Controll (11)م

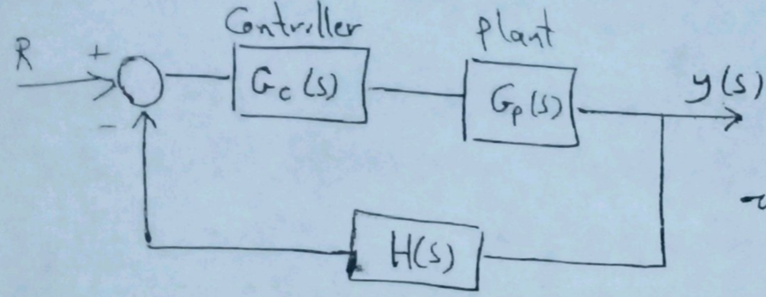
ثالث حاسبات

١. اخلاص

P_1

Lec-11 -

Freqency Response analysis



-where R: refrence i/p
 y : Control Variable (o/p)
 G_p : plant
 G_c : Controller
 H(s) : Feedback element

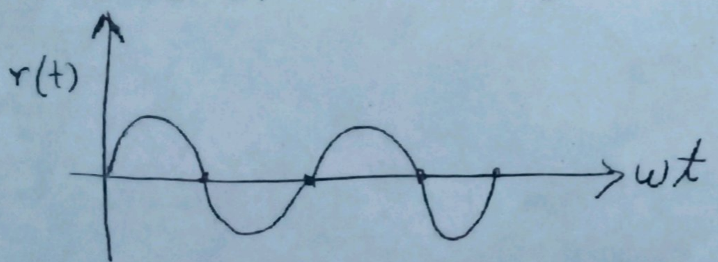
$H(s)G_c(s)G_p(s)$: is open loop T.F

$$T.F = \frac{G_c G_p}{1 + G_c G_p H} \Rightarrow \text{closed loop T.F}$$

There are two method to study the control systems:-

- Time Response
- Freqency Response

* The freqency Response means that the test i/p is Sinusoidal signal ($r(t) = A \sin wt$)



where w: is the Freqency that vary from $0 \longrightarrow \infty$ or vary over a certain Range

* The advantage of Freqency Response

1- we can use the data obtained from measurment on physical system without deriving its mathmatical model

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2- In many practical designs of control system both approaches (Root locus & Frequency Response) are employs

3- frequency Response tests are simple and can be made accurately by use of readily available sinusoidal signal and measurment equipment

4- The system may be design so that the effect of undesirable noise are neglgible, so that the analysis and design can be extended (يمكن توسيعه) to (يوصل) certain nonlinear control system

Some Notes

*1- The Freq. Response is taken only for the open loop T.F

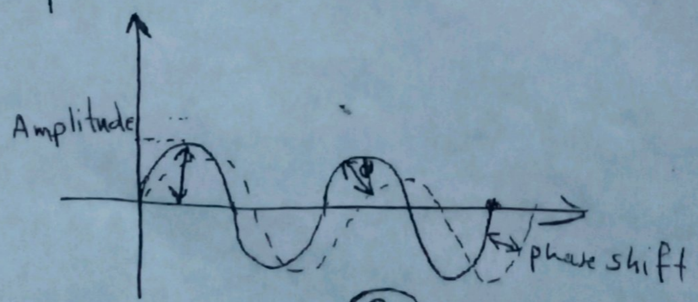
*2- The Freq. Response is calculated by replace $s=j\omega$ after the system reach the steady state

