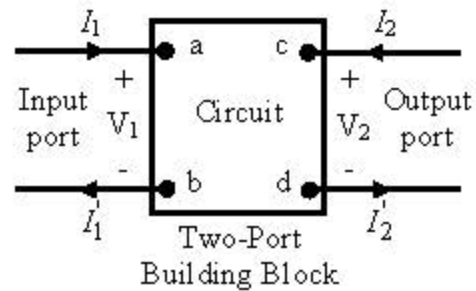


12.1 Two-Port Networks (Circuits):

A two-port network is an electrical network with two separate ports for input and output. It is a basic two-port building block with specified pair of terminals. A port is an access to the network and consists of a pair of terminals; the current entering one terminal leaves through the other terminal so that the net current entering the port equals zero.



There are several restrictions:

1. No storage energy within the circuit.
2. No independent sources within the circuit.
3. The current into the port must equal to the current out of the port $I_1 = I'_1$ and $I_2 = I'_2$.
4. All external connections are made to either input or output ports. No external connections between ports are allowed.

The study of two-port networks is for at least two reasons:

- a. Such networks are useful in communications, control systems, power systems, and electronics. For example, they are used in electronics to model transistors and to facilitate cascaded design.
- b. Knowing the parameters of a two-port network enables to treat it as a "black box" when embedded within a larger network.

12.2 The Terminal Equations:

To characterize a two-port network requires that we relate the terminal quantities I_1, V_1, I_2 and V_2 , out of which two are independent. The various terms that relate these voltages and currents are called parameters. We have a choice of six different ways in which to combine the four variables (I_1, V_1, I_2 and V_2). The six sets of equations are:

$$V_1 = z_{11}I_1 + z_{12}I_2 \quad V_2 = z_{21}I_1 + z_{22}I_2 \quad \dots (1)$$

$$I_1 = y_{11}V_1 + y_{12}V_2 \quad I_2 = y_{21}V_1 + y_{22}V_2 \quad \dots (2)$$

$$V_1 = a_{11}V_2 - a_{12}I_2 \quad I_1 = a_{21}V_2 - a_{22}I_2 \quad \dots (3)$$

$$V_2 = b_{11}V_1 - b_{12}I_1 \quad I_2 = b_{21}V_1 - b_{22}I_1 \quad \dots (4)$$

$$V_1 = h_{11}I_1 + h_{12}V_2 \quad I_2 = h_{21}I_1 + h_{22}V_2 \quad \dots (5)$$

$$I_1 = g_{11}V_1 + g_{12}I_2 \quad V_2 = g_{21}V_1 + g_{22}I_2 \quad \dots (6)$$

The coefficients of the current and/or voltage variables on the right-hand side of the above equations are called the parameters of the two-port circuit.

Note: The impedance (z) and admittance (y) parameters are grouped into the immittance parameters. The (a) and (b) parameters are called the transmission parameters. The (h) and (g) parameters are called hybrid parameters.

Immittance: denote a quantity that either impedance or admittance.

Transmission: give a measure of how the voltage and current are transmitted through the network.

Hybrid: relate cross-variables (V_1 and I_2) to (V_2 and I_1).

12.3 The Two-Port Parameters:

The parameters for any given circuit can be found either by computation or by measurement.

a. Z-Parameters (Ω):

Impedance parameters are commonly used in the synthesis of filters. They are also useful in the design and analysis of impedance-matching networks and power distribution networks. The terminal voltages can be related to the terminal currents as in equation 1 or in matrix form as:

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} z_{11} & z_{12} \\ z_{21} & z_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = [Z] \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

The values of the parameters can be evaluated by setting $I_1=0$ (input port open-circuited) or $I_2=0$ (output port open-circuited). Thus,

$$z_{11} = \left. \frac{V_1}{I_1} \right|_{I_2=0} = \text{input impedance seen looking into port "1" when port "2" is open.}$$

$$z_{12} = \left. \frac{V_1}{I_2} \right|_{I_1=0} = \text{transfer impedance, ratio of port "1" voltage to port "2" current when port "1" is open. = reversed impedance.}$$

$$z_{21} = \left. \frac{V_2}{I_1} \right|_{I_2=0} = \text{transfer impedance, ratio of port "2" voltage to port "1" current when port "2" is open. = forward impedance.}$$

$$z_{22} = \left. \frac{V_2}{I_2} \right|_{I_1=0} = \text{output impedance seen looking into port "2" when port "1" is open.}$$

b. Y-Parameters (Ω):

Admittance parameters are commonly used in the synthesis of filters. They are also useful in the design and analysis of impedance-matching networks and power distribution networks. The terminal currents can be related to the terminal voltages as in equation 2 or in matrix form as:

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = [Y] \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

The values of the parameters can be evaluated by setting $V_1=0$ (input port short-circuited) or $V_2=0$ (output port short-circuited). Thus,

$$y_{11} = \left. \frac{I_1}{V_1} \right|_{V_2=0} = \text{input admittance seen looking into port "1" when port "2" is short.}$$

$$y_{12} = \left. \frac{I_1}{V_2} \right|_{V_1=0} = \text{transfer admittance, ratio of port "1" current to port "2" voltage when port "1" is short. = reversed admittance.}$$

$$y_{21} = \left. \frac{I_2}{V_1} \right|_{V_2=0} = \text{transfer admittance, ratio of port "2" current to port "1" voltage when port "2" is short. = forward admittance.}$$

$$y_{22} = \left. \frac{I_2}{V_2} \right|_{V_1=0} = \text{output admittance seen looking into port "2" when port "1" is short.}$$

A-Parameters

$$\begin{aligned} a_{11} &= \left. \frac{V_1}{V_2} \right|_{I_2=0} \\ a_{12} &= - \left. \frac{V_1}{I_2} \right|_{V_2=0} \Omega \\ a_{21} &= \left. \frac{I_1}{V_2} \right|_{I_2=0} \text{U} \\ a_{22} &= - \left. \frac{I_1}{I_2} \right|_{V_2=0} \end{aligned}$$

B-Parameters

$$\begin{aligned} b_{11} &= \left. \frac{V_2}{V_1} \right|_{I_1=0} \\ b_{12} &= - \left. \frac{V_2}{I_1} \right|_{V_1=0} \Omega \\ b_{21} &= \left. \frac{I_2}{V_1} \right|_{I_1=0} \text{U} \\ b_{22} &= - \left. \frac{I_2}{I_1} \right|_{V_1=0} \end{aligned}$$

