Highway Pavement

Civil Engineering Department

4th stage, 2nd Semester, 2019-2020

2st Lecture: Vertical Alignment

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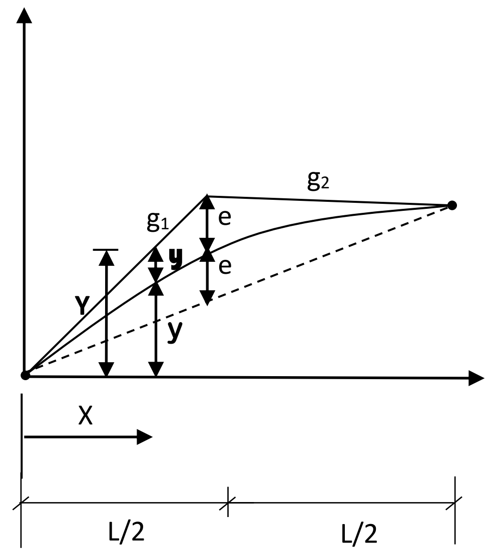
Elevation of point on curve

$y=ax^2+bx+c$ general equation of parabola

@ x = 0

y = 0 + 0 +c = Elev. Of PVC

∴ c = Elev. Of PVC



Symmetrical vertical curve

Take the first derivative,

$$\frac{dy}{dx} = 2ax + b$$

$$when\ x = 0, \quad \frac{dy}{dx} = g_1$$

$$\therefore\ g_1 = b$$

Take the second derivative,

$$\frac{d^2y}{dx^2} = 2a = r$$

r= rate of change in grade per station (or 100m)

$$\therefore\ a = \frac{r}{2}$$

Therefore to find the elevation of any point (E_P), for any distance (x), the equation is:

$$y = \frac{1}{2}\, r\, x^2 + g_1 x + E_{PVC}$$

Or

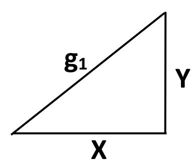
$$E_P = \frac{1}{2}\, r\, x^2 + g_1 x + E_{PVC}$$

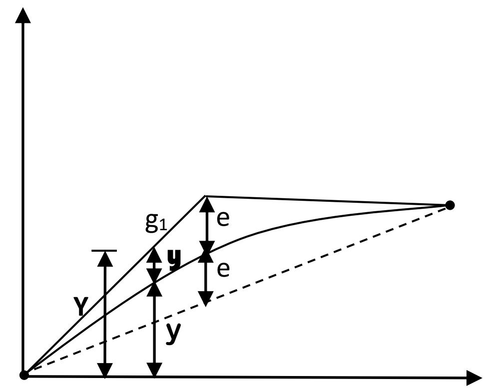
$y = \frac{1}{2}\, r\, x^2 + g_1 x + E_{PVC}$

$@\, x = 0 \;\rightarrow\; y = 0 \;\rightarrow c = 0$

$y = \frac{1}{2}\, r\, x^2 + g_1 x$

y = ± y + Y





* $\frac{1}{2}\, r\, x^2$ = **y** = the difference in elevation between the first tangent and curve.

For parabolic vertical curve, the rate of change in grade is constant, therefore:

$$\frac{d^2y}{dx^2} = r = constant$$

$$\frac{dy}{dx} = rx + c \qquad (by\, integration)$$

$$@\, x = 0 \;\rightarrow\; \frac{dy}{dx} = g_1$$

$$\therefore\; g_1 = c$$

$$\frac{dy}{dx} = rx + g_1$$

$$@\, X = L \;\rightarrow\; \frac{dy}{dx} = g_2$$

$$g_2 = rL + g_1$$

$$r = \frac{g_2 - g_1}{L}$$

A = algebraic difference in grades = $g_2 - g_1$, %

r= rate of change in grade per station (or 100m), (% / station) (- crest) (+ sag)

L= Length of vertical curve measured horizontally, (meter or station)

* $\frac{1}{2}\,r\,x^2$ = **y** = the difference in elevation between the first tangent and curve.