*3- The Freq. Response dose not dependent on the initial conditions

$$G(s) \xrightarrow[\text{تحوّل}]{} G(j\omega) = Me^{j\theta} = M\angle\theta$$

where M = amplitude ratio of the o/p and i/p sinusoidal

ϕ = phase shift between i/p sinusoidal and o/p



EX1: find the $y_{ss}(t)$ for $G(s) = \frac{k}{Ts+1}$, if the i/p $x(t) = X \sin wt$

sol $G(jw) = \frac{k}{Tjw+1} = M \angle \phi = |G(jw)| \angle \phi$

$|G(jw)| = \frac{k}{\sqrt{1+T^2w^2}}$ $\qquad \phi = -\tan^{-1}\frac{Tw}{1}$

$\sin wt = \frac{e^{jwt} - e^{-jwt}}{2j}$

$y_{ss}(t) = |G(jw)| \angle \phi * i/p = \left(\frac{k}{\sqrt{1+T^2w^2}} \angle -\tan^{-1} Tw\right) * X \sin wt$

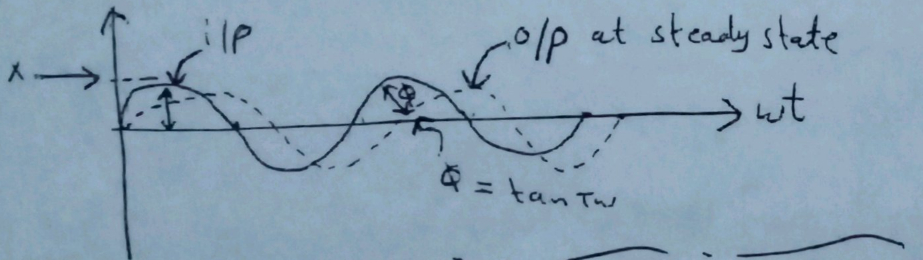
$y_{ss}(t) = M e^{j\theta} * i/p$
$= M e^{j\theta} * X \sin wt$

$= \frac{k}{\sqrt{1+T^2w^2}} e^{j\theta} * \frac{(e^{jwt} - e^{-jwt})X}{2j}$

$= \frac{k}{\sqrt{1+T^2w^2}} * X \frac{e^{(jwt+j\theta)} - e^{(-jwt+j\theta)}}{2j}$

$= \frac{Xk}{\sqrt{1+T^2w^2}} \sin(wt - \underbrace{\tan^{-1} Tw}_{\text{phase shift}})$

مقدار



EX2: Consider the network given by $G(s) = \frac{s + \frac{1}{T_1}}{s + \frac{1}{T_2}}$

find y_{ss} if the i/p is sinusoidal input

sol $G(s) = \frac{\frac{T_1 s + 1}{T_1}}{\frac{T_2 s + 1}{T_2}} \Rightarrow \frac{T_1 s + 1}{T_1} * \frac{T_2}{T_2 s + 1} = \boxed{\frac{T_2(T_1 s + 1)}{T_1(T_2 s + 1)}}$

Lec-11-

$$\therefore G(j\omega) = \frac{T_2(T_1 j\omega + 1)}{T_1(T_2 j\omega + 1)} \Longrightarrow G(j\omega) = |G(j\omega)| \angle \Phi$$

$$= \frac{T_2\sqrt{1 + T_1^2\omega^2}}{T_1\sqrt{1 + T_2\omega^2}} \angle \Phi$$

$$\because \Phi = + \tan^{-1}\frac{T_1\omega}{1} - \tan^{-1}\frac{T_2\omega}{1}$$

$$\therefore G(j\omega) = \frac{T_2\sqrt{1 + T_1^2\omega^2}}{T_1\sqrt{1 + T_2\omega^2}} \angle \tan^{-1} T_1\omega - \tan^{-1} T_2\omega$$

$$y_{ss}(t) = \left(G(j\omega) \angle \Phi\right) * X \sin \omega t$$

$$= \frac{X * T_2\sqrt{1 + T_1^2\omega^2}}{T_1\sqrt{1 + T_2\omega^2}} * \sin(\omega t + \tan^{-1} T_1\omega - \tan^{-1} T_2\omega)$$

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* The freqency response with the sinusoidal T.F can be represented by Three graphical approaches
- Bode diagram or logorithmic plot
- Nyquist plot or polar plot
- log-magnitude-versus-phase plot (Nichols plots)