H-Parameters

$$\begin{aligned} h_{11} &= \left. \frac{V_1}{I_1} \right|_{V_2=0} \Omega \\ h_{12} &= \left. \frac{V_1}{V_2} \right|_{I_1=0} \\ h_{21} &= \left. \frac{I_2}{I_1} \right|_{V_2=0} \text{U} \\ h_{22} &= \left. \frac{I_2}{V_2} \right|_{I_1=0} \Omega \end{aligned}$$

G-Parameters

$$\begin{aligned} g_{11} &= \left. \frac{I_1}{V_1} \right|_{I_2=0} \text{U} \\ g_{12} &= \left. \frac{I_1}{V_2} \right|_{V_1=0} \\ g_{21} &= \left. \frac{V_2}{V_1} \right|_{I_2=0} \Omega \\ g_{22} &= \left. \frac{V_2}{I_2} \right|_{V_1=0} \Omega \end{aligned}$$

12.4 To Find Z-Parameters:

The values of the parameters can be evaluated by setting $I_1=0$ (input port open-circuited) or $I_2=0$ (output port open-circuited). Thus, the voltages with respect to currents are:

$$V_1 = f_1(I_1, I_2)$$

$$V_2 = f_2(I_1, I_2)$$

$$V_1 = Z_{11}I_1 + Z_{12}I_2$$

$$V_2 = Z_{21}I_1 + Z_{22}I_2$$

Electric circuits are divided into two types:

1. Passive Network: Contains Resistances, Capacitances and Inductances.
2. Active Network: Contains Amplifiers (Diode, Transistor, Rectifier ...).

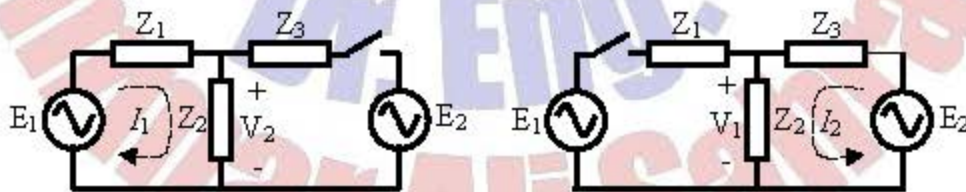
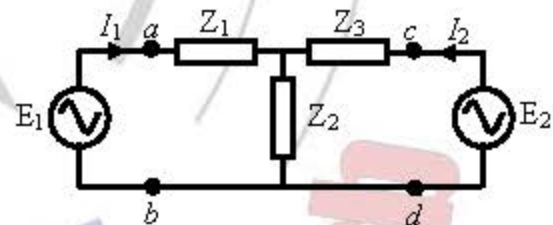
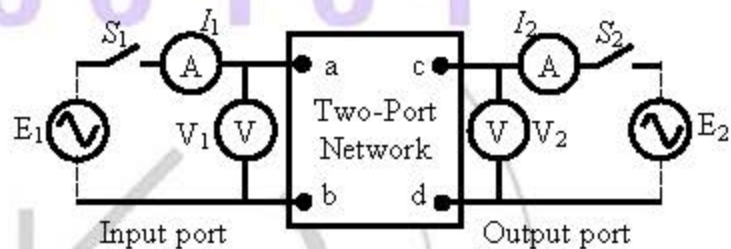
$$V_2 = I_1 Z_2$$

$$Z_{21} = \left. \frac{V_2}{I_1} \right|_{I_2=0} = \frac{I_1 Z_2}{I_1} = Z_2$$

$$V_1 = I_2 Z_2$$

$$Z_{12} = \left. \frac{V_1}{I_2} \right|_{I_1=0} = \frac{I_2 Z_2}{I_2} = Z_2$$

$$\therefore Z_{12} = Z_{21} = Z_2$$



14.5 Input and Reflected Impedance:

$$V_1 = Z_{11}I_1 + Z_{12}I_2$$

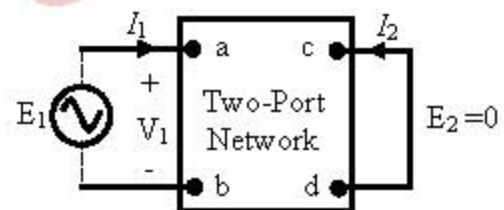
$$0 = Z_{21}I_1 + Z_{22}I_2 \Rightarrow I_2 = - \frac{Z_{21}}{Z_{22}} I_1$$

$$\therefore V_1 = Z_{11}I_1 - Z_{12} \frac{Z_{21}}{Z_{22}} I_1 = I_1 (Z_{11} - Z_{12} \frac{Z_{21}}{Z_{22}})$$

$$\text{or } \frac{V_1}{I_1} = Z_{in} = Z_{11} - Z_{12} \frac{Z_{21}}{Z_{22}}$$

it is the input impedance equal to the input impedance when the output is open circuit minus square of mutual (reflected) impedance divided by the output impedance when the input is open circuit.

$$Z_m = Z_{12} = Z_{21} = \text{mutual impedance}$$



$$Z_{in} = Z_{11} - \frac{Z_m^2}{Z_{22}} \text{ and in the same manner: } Z_{out} = Z_{22} - \frac{Z_m^2}{Z_{11}}$$

$$Z_{ref} = \frac{Z_m^2}{Z_{22}} = \text{reflected impedance}$$

14.6 To Find Y-Parameters:

The values of the parameters can be evaluated by setting $E_1=0$ (input port short-circuited) or $E_2=0$ (output port short-circuited). Thus, the currents with respect to voltages are:

$$I_1 = f_1(V_1, V_2)$$

$$I_2 = f_2(V_1, V_2)$$

$$I_1 = Y_{11}V_1 + Y_{12}V_2$$

$$I_2 = Y_{21}V_1 + Y_{22}V_2$$

In the passive networks $Y_{12} = Y_{21}$.