$@\ x = \frac{L}{2}$

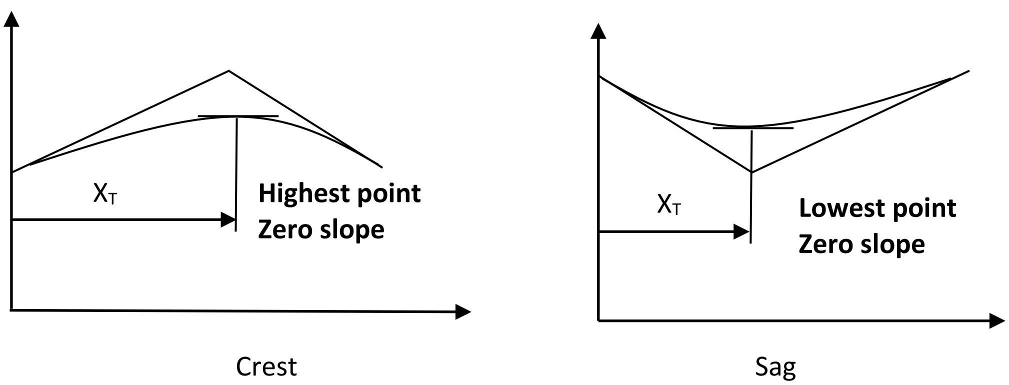
e = **y**

$e = \frac{1}{2}\,r\,x^2$

$$e = \frac{1}{2}\,\frac{g_2 - g_1}{L}\,\left(\frac{L}{2}\right)^2 \rightarrow e = \frac{L\,(g_2 - g_1)\%}{8} \quad or \quad e = \frac{L\,(g_2 - g_1)\%}{800}$$

L (st) L (m)

Highest or Lowest point on the VC.



Distance to highest and lowest points in crest and sag vertical curves

$X_{T\ (Turning)}$ = distance to the turning point (highest point: crest) (lowest point: sag)

$\frac{dy}{dx} = \ rx + g_1 = 0$

$$x_T = -\frac{g_1}{r} \ (from\, PVC) \quad or \quad = \frac{g_2}{r} \ (from\, PVT)$$

Elevation of major points of VC.

Elev. PVI =Elev. PVC ± g_1 (L/2)

Elev. PVT =Elev. PVI ± g_2 (L/2)

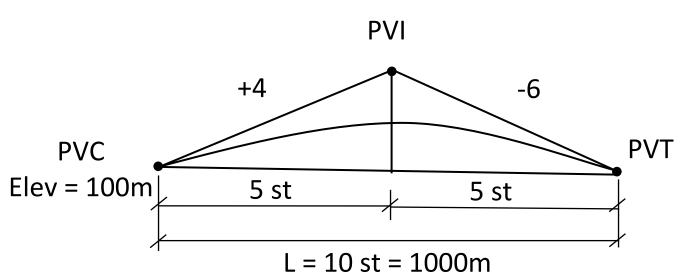
Station of major points of VC.

St. PVI = St. PVC + (L/2)

St. PVT = St. PVI + (L/2)

Example 1:

Find the elevation of the highest point (turning point) on the curve?



$$x_T = \frac{-g_1}{r}$$

$$r = \frac{g_2 - g_1}{L}$$

$$r = \frac{-6-4}{10} = -1$$

$$x_T = \frac{-4}{-1} = 4\ st$$

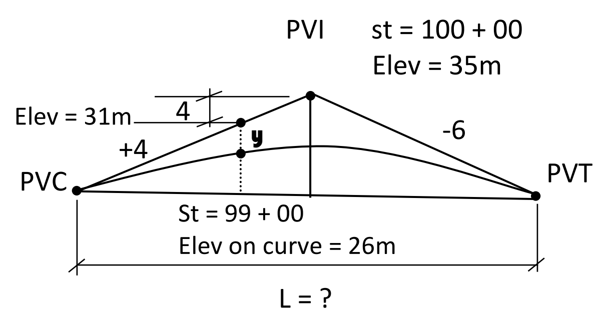
$$E_P = \frac{1}{2} r x^2 + g_1 x + Elev.PVC$$

$$E_P = \frac{1}{2}(-1)(4)^2 + 4 * 4 + 100$$

$$= 108m$$

Example 2:

Find the length of the curve?



y = 31 – 26 = 5 m

$$5 = \frac{1}{2} r\, x^2$$

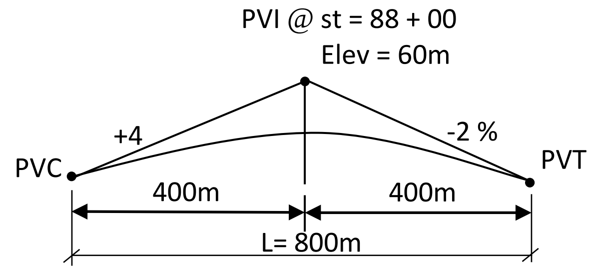
$$= \frac{1}{2}\,\frac{10}{L}\,(\frac{L}{2} - 1)^2$$

L = 7.5 st

$$r = \frac{|g_2 - g_1|}{L}$$

Example 3: Find the elevation on curve every one station, then find the station and elevation of highest point.

Solution:



St. PVI = St. PVC + (L/2)

Station of PVC = 88+00 – 4+00 = 84+00

Elev. PVI =Elev. PVC + g_1 (L/2)

Elevation of PVC = 60 – g_1 * (L/2)

= 60 – 4 *4 = 44m

St. PVT = St. PVI + (L/2)

Station of PVT = 88+00 + 4+00 = 92+00

Elev. PVT =Elev. PVI - g_2 (L/2)

Elevation of PVT = 60 – 2* 4 = 52m

$$r = \frac{g_2 - g_1}{L}$$

$$r = \frac{-2-(+4)}{8} = -\frac{6}{8} = -0.75$$

$$x_T = \frac{-g_1}{r} = \frac{-(+4)}{-\left(\frac{6}{8}\right)} = (5+33)st$$

$$\therefore\ X_T\ @\ station = (84+00)+(5+33)$$

$$= 89+33$$

$$E_{X_T} = E_{PVC} + g_1 x + \frac{1}{2} r\, x^2$$

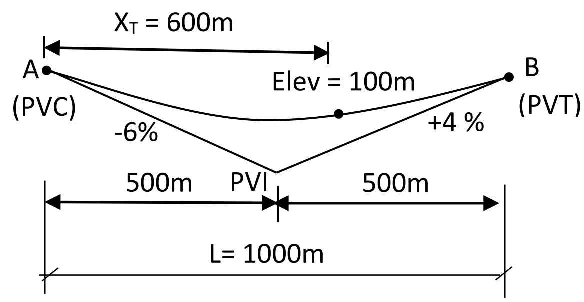
$$E_{X_T} = 44 + 4 * (5.33) + \frac{1}{2} * \left(-\frac{6}{8}\right) * (5.33)^2$$

$$= 54.67m$$

Station	X	$g_1\, X$	X^2	$\frac{1}{2} r\, X^2$	$g_1\, X + \frac{1}{2} r\, X^2$	**Elevation**
PVC 84+00	0	0	0	0	0	44+00
85+00	1	4	1	-0.375	3.625	47.625
86+00	2	8	4	-1.5	6.5	50.5
87+00	3	12	9	-3.375	8.625	52.625
PVI 88+00	4	16	16	$\frac{AL}{8} = e$ = -6	10	54
89+00	5	20	25	-9.375	10.625	54.625
* 89+33	5.33	21.32	28.4	-10.65	10.67	* 54.67
90+00 continue	6	24	36	-13.5	10.5	54.5

Example 4: Find the lowest point of the sag curve knowing that: g_1 = -6%, g_2 = +4%, elevation of intersection of two grades = 88 m and the length of symmetrical curve = 1000m.