Polar plot

<u>Polar plot</u> :- Represent mag. (magnitude) versus phase plot in polar cordinate as ω is varied from $0 \longrightarrow \infty$

* لا يجب الرسم بصورة دقيقة بتعويض كذا قيم لـ ω بل يكفيا نقطتين فقط وهي zero و ∞

$\tan^{-1} 0 = 0$
$\tan^{-1} \infty = 90$

* تسمى هذه الطريقة ايضا Nyquist plot

* Basic factor of $G(j\omega)$

1- Constant gain k: $\Rightarrow G(s) = k$ ← gain

$$G(j\omega) = k = |G(j\omega)| \angle \phi$$

$$|G(j\omega)| = |k| = k$$

$$\phi = \tan^{-1} \frac{0}{k} = 0$$

$$\therefore G(j\omega) = k \angle 0$$

2- n poles at the orign: $\Rightarrow G(s) = \frac{k}{s^n}$

$$G(j\omega) = \frac{k}{(j\omega)^n} = |G(j\omega)| \angle \phi$$

$$|G(j\omega)| = \frac{k}{\sqrt{\omega^2} * \sqrt{\omega^2} \cdots \sqrt{\omega}^{n}} = \frac{k}{\omega^n}$$

$$\phi = -90 * n$$

$$G(j\omega) = \frac{k}{\omega^n} \angle -90*n$$

3- n zeros at the orign: $\Rightarrow G(s) = k s^n$

$$G(j\omega) = k j\omega^n = |G(j\omega)| \angle \phi$$

$$|G(j\omega)| = k * \omega^n$$

$$\phi = 90 * n$$

$$G(j\omega) = k * \omega^n \angle 90*n$$

4- 1st order poles: $G(j\omega) = \frac{k}{(1 + Tj\omega)^n}$

$$|G(j\omega)| = \frac{k}{\sqrt{1 + T^2\omega^2} * \sqrt{1 + T^2\omega^2} \cdots \sqrt{1 + T^2\omega^2}}$$

$$= \frac{k}{(\sqrt{1 + T^2\omega^2})^n}$$

$$\phi = -\left(\tan^{-1}\frac{T\omega}{1}\right) * n$$

$$G(j\omega) = \frac{k}{\left(\sqrt{1+T^2\omega^2}\right)^n} \angle -\left(\tan^{-1}\frac{T\omega}{1}\right) * n$$

5- 1st order Zeros :

$$G(s) = k(1+Ts)^n$$

$$G(j\omega) = k\,(1+Tj\omega)^n = |G(j\omega)| \angle\phi$$

$$|G(j\omega)| = k\left(\sqrt{1+T^2\omega^2}\right)^n$$

$$\phi = \left(\tan^{-1} T\omega\right) * n$$

$$G(j\omega) = k\left(\sqrt{1+T^2\omega^2}\right)^n \angle\left(\tan^{-1}T\omega\right) * n$$

complex Conjugate poles or

6- Second order system :

$$G(s) = \frac{k\,\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

نحاول التخلص منه

$$G(s) = \frac{k}{\frac{1}{\omega_n^2}s^2 + \frac{2\zeta}{\omega_n}s + 1}$$

$$G(j\omega) = \frac{k}{-\frac{1}{\omega_n^2} * \omega^2 + \frac{2\zeta}{\omega_n}j\omega + 1} = \frac{k}{\underbrace{\left(1 - \frac{\omega^2}{\omega_n^2}\right)}_{\text{Real}} + \underbrace{\frac{2\zeta}{\omega_n}j\omega}_{\text{Img.}}}$$

$$|G(j\omega)| = \frac{k}{\sqrt{\left(1-\frac{\omega^2}{\omega_n^2}\right)^2 + \frac{4\zeta^2}{\omega_n^2} * \omega^2}}$$

$$\phi = -\tan^{-1}\frac{\frac{2\zeta}{\omega_n} * \omega}{1 - \frac{\omega^2}{\omega_n^2}}$$

