Lecture 2

The Equivalent Circuit and Torque of Three-Phase Induction Motor

The analysis of three phase induction motor is simplified by using their per-phase equivalent circuits. The derivation of the equivalent circuit is based on the consideration of the electromagnetic coupling between the stator and rotor when the latter is stationary (standstill), and when it is running.

The **exact** per phase equivalent circuit of three phase induction motor can be depicted as in Figure 1. This circuit represents the general per-phase equivalent circuit at any speed, as seen from the input terminals of the motor.

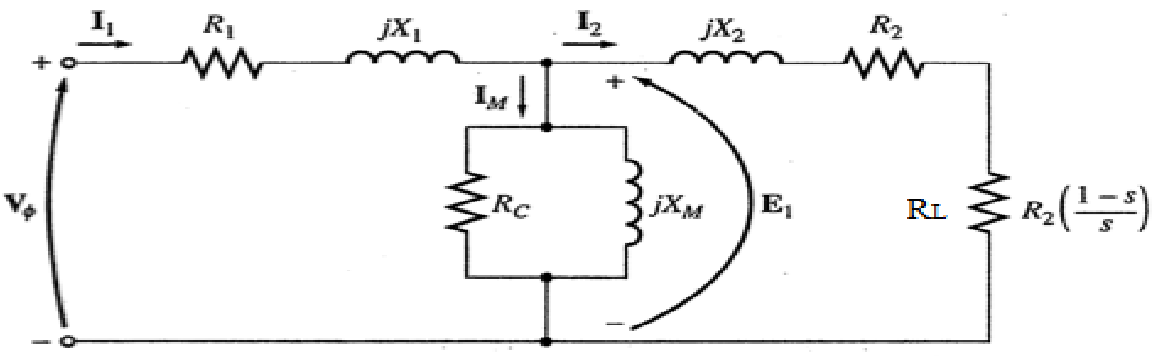

Figure 1 The Exact Per Phase Equivalent Circuit

The real component of the rotor impedance, as seen by the stator, can also be written as follows:

$$\frac{R_2}{S} = R_2 + R_2\,\frac{1-S}{S}$$

The second term on the right-hand side of the above equation is referred to the equivalent mechanical load (**R_L**) resistance. That means:

$$R_L = \frac{R_2(1-S)}{S}$$

Since the value of (R_c) is very high in compare with (X_m), in many times this resistance can be neglected. Also, the above circuit can be simplified by further approximation and in this case, the new (simplified) circuit is called an **approximate** equivalent circuit as shown in the figure 2.

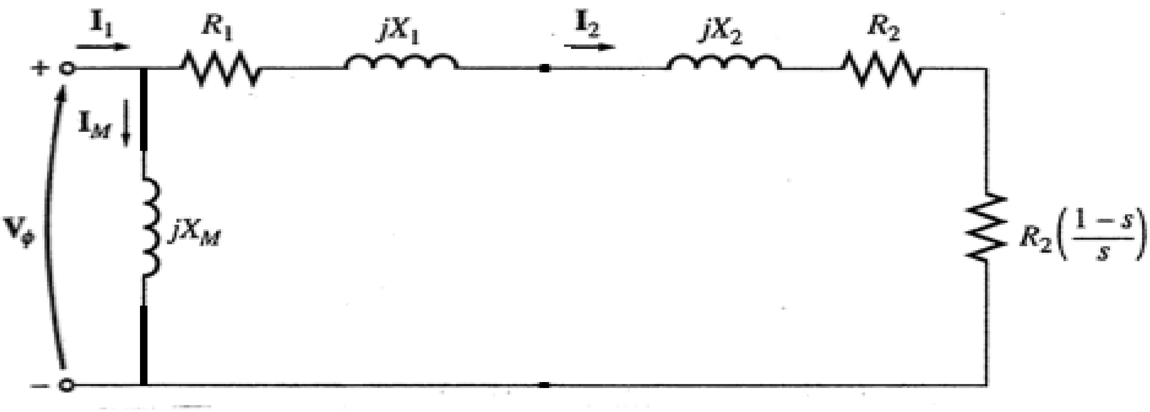

Figure 2 The Approximate Per Phase Equivalent Circuit

Torque of Three Phase Induction Motor:

The physical meaning of the torque for rotating bodies is the turning force through a radius and the unit is in Newton per meter (Nm) as shown in the figure 3 below:

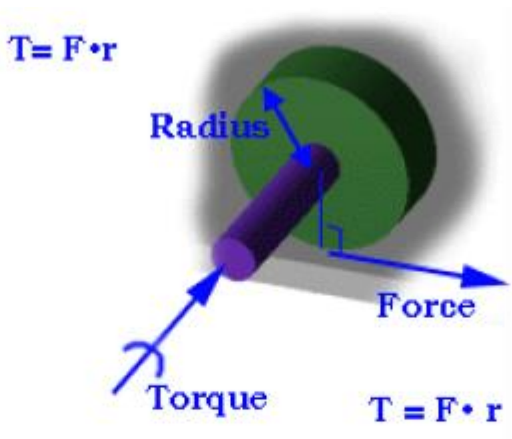

Figure 3 The Torque Representation for Rotating Bodies

Torque in three phase Induction Motor can be derived by using the equivalent circuit shown in Figure 4. In deriving the general expression for the torque of an induction motor, the effects of the magnetizing impedance (Z_m=R_c // X_m) will be neglected. Because this magnetizing impedance is relatively large and its effect on the torque developed by the motor at full load is negligible.

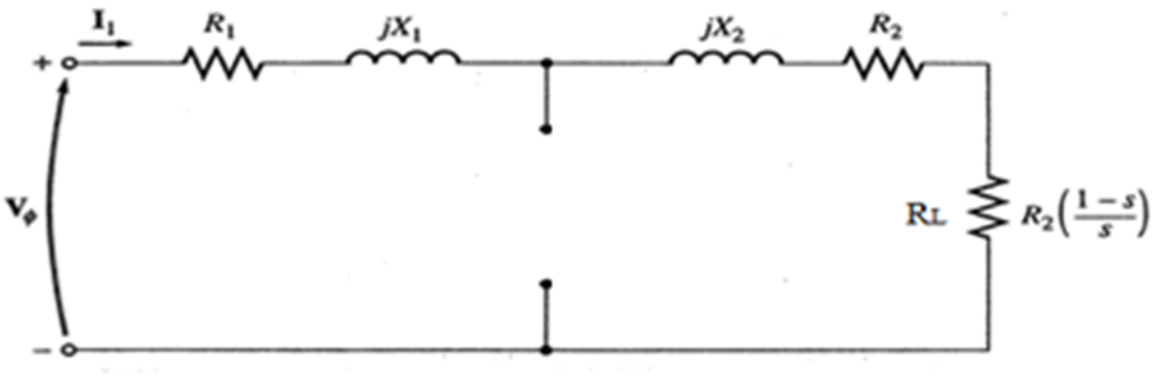

Figure 4 Per Phase Equivalent Circuit with Neglected Mag. Impedance

From basic definitions, the per-phase torque developed can be expressed as in (1).

$$\mathbf{T} = \frac{\textbf{power developed}}{\textbf{rotor speed}} = \frac{\mathbf{I_2}^2\,\mathbf{R_L}}{\mathbf{w_r}} \qquad (1)$$

The equivalent mechanical load resistance R_L :

$$\mathbf{R_L} = \mathbf{R_2}\frac{\mathbf{1-S}}{\mathbf{S}} \qquad (2)$$

Where **S** is induction motor slip which is calculated as follows:

$$\mathbf{S} = \frac{\mathbf{Ws-W_r}}{\mathbf{W_s}} \qquad (3)$$

$$\boldsymbol{W_s} = \frac{\boldsymbol{2\pi N_s}}{\mathbf{60}} \quad \text{in rad/sec,}$$

$$\mathbf{W_r} = \frac{\mathbf{2\pi Nr}}{\mathbf{60}} \quad \text{in rad/sec,}$$

$$\mathbf{W_r} = \mathbf{W_s}(\mathbf{1-S}) \qquad (4)$$

Substituting (2), and (4) into (1), we get

$$\mathbf{T} = \frac{\mathbf{I_2}^2\,\mathbf{R_2}\frac{\mathbf{1-S}}{\mathbf{S}}}{\mathbf{W_s}\ (\mathbf{1-S})} \qquad (5)$$

$$\mathbf{T} = \frac{I_2{}^2 R_2}{W_s\ S} \quad \text{N.m / phase} \qquad (6)$$

From the equivalent circuit, the magnitude of the current $\mathbf{I_2}$ is

$$I_2 = \left|\frac{V}{Z}\right| = \frac{V}{\sqrt{\left(R_1+\frac{R_2}{S}\right)+ (X_1+X_2)^2}} \qquad (7)$$

From the above, the torque developed in terms of the motor's parameters can be expressed as:

$$\mathbf{T} = \frac{V^2}{\left[\left(R_1+\frac{R_2}{S}\right)^2+(X_1+X_2)^2\right]} \left(\frac{R_2}{W_s\ S}\right) \quad \text{N.m / phase} \qquad (8)$$

So, the torque for **three** phase induction motor can be calculated according to the general expression as follows:

$$\mathbf{T} = \frac{3\ V^2\ \frac{R_2}{S}}{W_s\ \left[\left(R_1+\frac{R_2}{S}\right)^2+(X_1+X_2)^2\right]} \quad \text{N.m} \qquad (9)$$

The torque can also be represented by the expression below:

$$\mathbf{T} = \frac{K\ S\ R_2\ E_2{}^2}{R_2{}^2 + S^2\ X_2{}^2} \qquad (10)$$

Where, K is a function of the physical parameters of the motor.