Solution:



$$r = \frac{g_2 - g_1}{L}$$

$$r = \frac{4-(-6)}{10} = 1\ \frac{\%}{sta}$$

$$x_T = \frac{-g_1}{r} = \frac{-(-6)}{1} = 6.00\ stations\ from\ PVC$$

Elev. PVI =Elev. PVC - g_1 (L/2)

$$E_A = E_{PVC} = 88 + 6 * 5 = 118m$$

$$E_{x_T} = E_{PVC} + g_1 x + \frac{1}{2} r\, x^2$$

$$E_{x_T} = 118 + (-6 * 6) + \frac{1}{2} * (1) * (6)^2$$

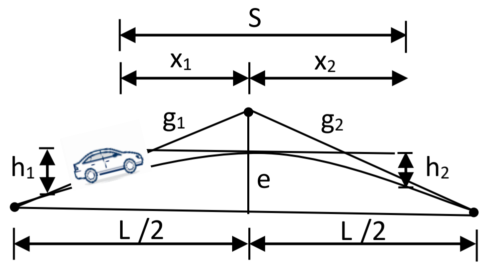
$$= 118 - 36 + 18 = 100m$$

Minimum length of parabolic curve:

⟶ ***Crest curve:***

(1) ***Safety requirement*** ⟶ ***to provide sight distance (clear sight either for stopping or passing)***

a) **Sight distance is included within the limits of the curve** ($S \leq L_{min}$)



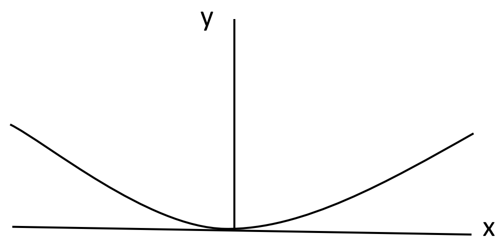
$$y = c\,x^2$$

$$h_1 = c\,x_1^2$$

$$h_2 = c\,x_2^2$$

$$e = c\left(\frac{L}{2}\right)^2$$

$$\frac{h_1}{e} = \frac{c\,x_1^2}{c\left(\frac{L}{2}\right)^2} = \frac{4\,x_1^2}{L^2}$$



y = c x²

$$\frac{h_2}{e} = \frac{4\,x_2^2}{L^2}$$

$$e = \frac{AL}{800}$$ Where: $A = |g_2 - g_1|$

$$S = X_1 + X_2$$

% m

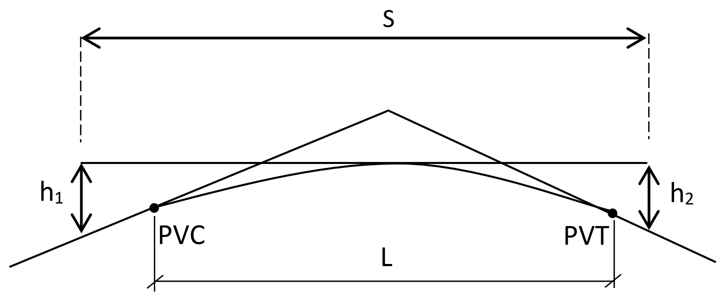
$$L_{min}(m) = \frac{AS^2}{200\,(\sqrt{h_1} + \sqrt{h_2})^2}$$

h_1: height of driver eye above pavement surface (m)

h_2: height of hazardous object (m)

b) **Sight distance is greater than the length of the curve (S > L min)**

$$L_{min}(m) = 2S - \frac{200\,(\sqrt{h_1} + \sqrt{h_2})^2}{A}$$



* ***For safety:***

→ *S stopping: h_1 = 1.08m, h_2 = 0.6m (height of object)*

$$200\,(\sqrt{h_1} + \sqrt{h_2})^2 = 658$$

→ *S passing: h_1 = 1.08 m, h_2 = 1.08 m (height of vehicle)*

$$200\,(\sqrt{h_1} + \sqrt{h_2})^2 = 864$$

* We can use ***K*** value for the calculation of L_{min}

K: rate of vertical curvature: m%

K is the length of curve per 1 percent change in grade

K = F (V)