7- complex Conjugate Zeros :

$$G(s) = \frac{k}{\omega_n^2}\left(s^2 + 2\zeta\omega_n s + \omega_n^2\right)$$

$$G(s) = k\left(\frac{1}{\omega_n^2}s^2 + \frac{2\zeta}{\omega_n}s + 1\right)$$

$$G(j\omega) = k\left(\left(1 - \frac{\omega^2}{\omega_n^2}\right) + \frac{2\zeta}{\omega_n}j\omega\right)$$

$$|G(j\omega)| = k\sqrt{\left(1-\frac{\omega^2}{\omega_n^2}\right)^2 + \frac{4\zeta^2}{\omega_n^2} * \omega_n^2}$$

$$\phi = \tan^{-1}\frac{\frac{2\zeta}{\omega_n} * \omega}{1 - \frac{\omega^2}{\omega_n^2}}$$

Ex1: Draw Polar plot for the following T.f

$$G(s) = \frac{k}{(1+s)(1+2s)}$$

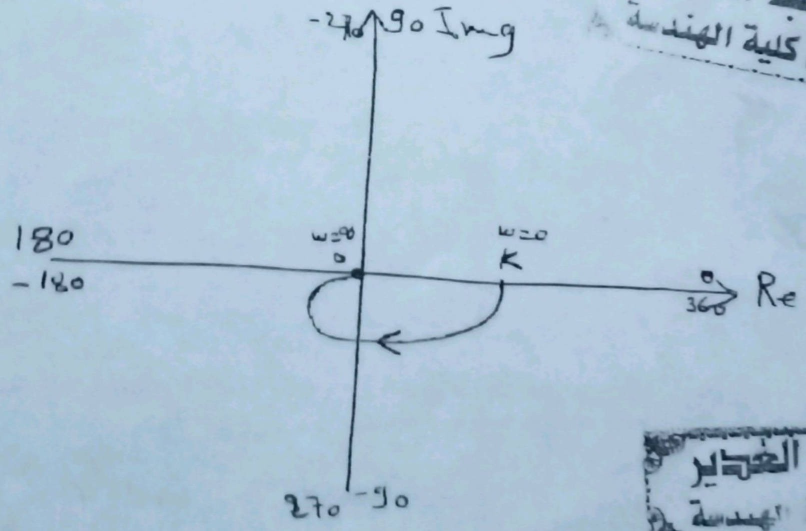
sol put $s = j\omega$ $\therefore G(j\omega) = \frac{k}{(1+j\omega)(1+2j\omega)}$

$G(j\omega) = |G(j\omega)| \angle \phi$ $|G(j\omega)| = \frac{k}{\sqrt{1+\omega^2}\ \sqrt{1+4\omega^2}}$

$$\phi = -\tan^{-1}\omega - \tan^{-1}2\omega$$

$G(j0) = k \angle 0$

$G(j\infty) = \frac{k}{(j\infty)(2j\infty)} = 0 \angle -180$



Ex2: $G(s) = \frac{k}{(1+s)(1+0.5s)(1+2s)}$

Draw polar plot

sol $G(j\omega) = \frac{k}{(1+j\omega)(1+0.5j\omega)(1+2j\omega)}$

$$G(j\omega) = |G(j\omega)| \angle \Phi \Longrightarrow |G(j\omega)| = \frac{k}{\sqrt{1+\omega^2}\sqrt{1+(0.5)^2\omega^2}\sqrt{1+4\omega^2}}$$

$G(j0) = k \angle 0$ where $|G(j0)| = \frac{k}{(1+0)(1+0.5\times 0)(1+2\times 0)} = k$

$$\Phi\Big|_{\omega=0} = -\tan^{-1}0 - \tan^{-1}0.5\times 0 - \tan^{-1}2\times 0$$