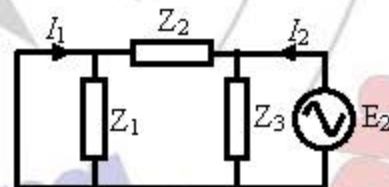
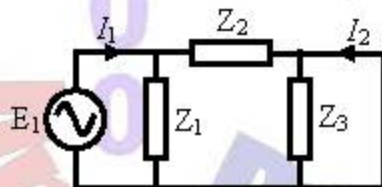
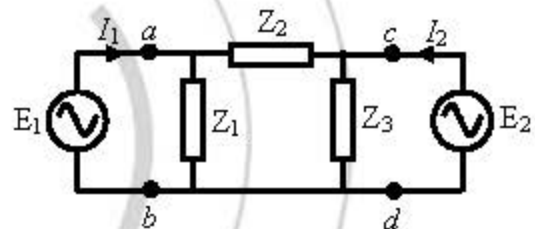
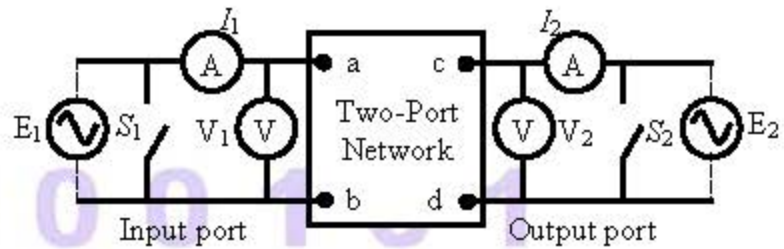
$$I_2 = \frac{V_1}{Z_2}$$

$$Y_{21} = \left. \frac{I_2}{V_1} \right|_{V_2=0} = \frac{-V_1/Z_2}{V_1} = \frac{-1}{Z_2}$$

$$I_1 = \frac{V_2}{Z_2}$$

$$Y_{12} = \left. \frac{I_1}{V_2} \right|_{V_1=0} = \frac{-V_2/Z_2}{V_2} = \frac{-1}{Z_2}$$

$$\therefore Y_{12} = Y_{21} = \frac{-1}{Z_2}$$



14.7 Input and Reflected Admittance:

$$I_1 = Y_{11}V_1 + Y_{12}V_2$$

$$0 = Y_{21}V_1 + Y_{22}V_2 \Rightarrow V_2 = -\frac{Y_{21}}{Y_{22}}V_1$$

$$\therefore I_1 = Y_{11}V_1 - Y_{12}\frac{Y_{21}}{Y_{22}}V_1$$

$$= V_1(Y_{11} - Y_{12}\frac{Y_{21}}{Y_{22}})$$

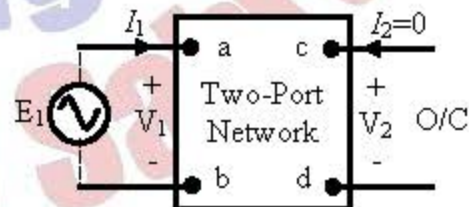
$$\text{or } \frac{I_1}{V_1} = Y_{in} = Y_{11} - Y_{12}\frac{Y_{21}}{Y_{22}}$$

it is the input admittance equal to the input admittance when the output is short circuit minus square of mutual (reflected) admittance divided by the output admittance when the input is short circuit.

$$Y_m = Y_{12} = Y_{21} = \text{mutual admittance}$$

$$Y_{in} = Y_{11} - \frac{Y_m^2}{Y_{22}} \text{ and in the same manner: } Y_{out} = Y_{22} - \frac{Y_m^2}{Y_{11}}$$

$$Y_{ref} = \frac{Y_m^2}{Y_{22}} = \text{reflected admittance}$$





Example: For the circuit shown, find Z, Y, A, B, H and G parameters?

Solution:

$$Z_{11} = \left. \frac{V_1}{I_1} \right|_{I_2=0} = \frac{(5+15) \times 20}{(5+15)+20} = 10\Omega \text{ (O/P port is O/C)}$$

$$Y_{11} = \left. \frac{I_1}{V_1} \right|_{V_2=0} = \frac{1}{5} + \frac{1}{20} = \frac{1}{4} = 0.25\text{U} \text{ (15}\Omega \text{ is S/C)}$$

$$\text{Using KVL } V_2 = V_1 \times \frac{15}{15+5} = 0.75V_1 \text{ (O/P port is O/C)}$$

$$a_{11} = \left. \frac{V_1}{V_2} \right|_{I_2=0} = \frac{15+5}{15} = \frac{4}{3} = 1.333$$

$$\text{Using KVL } V_1 = V_2 \times \frac{20}{5+20} = 0.8V_2 \text{ (I/P port is O/C)}$$

$$b_{11} = \left. \frac{V_2}{V_1} \right|_{I_1=0} = \frac{5+20}{20} = 1.25$$

$$h_{11} = \left. \frac{V_1}{I_1} \right|_{V_2=0} = \frac{5 \times 20}{5+20} = 4\Omega \text{ (O/P port is S/C)}$$

$$g_{11} = \left. \frac{I_1}{V_1} \right|_{I_2=0} = \frac{1}{(5+15) \parallel 20} = 0.1\text{U} \text{ (O/P port is O/C)}$$

$$Z_{21} = \left. \frac{V_2}{I_1} \right|_{I_2=0} = \frac{V_1 \times \frac{15}{15+5}}{V_1/Z_{11}} = \frac{0.75V_1}{V_1/10} = 7.5\Omega \text{ (O/P port is O/C)}$$

$$Y_{21} = \left. \frac{I_2}{V_1} \right|_{V_2=0} = \frac{-V_1/5}{V_1} = \frac{-1}{5} = -0.2\text{U} \text{ (O/P port is S/C)}$$

$$\text{Using KCL } I_L = I_1 \times \frac{20}{20+5+15} = 0.5I_1 \text{ (O/P port is O/C) and } V_2 = 15I_L = \frac{15}{2}I_1$$

$$a_{21} = \left. \frac{I_1}{V_2} \right|_{I_2=0} = \frac{I_1}{\frac{15}{2}I_1} = \frac{2}{15} = 0.133\text{U}$$

$$\text{Using KCL } I_L = I_2 \times \frac{15}{15+20+5} = \frac{3}{8}I_2 \text{ (I/P port is O/C) and } V_1 = 20I_L = \frac{15}{2}I_2$$

$$b_{21} = \left. \frac{I_2}{V_1} \right|_{I_1=0} = \frac{I_2}{\frac{15}{2}I_2} = \frac{2}{15}\text{U}$$