Torque-Speed Characteristic of Three Phase Induction Motor:

The torque developed by three phase induction motors varies with the speed of the motor when it accelerates from full stop or zero speed, to maximum operating speed. Figure 5 shows the changes in the developed torque of the motor in accordance with motor speed.

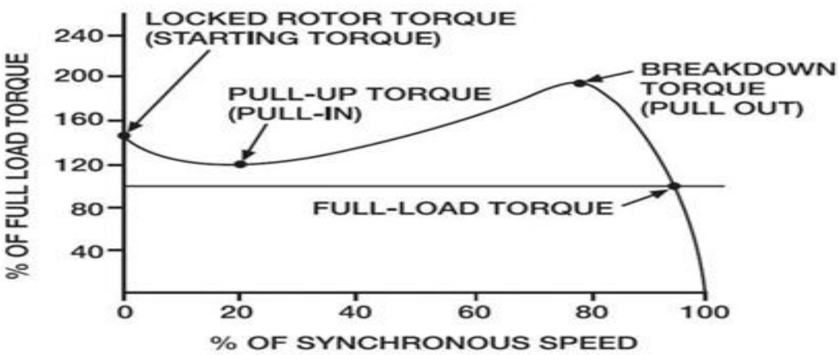

Figure 5 Torque-Speed Characteristic of 3-Phase Induction Motor

Locked Rotor (Starting) Torque:

The locked rotor torque or starting torque is the developed torque by the motor when it starts from rest or zero speed. A high starting torque is more important for starting heavy or hard types of loads such as positive displacement pumps, cranes etc. A lower starting torque can be accepted in applications such as centrifugal fans or pumps when the required starting torque is low or close to zero.

Pull-up Torque:

The Pull-up Torque is the minimum torque developed by the motor when it runs from zero to full-load speed before it reaches the break-down torque point. When the motor starts and begins to accelerate the torque in general decrease until it reaches a low point at a certain speed (at the pull-up torque point) before the torque increases until it reaches the highest torque at a higher speed (at the break-down torque point). The pull-up torque may be critical for applications that needs power to go through some temporary barriers achieving the working conditions.

Break-down (or Maximum) Torque:

Break-down torque is the highest torque available before the torque decreases when the machine continues to accelerate at running (working) conditions. The maximum torque occurs when rotor resistance and rotor reactance are equal, i.e. $R_2 = S X_2$,

So, $S_m = R_2/X_2$, Where, S_m is the slip at maximum torque.

Full-load (or Rated) Torque:

The Full-load torque is the torque required to produce the rated power of the electrical motor at full-load speed. Figure 6 shows a typical torque–speed curve showing two different loads which have the same steady running speed (N). The solid line is the torque–speed curve of the motor, while the dotted lines represent two different load characteristics. Load (A) is typical of a simple hoist, which applies constant torque to the motor at all speeds, while load (B) might represent a fan.

Tacc represent the accelerating torque, which is the difference between the torque developed by the motor and the torque required to run the load at that speed.

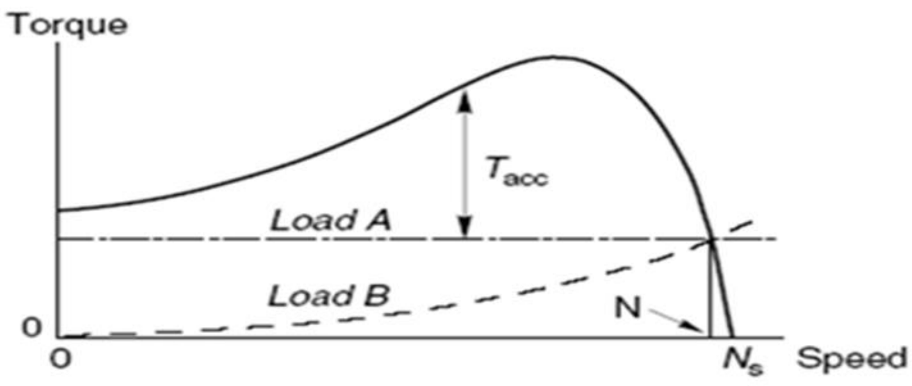

Figure 6 Typical Torque Speed Curve with Two Different Loads

The intersection of these characteristics gives the operating torque and speed of the motor. **It is extremely important to accurately calculate the torque-speed characteristic of the driven load in order to ensure the selection of the suitable motor for this load**.

