0.525)] = 7.14$ ksi

5.2. Shear Limit State of Beam Webs

The nominal shear capacity of a W-shape with unstiffened web depends on the slenderness of the web, and the web slenderness parameter (λ) is defined as:

$\lambda = h/t_w$

where:

h: is clear distance between flanges less the corner radius at each flange for rolled shapes or clear distance between flanges for built-up welded sections.

As the web slenderness parameter of a beam increases, web failure occurs by either

- Plastic yielding of the web in beams with a compact web
- Inelastic web buckling of the web in beams with a noncompact web
- Elastic web buckling of the web in beams with a slender web

The relationship between nominal shear strength and web slenderness is shown in Figure 5.3.

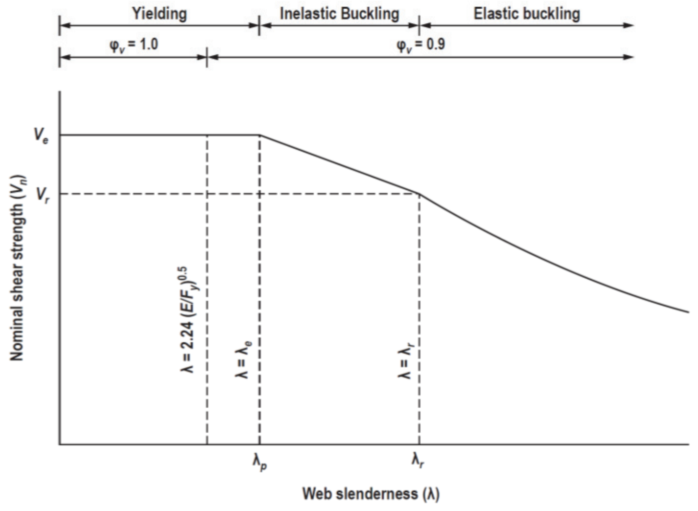

Figure 5.3: Nominal Shear Strength and Web Slenderness, Beams with Unstiffened Webs

5.2.1. <u>Web Yielding</u>

The web yielding is achieved when the slenderness parameter of the web (λ), is less than the plastic slenderness limit (λ_p) which can be given by:

$$\lambda_p = 2.46(E/F_y)^{0.5}$$

- $\lambda \leq 2.24(E/F_y)^{0.5}$ $\qquad\qquad \phi_v = 1.0$
- $2.24(E/F_y)^{0.5} < \lambda \leq 2.46(E/F_y)^{0.5}$ $\qquad \phi_v = 0.9$

The nominal shear strength is given by:

$$V_n = V_p$$

$$V_p = F_v . A_w$$

$$F_v = 0.6F_y$$

$$\therefore V_n = 0.6F_y . A_w$$

where:

F_v: shear yield strength of the steel.

ϕ_v: shear resistance factor of the steel

The design shear strength is:

5.3. <u>Design Flexural Strength</u>

After the nominal shear strength is determined, the design shaer strength may be obtained from AISC Manual Table 3-2 as:

$$\phi_v V_n \geq V_u$$

where:

$\phi_v V_n$: design flexural strength.

V_u: required shear strength using LRFD load combinations.

ϕ_v: resistance factor for sh (ϕ_f)

Table 3-2, for a value of $C_v = 1.0$, provides values of $\phi_v V_n$ to ensure yielding failure of the section.

Example 5.2

A W16x89 beam, with a yield stress of 50 ksi, is simply supported over a span of 20 ft. The beam supports a uniformly distributed dead load of w_D = 2 kip/ft, that includes the self-weight of the beam, and a uniformly distributed live load of w_L = 6 kip/ft. The beam is laterally braced at the supports and at the third points of the span. Determine if the beam is adequate for shear.

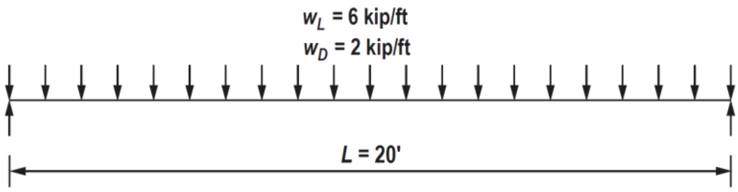

Figure 5.4: Details for Example 5.2

Solution 5.2

$$w_u = 1.25w_D + 1.75w_L$$
$$= 1.25(2) + 1.75(6) = 13 \text{ kip/ft}$$
$$V_u = w_u.\text{L}/2$$
$$= 13(20)/2 = 130 \text{ kip}$$

From page 3-25, select W16x89 made from 50 ksi steel with:

$\phi_v V_n = 265$ kip > 130 kip … satisfactory

• Check

Properties of W16x89

Table 1-1	Section	d	t_w	h/t_w
page 1-22	W16×89	16.8	0.525	27

$$\lambda = h/t_w = 27$$
$$\lambda = 2.24(E/F_y)^{0.5}$$
$$= 2.24(29000/50)^{0.5} = 53.95 > 27$$
$$\rightarrow \phi_v = 1$$
$$\phi_v V_n = 0.6F_y.A_w$$
$$= 0.6(50)[16.8(0.525)] = 264.5 \text{ kip} \quad \therefore \text{OK}$$