$$h_{21} = \left. \frac{I_2}{I_1} \right|_{V_2=0} = -\frac{20}{20+5} = -\frac{4}{5} \text{ using KCL (O/P port is S/C)}$$

$$g_{21} = \left. \frac{V_2}{V_1} \right|_{I_2=0} = \frac{15}{5+15} = \frac{3}{4} \text{ using KVL (O/P port is O/C)}$$

$$Z_{22} = \left. \frac{V_2}{I_2} \right|_{I_1=0} = \frac{(20+5) \times 15}{(20+5)+15} = 9.375\Omega \text{ (I/P port is O/C)}$$

$$Y_{22} = \left. \frac{I_2}{V_2} \right|_{V_1=0} = \frac{1}{5} + \frac{1}{15} = \frac{4}{15} = 0.267\text{U} \text{ (I/P port is S/C)}$$

$$\text{Using KCL } I_2 = -I_1 \times \frac{20}{20+5} \text{ (O/P port is S/C)}$$

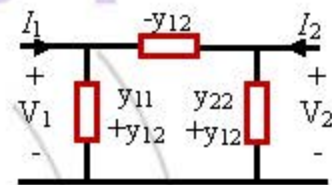
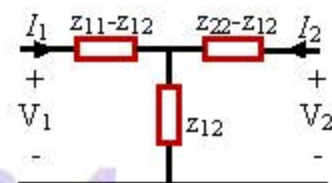
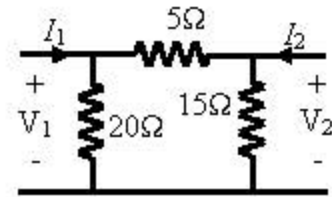
$$a_{22} = -\left. \frac{I_1}{I_2} \right|_{V_2=0} = \frac{20+5}{20} = 1.25$$

$$\text{Using KCL } I_1 = -I_2 \times \frac{15}{15+5} \text{ (I/P port is S/C)}$$

$$b_{22} = -\left. \frac{I_2}{I_1} \right|_{V_1=0} = \frac{15+5}{15} = \frac{4}{3}$$

$$h_{22} = \left. \frac{I_2}{V_2} \right|_{I_1=0} = \frac{1}{(5+20) \parallel 15} = 0.1067\text{U} \text{ (I/P port is O/C)}$$

$$g_{22} = \left. \frac{V_2}{I_2} \right|_{V_1=0} = \frac{5 \times 15}{5+15} = 3.75\Omega \text{ (I/P port is S/C)}$$



Using KVL $V_1 = V_2 \times \frac{20}{5+20} = 0.8V_2$ and $I_2 = \frac{V_2}{Z_{22}} = \frac{V_2}{9.375}$ (I/P port is O/C)

$$Z_{12} = \left. \frac{V_1}{I_2} \right|_{I_1=0} = \frac{0.8V_2}{V_2/9.375} = 7.5\Omega$$

$$Y_{12} = \left. \frac{I_1}{V_2} \right|_{V_1=0} = \frac{-V_2/5}{V_2} = -\frac{1}{5} = -0.2\text{S} \text{ (I/P port is S/C)}$$

$$a_{12} = -\left. \frac{V_1}{I_2} \right|_{V_2=0} = -\frac{-5I_2}{I_2} = 5\Omega \text{ (O/P port is S/C)}$$

$$b_{12} = -\left. \frac{V_2}{I_1} \right|_{V_1=0} = -\frac{-5I_1}{I_1} = 5\Omega \text{ (I/P port is S/C)}$$

$$h_{12} = \left. \frac{V_1}{V_2} \right|_{I_1=0} = \frac{20}{20+5} = \frac{4}{5} \text{ (I/P port is O/C)}$$

$$g_{12} = \left. \frac{I_1}{I_2} \right|_{V_1=0} = -\frac{15}{5+15} = -\frac{3}{4} \text{ (I/P port is S/C)}$$

$$V_1 = 10I_1 + 7.5I_2$$

$$V_2 = 7.5I_1 + 9.375I_2$$

$$I_1 = 0.25V_1 - 0.2V_2$$

$$I_2 = -0.2V_1 + 0.267V_2$$

$$V_1 = 1.333V_2 - 5I_2$$

$$I_1 = 0.133V_2 - 1.25I_2$$

$$V_2 = 1.25V_1 - 5I_1$$

$$I_2 = 0.133V_1 - 1.333I_1$$

$$V_1 = 4I_1 + 0.8V_2$$

$$I_2 = -0.8I_1 + 0.1067V_2$$

$$I_1 = 0.1V_1 - 0.75I_2$$

$$V_2 = 0.75V_1 + 3.75I_2$$

Example: For the circuit shown, find the Z and Y parameters?

Solution:

$$Z_{11} = \left. \frac{V_1}{I_1} \right|_{I_2=0} = (10 + 4) \parallel (5 + 6) = 6.16\Omega$$

$$V_g - V_c = 10I_a$$

$$V_g - V_d = 5I_b$$

$$\therefore V_2 = V_d - V_c = 10I_a - 5I_b$$

$$I_a = I_1 \times \frac{11}{25}$$

$$I_b = I_1 \times \frac{14}{25}$$

$$Z_{21} = \left. \frac{V_2}{I_1} \right|_{I_2=0} = \frac{8}{5} = 1.6\Omega$$

$$Z_{22} = \left. \frac{V_2}{I_2} \right|_{I_1=0} = (5 + 10) \parallel (6 + 4) = 6\Omega$$