عند zero تكون الزاوية zero

$$= 0$$

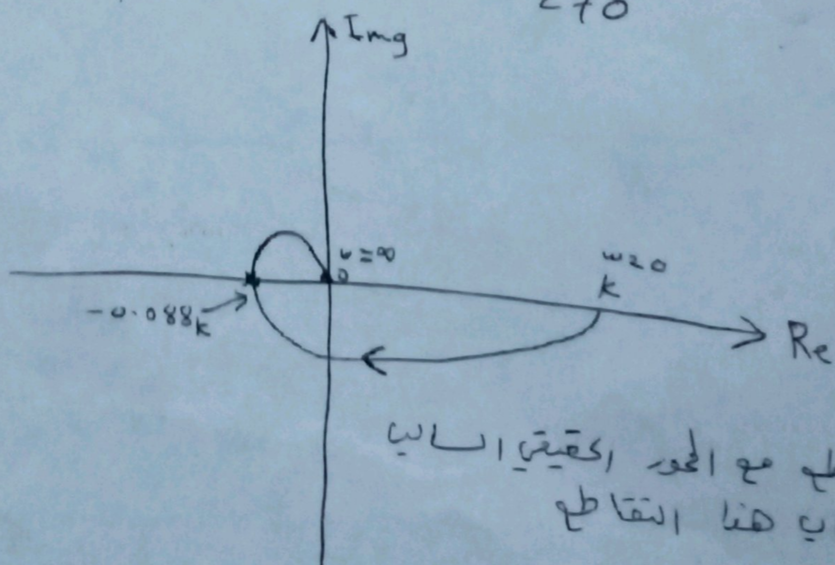
$G(j\infty) = 0 \angle -270$ where $G(j\infty) = \frac{k}{(1+\infty)(1+0.5\times\infty)(1+2\times\infty)} = 0$

عند ∞ فان الزاوية تكون 90

$$\Phi\Big|_{\omega=\infty} = -\tan^{-1}\infty - \tan^{-1}0.5\times\infty - \tan^{-1}2\times\infty$$

$$= -90 - 90 - 90$$

$$= -270$$



* في حالة وجود تقاطع مع المحور الحقيقي السالب
يجب حساب هنا التقاطع

$$G(j\omega) = X(\omega) + jY(\omega)$$

$$G(j\omega) = \frac{k}{(1+j\omega)(1+2j\omega)(1+0.5j\omega)} \cdot * \frac{(1-j\omega)(1-2j\omega)(1-0.5j\omega)}{(1-j\omega)(1-2j\omega)(1-0.5j\omega)}$$

$$G(j\omega) = \frac{k(1-3j\omega-2\omega^2)(1-0.5j\omega)}{(1+\omega^2)(1+4\omega^2)(1+0.25\omega^2)}$$

$$= \frac{k(1-0.5j\omega-3j\omega-1.5\omega^2+j\omega^3)}{(1+\omega^2)(1+4\omega^2)(1+0.25\omega^2)}$$

$$G(j\omega) = \underbrace{\frac{k(1-3.5\omega^2)}{(1+\omega^2)(1+4\omega^2)(1+0.25\omega^2)}}_{\text{Real part}} + j\underbrace{\frac{k(\omega^3-3.5\omega)}{(1+\omega^2)(1+4\omega^2)(1+0.25\omega^2)}}_{\text{Img. part}}$$

$$\frac{k(\omega^3-3.5\omega)}{(1+\omega^2)(1+4\omega^2)(1+0.25\omega^2)} = 0 \Rightarrow \omega^3-3.5\omega = 0$$

$$\omega(\omega^2-3.5) = 0$$

$$\omega = 0$$

$$\omega = \mp 1.87$$

يوخذ الجزء الموجب فقط $\boxed{\omega = 1.87}$ rad/sec

* w التي تعطي التقاطع مع المحور الحقيقي السالب يجب ان تكون قيمة حقيقية اكبر من صفر

sub the value of w in Real part of G(jw)