Ways of Improving Starting Torque of 3-ph Induction Motor:

For a certain range of rotor resistance, the starting torque (T_s) of the motor is proportional to its rotor resistance. Mathematically ($\mathbf{T_s \alpha\ R_2}$) So, if (**R_2**) is increased at motor starting that will make rotor circuit being more resistive and the starting torque is increased.

Adding External Resistance to the Rotor Circuit

This method of improving starting torque is applicable for Wound Rotor type induction motor only. It is possible to increase the starting torque of the motor by adding an external resistance to the rotor resistance via slip rings as in Figure 7.

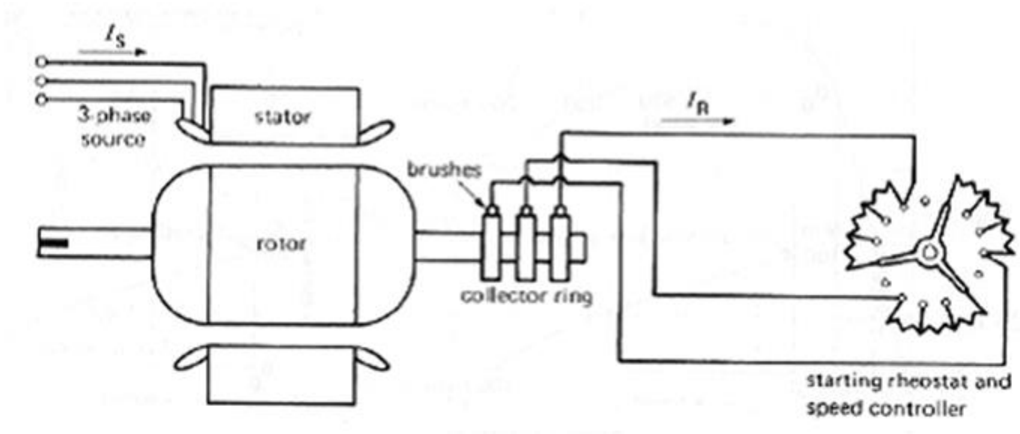

Figure 7 Effect of Adding an External Rotor Resistance for WRIM

Deep Bar Design:

Leakage reactance (**X_2**) represents the rotor leakage reactance (reactance due to the rotor's flux lines that do not couple with the stator windings.) Generally, the farther away the rotor bar is from the stator, the greater (**X_2**) will be. Since a smaller percentage of the bar's flux will reach the stator. Thus, if the bars of a cage rotor are placed near the surface of the rotor, they will have small leakage flux and (**X_2**) will be small. The basic concept of deep bar design is illustrated in Figure 8.

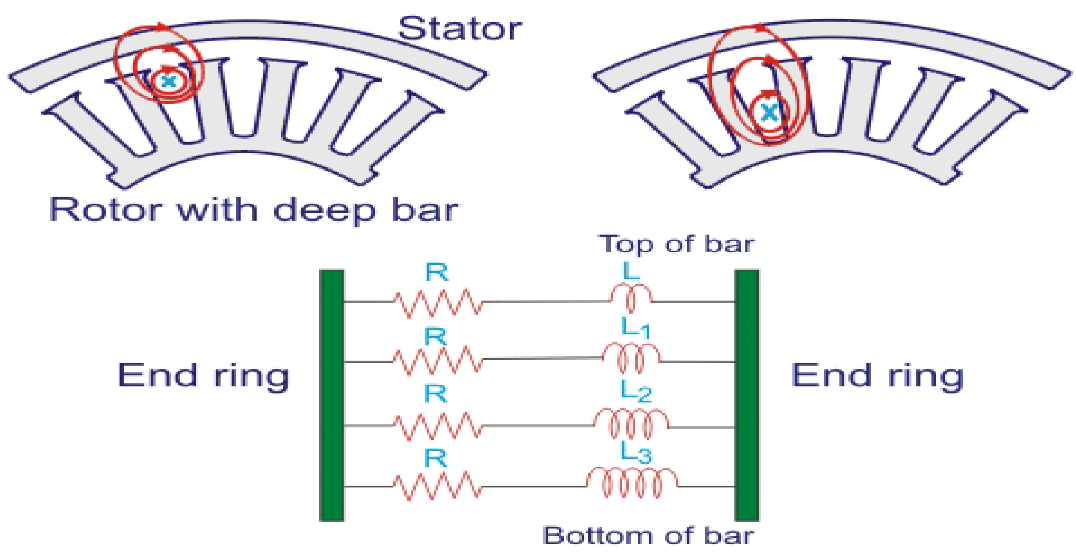

Figure 8 Deep Bar Design Rotor for SCIM

At high slips (starting condition), the reactance is large compared to the resistances in the rotor bars, so all the current is forced to flow in the low-reactance part of the bar near the stator. Since the effective cross section is lower, the rotor resistance is higher. Thus, the starting torque is increased, and the starting current is reduced