$$V_g - V_a = 5I_a$$

$$V_g - V_b = 6I_b$$

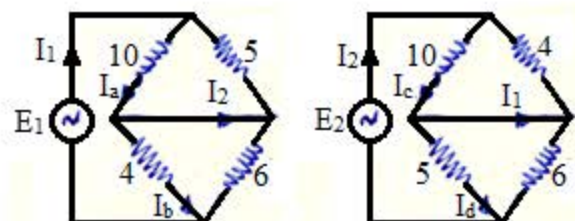
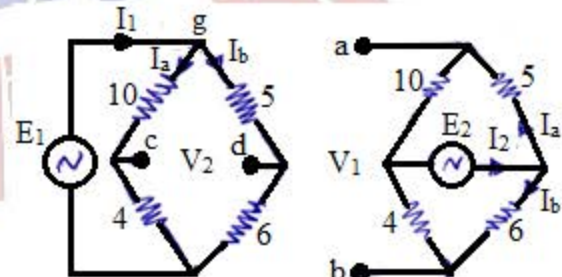
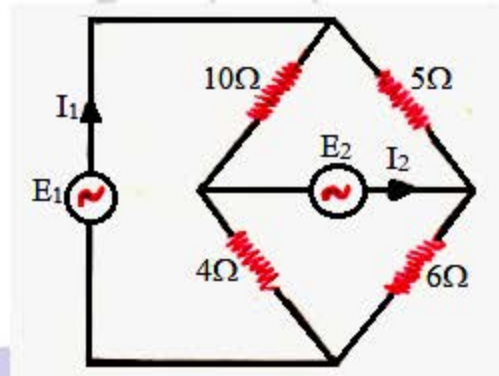
$$\therefore V_1 = V_a - V_b = 6I_b - 5I_a$$

$$I_a = I_2 \times \frac{10}{25}$$

$$I_b = I_2 \times \frac{15}{25}$$

$$Z_{12} = \left. \frac{V_1}{I_2} \right|_{I_1=0} = \frac{8}{5} = 1.6\Omega$$

$$Y_{11} = \left. \frac{I_1}{V_1} \right|_{V_2=0} = \frac{1}{(10 \parallel 5) + (4 \parallel 6)} = 0.1744\text{S}$$



$$Y_{22} = \left. \frac{I_2}{V_2} \right|_{V_1=0} = \frac{1}{(5 \parallel 6) + (10 \parallel 4)} = 0.179 \text{U}$$

$$I_a = I_1 \times \frac{5}{5+10} = \frac{1}{3} I_1$$

$$I_b = I_1 \times \frac{6}{6+4} = \frac{3}{5} I_1$$

$$I_2 = I_a - I_b = I_1 \left(\frac{1}{3} - \frac{3}{5} \right) = -\frac{4}{15} I_1$$

$$= -0.0465 V_1$$

For the same condition $I_1 = Y_{11} V_1$

$$Y_{21} = \left. \frac{I_2}{V_1} \right|_{V_2=0} = -0.0465 \text{U}$$

$$I_c = I_2 \times \frac{4}{4+10} = \frac{2}{7} I_2$$

$$I_d = I_2 \times \frac{6}{6+5} = \frac{6}{11} I_2$$

$$I_1 = I_c - I_d = I_2 \left(\frac{2}{7} - \frac{6}{11} \right) = \frac{-40}{154} I_2 = -0.0465 V_2$$

For the same condition $I_2 = Y_{22} V_2$

$$Y_{12} = \left. \frac{I_1}{V_2} \right|_{V_1=0} = -0.0465 \text{U}$$

$$V_1 = 6.16 I_1 + 1.6 I_2$$

$$V_2 = 1.6 I_1 + 6 I_2$$

$$I_1 = 0.1744 V_1 - 0.0465 V_2$$

$$I_2 = -0.0465 V_1 + 0.179 V_2$$

Example: For the circuit shown, write the equations of voltages w. r. t. currents and the equations of currents w. r. t. voltages.

Solution:

$$Z_{11} = \left. \frac{V_1}{I_1} \right|_{I_2=0} = 80 + 100 = 180 \Omega$$

$$Y_{11} = \left. \frac{I_1}{V_1} \right|_{V_2=0} = \frac{1}{80 + (100 \parallel 80)} = 0.008 \text{U}$$

$$Z_{21} = \left. \frac{V_2}{I_1} \right|_{I_2=0} = 100 \Omega$$

$$I_2 = -I_1 \times \frac{100}{100+80} \text{ and } I_1 = 0.008 V_1$$

$$Y_{21} = \left. \frac{I_2}{V_1} \right|_{V_2=0} = -0.00446 \text{U}$$

$$Z_{22} = \left. \frac{V_2}{I_2} \right|_{I_1=0} = 80 + 100 = 180 \Omega$$

$$Y_{22} = \left. \frac{I_2}{V_2} \right|_{V_1=0} = \frac{1}{80 + (100 \parallel 80)} = 0.008 \text{U}$$

$$Z_{12} = \left. \frac{V_1}{I_2} \right|_{I_1=0} = 100 \Omega$$

$$I_1 = -I_2 \times \frac{100}{100+80} \text{ and } I_2 = 0.008 V_2$$

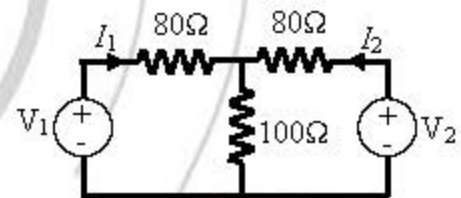
$$Y_{12} = \left. \frac{I_1}{V_2} \right|_{V_1=0} = -0.00446 \text{U}$$

$$V_1 = 180 I_1 + 100 I_2$$

$$V_2 = 100 I_1 + 180 I_2$$

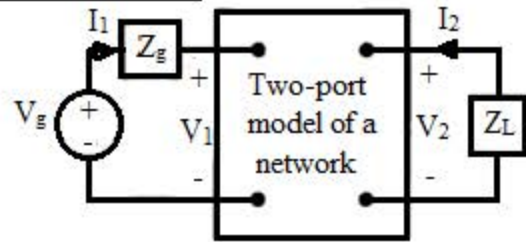
$$I_1 = 0.008 V_1 - 0.00446 V_2$$

$$I_2 = -0.00446 V_1 + 0.008 V_2$$



14.8 Analysis of the Terminated Two-Port Circuit:

In the typical application of a two-port model, the circuit is driven at port 1 and loaded at port 2. The figure beside shows the s-domain circuit diagram for a typically terminated two-port model. Here, Z_g represents the internal impedance of the source, V_g the internal voltage of the source, and Z_L the load impedance. Analysis of this circuit involves expressing the terminal currents and voltages as functions of the two-port parameters, V_s , Z_g , and Z_L .