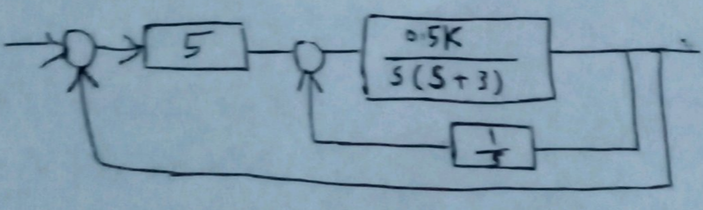
$\therefore$ Rel $= X(\omega) = -0.088k$

H.w1: A unity Feedback Control system with open loop T.F

$$G(s) = \frac{k}{s(1+s)(1+3s)}$$

Draw Polar plot

H.w2 Draw polarplot For the system shown in Fig(*)



Non minimum phase systems:

The sys. is called nonminimum phase sys. if there are one poles (or Zeros) lie in the right half plane of s-plane

*** Minimum phase system** :- The sys. is called "Minimum phase system" if there is no poles or Zeros of The T.F lie in the right half plane of the s-plane

رفة استقرارية النظام عن طريق polarplot :-

Min

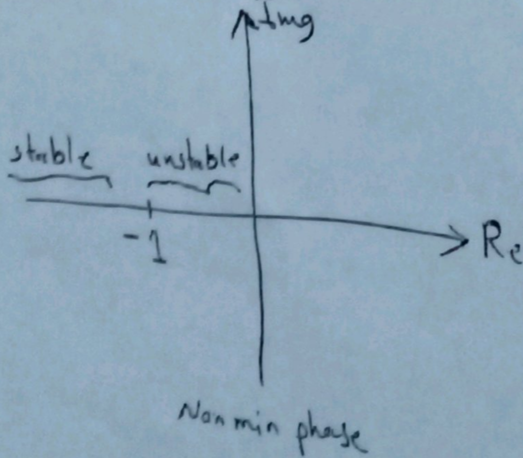
التقاطع مع المحور الحقيقي قبل نقطة 1- يكون stable