L min (m) = K.A

V (Km/hr)	Minimum (K) for Crest curve	
	Safe stopping	Safe passing
130	124	769
110	74	617
80	26	338
50	7	138

Example 1: Find the length of curve to provide safe stopping distance (V_0 = 80 Km/hr) (rural area) (wet pavement condition)

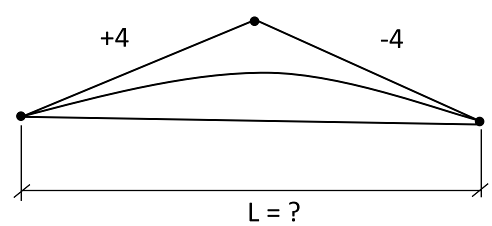
Solution:

1) V_o = 80 Km/hr → K stopping = 26

$A = |g_2 - g_1| = |-4-4| = 8$

$\therefore L_{min} = K * A$

$= 26 * 8 = 208m$



2) By equations:-

V = 80 Km/hr

$$S = 0.278\ Vt + \frac{V^2}{254\ (f_b \pm g)}$$

$$S = 0.278 * 80 * 2.5 + \frac{80^2}{254\ (0.35 - 0.04)} = 137m$$

Let $S \leq L_{min}$

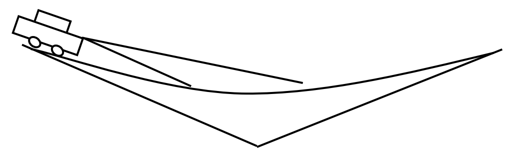
$$L_{min} = \frac{A\ S^2}{658} = \frac{8 *\ 137^2}{658} = 228m$$

عند التصميم لل (passing) نحسب d_1, d_2, d_3, d_4 و نعوض المجموع بـ S

→ Sag curve:

A- Min length of curve for → Safety requirements ①
→ Comfort requirements ②
ناخذ الاكبر