Six characteristics of the terminated two-port circuit define its terminal behavior:

- The input impedance $Z_{in} = \frac{V_1}{I_1}$
- The input admittance $Y_{in} = \frac{I_1}{V_1}$
- The output current I_2
- The Thevenin voltage and impedance (V_{Th} , Z_{Th}) with respect to port 2
- The current gain $\frac{I_2}{I_1}$
- The voltage gain $\frac{V_2}{V_1}$ or $\frac{V_2}{V_g}$

Using Z-Parameters:

The derivation of any one of the desired expressions involves the algebraic manipulation of the two-port equations along with the two constraint equations imposed by the terminations. If we use the z-parameter equations, the four that describe the circuit in above figure are

$$\begin{aligned} V_1 &= Z_{11}I_1 + Z_{12}I_2 & V_2 &= Z_{21}I_1 + Z_{22}I_2 \\ V_1 &= V_g - Z_g I_1 & V_2 &= -Z_L I_2 \end{aligned}$$

1. The Terminated Two-Port Equations in Terms of the Z-Parameters:

$$Z_{in} = Z_{11} - \frac{Z_{12}Z_{21}}{Z_{22} + Z_L}$$

$$\frac{I_2}{I_1} = -\frac{Z_{21}}{Z_{22} + Z_L}$$

$$I_2 = -\frac{Z_{21}V_g}{(Z_{11} + Z_g)(Z_{22} + Z_L) - Z_{12}Z_{21}}$$

$$\frac{V_2}{V_1} = \frac{Z_{21}Z_L}{Z_{11}Z_L + \Delta Z} \quad \text{where } \Delta Z = Z_{11}Z_{22} - Z_{12}Z_{21}$$

$$V_{th} = \frac{Z_{21}}{Z_{11} + Z_g} V_g$$

$$Z_{th} = Z_{22} - \frac{Z_{12}Z_{21}}{Z_{11} + Z_g}$$

$$\frac{V_2}{V_g} = \frac{Z_{21}Z_L}{(Z_{11} + Z_g)(Z_{22} + Z_L) - Z_{12}Z_{21}}$$

2. The Terminated Two-Port Equations in Terms of the Y-Parameters:

$$Y_{in} = Y_{11} - \frac{Y_{12}Y_{21}Z_L}{1+Y_{22}Z_L}$$

$$\frac{I_2}{I_1} = \frac{Y_{21}}{Y_{11} + \Delta Y Z_L}$$

$$I_2 = - \frac{Y_{21}V_g}{1+Y_{22}Z_L+Y_{11}Z_g+\Delta Y Z_g Z_L}$$

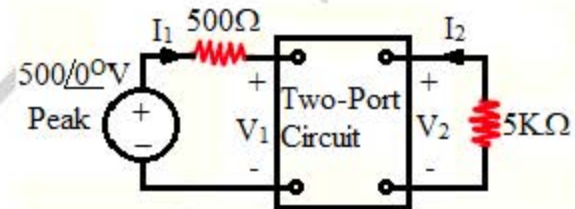
$$\frac{V_2}{V_1} = - \frac{Y_{21}Z_L}{1+Y_{22}Z_L} \quad \text{where } \Delta Y = Y_{11}Y_{22} - Y_{12}Y_{21}$$

$$V_{th} = - \frac{Y_{21}}{Y_{22} + \Delta Y Z_g} V_g$$

$$Z_{th} = \frac{1+Y_{11}Z_g}{Y_{22} + \Delta Y Z_g}$$

$$\frac{V_2}{V_g} = \frac{Y_{21}Z_L}{Y_{12}Y_{21}Z_g Z_L - (1+Y_{11}Z_g)(1+Y_{22}Z_L)}$$

Example: The two-port circuit shown described by: $z_{11}=100\Omega$, $z_{12}=-500\Omega$, $z_{21}=1K\Omega$, & $z_{22}=10K\Omega$. Find: V_2 , P_L , average power delivered to input port, load impedance for maximum average power transfer, and maximum average power delivered to the load and input ports?



Solution:

$$\frac{V_2}{V_g} = \frac{Z_{21}Z_L}{(Z_{11}+Z_g)(Z_{22}+Z_L)-Z_{12}Z_{21}} = \frac{1000 \times 5000}{(100+500)(1000+5000)-(-500)(1000)} = \frac{10}{19}$$

$$\therefore V_2 = \frac{10}{19} \times 500 \angle 0^\circ = 263.16 \angle 0^\circ V$$

$$P_L = P_2 = \frac{|V_2|^2}{2Z_L} = \frac{(263.16)^2}{2 \times 5000} = 6.93W$$

$$Z_{in} = Z_{11} - \frac{Z_{12}Z_{21}}{Z_{22}+Z_L} = 100 - \frac{(-500)(1000)}{10000+5000} = 133.33\Omega$$

$$I_1 = \frac{V_g}{Z_{in}+Z_g} = \frac{500 \angle 0^\circ}{133.33+500} = 789.47mA$$

$$P_{in} = P_1 = \frac{1}{2} |I_1|^2 Z_{in} = \frac{1}{2} \times (0.78947)^2 \times 133.33 = 41.55W$$

$$Z_{th} = Z_{22} - \frac{Z_{12}Z_{21}}{Z_{11}+Z_g} = 10000 - \frac{(-500)(1000)}{100+500} = 10833.33\Omega$$

$$Z_L = Z_{th}^* = 10833.33\Omega$$

$$\frac{V_2}{V_g} = \frac{Z_{21}Z_L}{(Z_{11}+Z_g)(Z_{22}+Z_L)-Z_{12}Z_{21}} = \frac{1000 \times 10833.33}{(100+500)(1000+10833.33)-(-500)(1000)} = 0.8333$$

$$\therefore V_2 = 0.8333 \times 500 \angle 0^\circ = 416.67 \angle 0^\circ V$$

$$P_m = \frac{|V_2|^2}{2Z_L} = \frac{(416.67)^2}{2 \times 10833.33} = 8.01W$$

$$Z_{in} = Z_{11} - \frac{Z_{12}Z_{21}}{Z_{22}+Z_L} = 100 - \frac{(-500)(1000)}{10000+10833.33} = 124\Omega$$

$$I_1 = \frac{V_g}{Z_{in}+Z_g} = \frac{500 \angle 0^\circ}{124+500} = 801.3mA$$

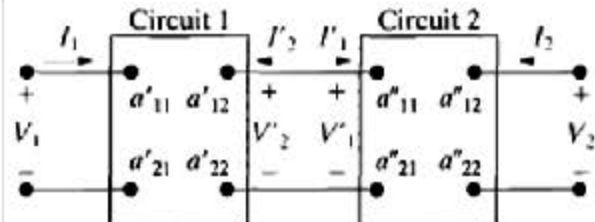
$$P_{in} = P_1 = \frac{1}{2} |I_1|^2 Z_{in} = \frac{1}{2} \times (0.8013)^2 \times 124 = 40W$$

14.9 Interconnected Two-Port Circuits:

Synthesizing a large, complex system is usually made easier by first designing subsections of the system. Interconnecting these simpler, easier-to-design units then completes the system. If the subsections are modeled by two-port circuits, synthesis involves the analysis of interconnected two-port circuits.