اذا كان التقاطع مع المحور الحقيقي بعد النقطة 1- يكون unstable

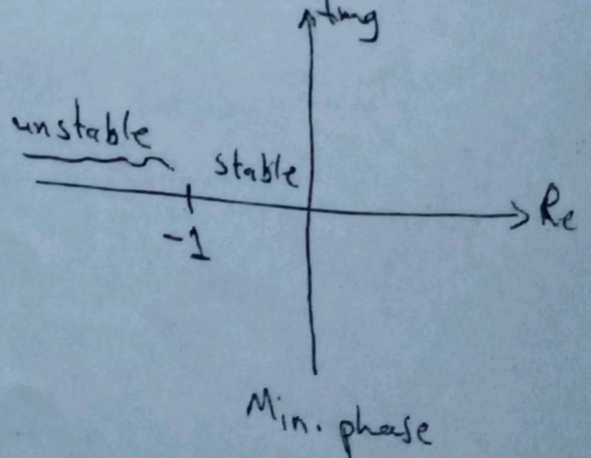
Non Min

اذا كان التقاطع مع المحور الحقيقي بعد النقطة -1- يكون stable

= = = = = قبل = -1 يكون unstable



Non min phase



Min. phase

Ex3: For the control sys. with open loop T.F

$$G(s) = \frac{5(1+2s)}{s^2(1+s+s^2)}$$

Draw polar plot and check weather the system is stable or not

P10

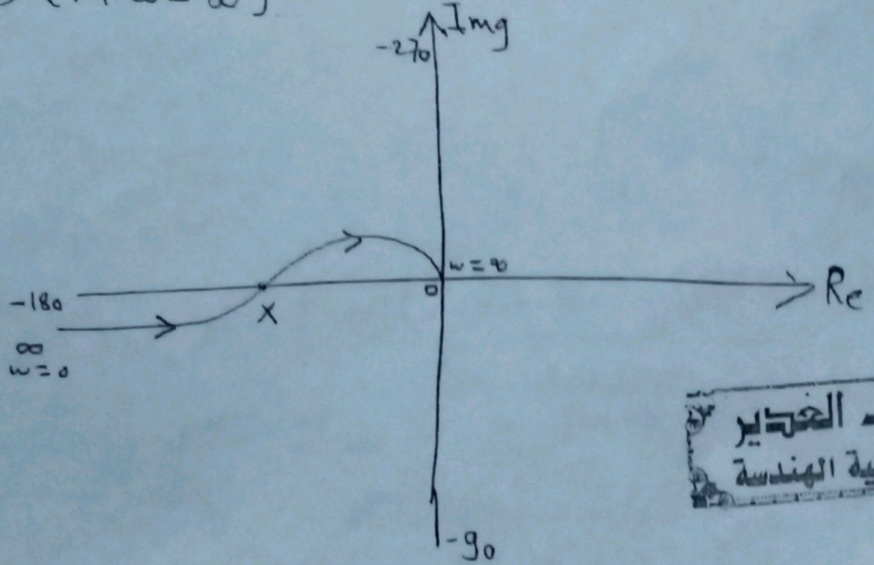
sol $G(jw) = \dfrac{5(1+2jw)}{(jw)^2(1+jw-w^2)}$

$$|G(jw)| = \frac{5\sqrt{1+4w^2}}{w^2 \times \sqrt{(1-w^2)^2+w^2}}$$

$$\phi = -180 - \tan^{-1}\frac{w}{1-w^2} + \tan^{-1} 2w$$

$$G(j0) = \frac{5(1+2\times0)}{(0)^2(1+0-0^2)} = \infty \angle -180$$

$$G(j\infty) = \frac{5(1+2\times\infty)}{(\infty)^2(1+\infty-\infty^2)} = 0 \angle -270$$



لايجاد نقطة التقاطع X

$$G(jw) = X(w) + jY(w)$$

$$= \frac{5(1+2jw)}{(jw)^2(1+jw-w^2)} * \frac{(-jw)^2(1-jw-w^2)}{(-jw)^2(1-jw-w^2)}$$

يتم تغيير اشارة Img فقط

$$G(j\omega) = \frac{5(1+2j\omega) * \left[-\omega^2 + \omega^4 + j\omega^3\right]}{\omega^4\left[(1-\omega^2)^2 + \omega^2\right]}$$

$$= \frac{5\left[-\omega^2 + \omega^4 + j\omega^3 - 2j\omega + 2j\omega^5 - 2\omega^4\right]}{\omega^4\left[(1-\omega^2)^2 + \omega^2\right]}$$

$$= \underbrace{\frac{5(-\omega^2 + \omega^4 - 2\omega^4)}{\omega^4\left[(1-\omega^2)^2 + \omega^2\right]}}_{X} + \underbrace{j\,\frac{-5(2\omega^5 - \omega^3)}{\omega^4\left[(1-\omega^2)^2 + \omega^2\right]}}_{j\,y}$$

$$y = 0 \Rightarrow 2\omega^5 - \omega^3 = 0 \Rightarrow \omega^3(2\omega^2 - 1) = 0$$

$\omega = 0$

$2\omega^2 - 1 = 0$

$\omega = \mp\sqrt{\frac{1}{2}}$

$\omega = 0.707$ rad/sec يعني الوحدة [illegible]

sub $\omega = 0.707$ in Rel (X) part, we get

$Rel = -20$

$\underline{\underline{-20}} < -1$

$\therefore$ the system unstable

<u>H.w</u>: for the control system with O.L.T.F $G(s) = \frac{1}{1+TS}$, Prove that the polar plot is a semicircle with center $(0.5, 0)$ and ~~radius~~ radius equal 0.5

<u>note</u> $\boxed{\text{use } \underset{Rel}{X^2} + \underset{Img}{y^2} = ?}$