B- Max length of curve for → drainage requirement → ناخذ الاقل



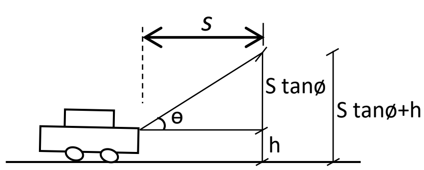
1. Safety requirements

Distance of Head light = *S stopping*

Passing هنا ناخذ مسافة التوقف فقط و ليس الاجتياز

a) $S \le L_{min}$

A = algebraic difference in grades = $|g_2 - g_1|$ = المجموع الجبري للميلين



h: height of head light beam = 0.6m

ɵ: upward divergence of head light beam center ≈ 1°

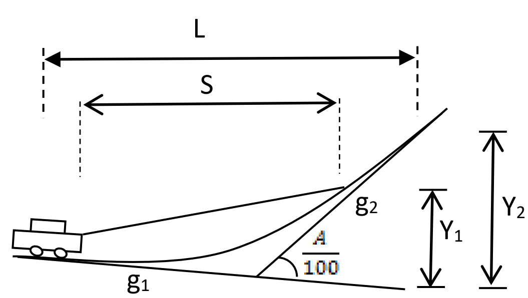
$$y = cx^2$$

$$Y_1 = cS^2$$

$$Y_2 = cL^2$$

$$\frac{Y_1}{Y_2} = \frac{S^2}{L^2}$$

$$Y_1 = Y_2 \frac{S^2}{L^2}$$



$$Y_1 = \frac{A}{100}\frac{L}{2}\frac{S^2}{L^2} = h + S\,tan\theta$$

$$L_{min}(m) = \frac{A\,S^2}{200\,(h + S\tan\theta)}$$

A: $|g_2 - g_1|$, %

h: 0.6 m

ɵ: 1°, tan ɵ = 0.0175

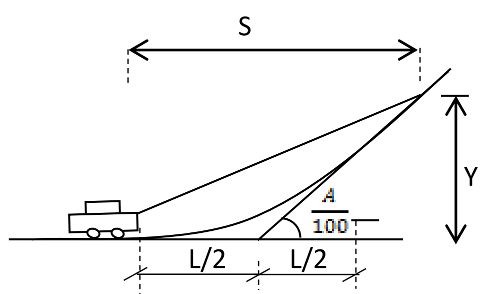
The required sight distance must be provided $S_{stopping}$

$$S = 0.278\,Vt + \frac{V^2}{254\,(f_b \pm g)}$$

b) $S > L_{min}$

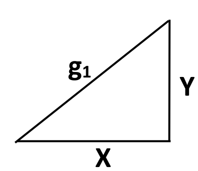
$$\left(S - \frac{L}{2}\right) * \frac{A}{100} = h + S\,tan\theta$$

$$L_{min}(m) = 2S - \frac{200\,(h + S\tan\theta)}{A}$$

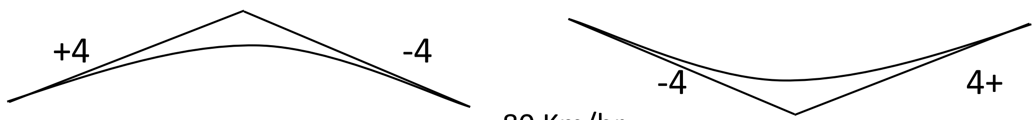


- *We can calculate L from K value*

  $L_{min} = K*A$



V (Km/hr)	***Minimum K for sag curve (safety)***
50	*13*
80	*30*
110	*55*
130	*73*



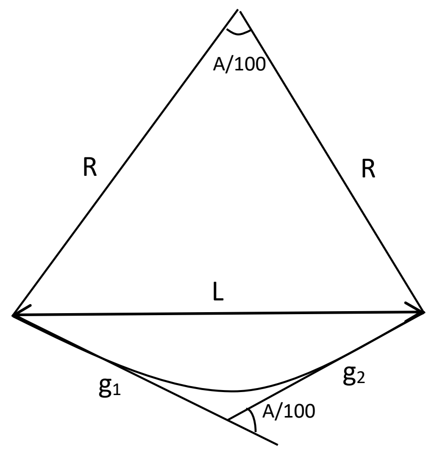
نفس السرعة يكون التقعر اطول لان K اعلى لنفس القيمة

2. ***Comfort (convenience) requirement***:

- Convenience = gravitational force + centrifugal force

نفترض القوس جزء من دائرة لاغراض الاشتقاق

Parabolic curve ≈ circular curve



Centrifugal force $a = \frac{v^2}{R} = \frac{(0.278^2 V^2)}{R}$

$$L = R * \frac{A}{100}$$

$$= \frac{v^2}{a} * \frac{A}{100}$$

$$= \frac{A\, v^2}{100a}$$

(m/sec) * 0.278^2

خطأ موجود هنا و بالفديو (المفروض الاشارة تقسيم و ليس ضرب)

%

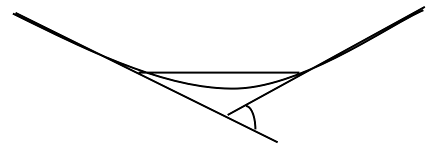
Km/hr

$$L_{min}(m) = \frac{A\, V^2}{1300\, a}$$

m/sec^2

For comfort: a (0.3-0.4)

3. <u>Max L for drainage:</u>



$L_{max} = A * k \longrightarrow (0.44\text{-}0.66)$

Low grades, better efficiency of surface drainage

Where K exceed 0.66 efficient drainage system is required.