1. Cascade Interconnection:

We analyze the cascade connection by using the circuit shown in figure beside, where a single prime denotes parameters in the first circuit and a double prime denotes parameters in the second circuit.



$$V_1 = a'_{11}V_2' - a'_{12}I_2' \quad \dots (1)$$

$$I_1 = a'_{21}V_2' - a'_{22}I_2' \quad \dots (2)$$

Apply equations 1 & 2 to individual circuits:

$$\text{First circuit: } V_1 = a'_{11}V_2' - a'_{12}I_2' \quad \dots (3)$$

$$I_1 = a'_{21}V_2' - a'_{22}I_2' \quad \dots (4)$$

$$\text{Where: } V_2' = V_1' \text{ \& } I_2' = -I_1'$$

Substitute in equations 3 & 4 yields:

$$V_1 = a'_{11}V_1' + a'_{12}I_1' \quad \dots (5)$$

$$I_1 = a'_{21}V_1' + a'_{22}I_1' \quad \dots (6)$$

$$\text{Second circuit: } V_1' = a''_{11}V_2 - a''_{12}I_2 \quad \dots (7)$$

$$I_1' = a''_{21}V_2 - a''_{22}I_2 \quad \dots (8)$$

$$\text{Where: } V_2' = V_1' \text{ \& } I_2' = -I_1'$$

Substitute equations 7 & 8 in equations 5 & 6 yields:

$$V_1 = (a'_{11}a''_{11} + a'_{12}a''_{21})V_2 - (a'_{11}a''_{12} + a'_{12}a''_{22})I_2$$

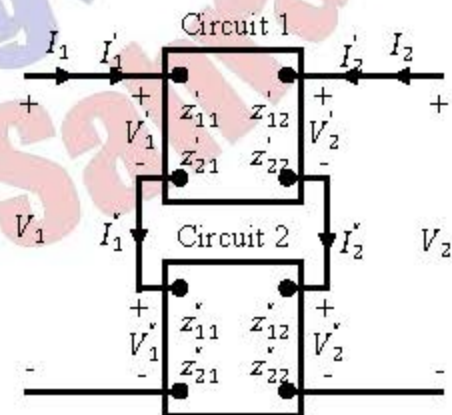
$$I_1 = (a'_{21}a''_{11} + a'_{22}a''_{21})V_2 - (a'_{21}a''_{12} + a'_{22}a''_{22})I_2$$

$$\text{Where: } a_{11} = a'_{11}a''_{11} + a'_{12}a''_{21} \quad a_{12} = a'_{11}a''_{12} + a'_{12}a''_{22}$$

$$a_{21} = a'_{21}a''_{11} + a'_{22}a''_{21} \quad a_{22} = a'_{21}a''_{12} + a'_{22}a''_{22}$$

2. Series Interconnection:

We analyze the series connection by using the circuit shown in figure beside, where a single prime denotes parameters in the first circuit and a double prime denotes parameters in the second circuit.



$$V_1 = z'_{11}I_1 + z'_{12}I_2 \quad \dots (1)$$

$$V_2 = z'_{21}I_1 + z'_{22}I_2 \quad \dots (2)$$

Apply equations 1 & 2 to individual circuits:

First circuit:

$$V_1' = z'_{11}I_1' + z'_{12}I_2' \quad \dots (3)$$

$$V_2' = z'_{21}I_1' + z'_{22}I_2' \quad \dots (4)$$

$$\text{Where: } I_1 = I_1' \text{ \& } I_2 = I_2'$$

Substitute in equations 3 & 4 yields:

$$V_1' = z'_{11}I_1 + z'_{12}I_2 \quad \dots (5)$$

$$V_2' = z_{21}'I_1 + z_{22}'I_2 \quad \dots (6)$$

$$\text{Second circuit: } V_1'' = z_{11}''I_1 + z_{12}''I_2 \quad \dots (7)$$

$$V_2'' = z_{21}''I_1 + z_{22}''I_2 \quad \dots (8)$$

$$\text{Where: } V_1 = V_1' + V_1'' \text{ \& } V_2 = V_2' + V_2''$$

$$I_1 = I_1' = I_1'' \text{ \& } I_2 = I_2' = I_2''$$

Substitute equations 7 & 8 in equations 5 & 6 yields:

$$V_1 = (z_{11}' + z_{11}'')I_1 + (z_{12}' + z_{12}'')I_2$$

$$V_2 = (z_{21}' + z_{21}'')I_1 + (z_{22}' + z_{22}'')I_2$$

$$\text{Where: } z_{11} = z_{11}' + z_{11}'' \quad z_{12} = z_{12}' + z_{12}''$$

$$z_{21} = z_{21}' + z_{21}'' \quad z_{22} = z_{22}' + z_{22}''$$

3. Parallel Interconnection:

We analyze the parallel connection by using the circuit shown in figure beside, where a single prime denotes parameters in the first circuit and a double prime denotes parameters in the second circuit.

$$I_1 = y_{11}V_1 + y_{12}V_2 \quad \dots (1)$$

$$I_2 = y_{21}V_1 + y_{22}V_2 \quad \dots (2)$$

Apply equations 1 & 2 to individual circuits:

First circuit:

$$I_1' = y_{11}'V_1' + y_{12}'V_2' \quad \dots (3)$$

$$I_2' = y_{21}'V_1' + y_{22}'V_2' \quad \dots (4)$$

$$\text{Where: } V_1 = V_1' \text{ \& } V_2 = V_2'$$

Substitute in equations 3 & 4 yields:

$$I_1' = y_{11}'V_1 + y_{12}'V_2 \quad \dots (5)$$

$$I_2' = y_{21}'V_1 + y_{22}'V_2 \quad \dots (6)$$

$$\text{Second circuit: } I_1'' = y_{11}''V_1'' + y_{12}''V_2'' \quad \dots (7)$$

$$I_2'' = y_{21}''V_1'' + y_{22}''V_2'' \quad \dots (8)$$

$$\text{Where: } I_1 = I_1' + I_1'' \text{ \& } I_2 = I_2' + I_2''$$

$$V_1 = V_1' = V_1'' \text{ \& } V_2 = V_2' = V_2''$$

Substitute equations 7 & 8 in equations 5 & 6 yields:

$$I_1 = (y_{11}' + y_{11}'')V_1 + (y_{12}' + y_{12}'')V_2$$

$$I_2 = (y_{21}' + y_{21}'')V_1 + (y_{22}' + y_{22}'')V_2$$

$$\text{Where: } y_{11} = y_{11}' + y_{11}'' \quad y_{12} = y_{12}' + y_{12}''$$

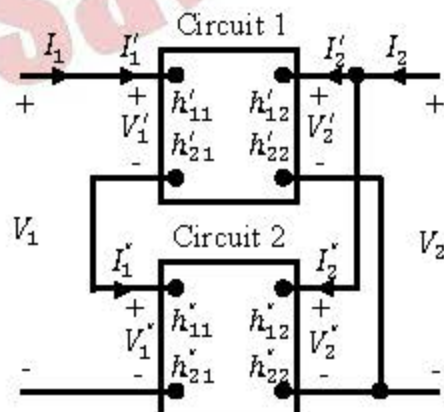
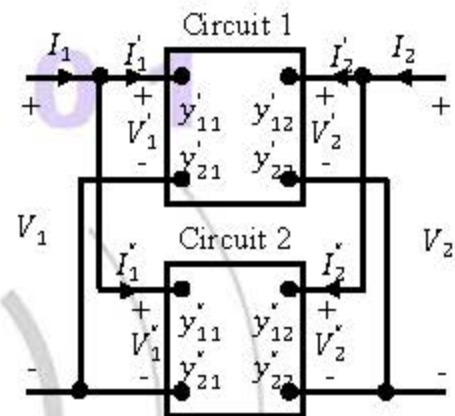
$$y_{21} = y_{21}' + y_{21}'' \quad y_{22} = y_{22}' + y_{22}''$$

4. Serial-Parallel Interconnection:

We analyze the serial-parallel connection by using the circuit shown in figure beside, where a single prime denotes parameters in the first circuit and a double prime denotes parameters in the second circuit.

$$V_1 = h_{11}I_1 + h_{12}V_2 \quad \dots (1)$$

$$I_2 = h_{21}I_1 + h_{22}V_2 \quad \dots (2)$$



Apply equations 1 & 2 to individual circuits:

First circuit:

$$V_1' = h_{11}'I_1' + h_{12}'V_2' \quad \dots (3)$$

$$I_2' = h_{21}'I_1' + h_{22}'V_2' \quad \dots (4)$$

Where: $I_1 = I_1'$ & $V_2 = V_2'$

Substitute in equations 3 & 4 yields:

$$V_1' = h_{11}'I_1 + h_{12}'V_2 \quad \dots (5)$$

$$I_2' = h_{21}'I_1 + h_{22}'V_2 \quad \dots (6)$$

Second circuit: $V_1'' = h_{11}''I_1'' + h_{12}''V_2'' \quad \dots (7)$

$$I_2'' = h_{21}''I_1'' + h_{22}''V_2'' \quad \dots (8)$$

Where: $V_1 = V_1' + V_1''$ & $I_2 = I_2' + I_2''$

$I_1 = I_1' = I_1''$ & $V_2 = V_2' = V_2''$

Substitute equations 7 & 8 in equations 5 & 6 yields:

$$V_1 = (h_{11}' + h_{11}'')I_1 + (h_{12}' + h_{12}'')V_2$$

$$I_2 = (h_{21}' + h_{21}'')I_1 + (h_{22}' + h_{22}'')V_2$$

Where: $h_{11} = h_{11}' + h_{11}''$ $h_{12} = h_{12}' + h_{12}''$

$h_{21} = h_{21}' + h_{21}''$ $h_{22} = h_{22}' + h_{22}''$

5. Parallel-Serial Interconnection:

We analyze the parallel-serial connection by using the circuit shown in figure beside, where a single prime denotes parameters in the first circuit and a double prime denotes parameters in the second circuit.

$$I_1 = g_{11}V_1 + g_{12}I_2 \quad \dots (1)$$

$$V_2 = g_{21}V_1 + g_{22}I_2 \quad \dots (2)$$

Apply equations 1 & 2 to individual circuits:

First circuit:

$$I_1' = g_{11}'V_1' + g_{12}'I_2' \quad \dots (3)$$

$$V_2' = g_{21}'V_1' + g_{22}'I_2' \quad \dots (4)$$

Where: $V_1 = V_1'$ & $I_2 = I_2'$

Substitute in equations 3 & 4 yields:

$$I_1' = g_{11}'V_1 + g_{12}'I_2 \quad \dots (5)$$

$$V_2' = g_{21}'V_1 + g_{22}'I_2 \quad \dots (6)$$

Second circuit: $I_1'' = g_{11}''V_1'' + g_{12}''I_2'' \quad \dots (7)$

$$V_2'' = g_{21}''V_1'' + g_{22}''I_2'' \quad \dots (8)$$

Where: $I_1 = I_1' + I_1''$ & $V_2 = V_2' + V_2''$

$V_1 = V_1' = V_1''$ & $I_2 = I_2' = I_2''$

Substitute equations 7 & 8 in equations 5 & 6 yields:

$$I_1 = (g_{11}' + g_{11}'')V_1 + (g_{12}' + g_{12}'')I_2$$

$$V_2 = (g_{21}' + g_{21}'')V_1 + (g_{22}' + g_{22}'')I_2$$

Where: $g_{11} = g_{11}' + g_{11}''$ $g_{12} = g_{12}' + g_{12}''$

$g_{21} = g_{21}' + g_{21}''$ $g_{22} = g_{22}' + g_{22}''$