At low slips (running condition) the rotor's frequency is very small, and the reactance of all the parallel paths are small compared to their resistances. The impedances of all parts of the bar are approximately equal, so current flows through all the parts of the bar equally. The resulting large cross-sectional area makes the rotor resistance quite small, resulting in good efficiency at low slips.

Double cage design:

In a double cage induction motor two set of bars are used as shown in figure 9. One set with a small resistance is put deep to the rotor. A second cage with a high resistance is put close to the stator. At starting the leakage reactance is smaller at the bars close to the stator (those with the high resistance). Current flows mostly on the outer bars resulting in a high starting torque. As the rotor speeds up the frequency decreases the leakage reactance of both bars, and more current flows in the deep bars with the lower resistance. The torque speed characteristic of the inner cage is that of a normal induction motor, as in Figure 9.

At starting, the outer cage produces the torque, but when running the inner cage produces the torque. The combined characteristic of inner and outer cages is shown in this Figure. The double cage induction motor is highly efficient when running.

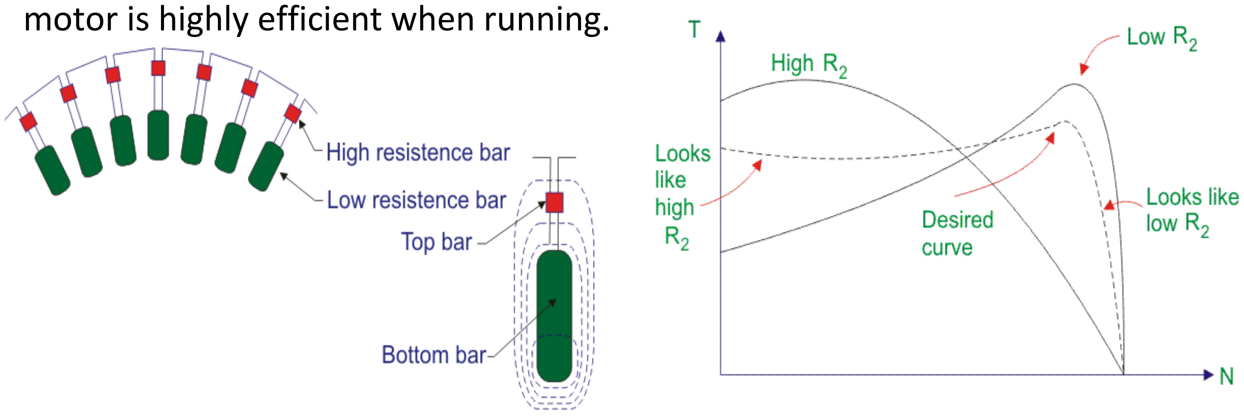

Figure 9 Double Cage Rotor Design and Characteristics for SCIM

NEMA Motor Design Classes:

NEMA classifies poly phase induction motors in four different design codes (A, B, C and D) according to locked rotor torque and current, breakdown torque, pull up torque, and percent slip.

NEMA Design Class A:

Rotor bars are quite large and are placed near the surface of the rotor. Low resistance (due to its large cross section) and a low leakage reactance $\mathbf{X_2}$ (due to the bar's location near the stator) .Since $\mathbf{R_2}$ is small, starting torque will be small. This design is the standard motor design. **Typical applications as driving fans, pumps, and other machine tools.**

NEMA Design class B:

It contain deep-bar rotor. **The applications of this motor design type are similar to class A,** and have largely replaced type A.

NEMA Design class C:

It contain double-cage rotor. At starting conditions, only the small bars are effective, and the rotor resistance is high. Hence, high starting torque. **Used in high starting torque loads such as loaded pumps, compressors, and conveyors.**

NEMA Design class D:

Rotor with small bars placed near the surface of the rotor (higher-resistance material) High resistance (due to its small cross section) and a low leakage reactance $\mathbf{X_2}$ (due to the bar's location near the stator). Like a wound-rotor induction motor with extra resistance inserted into the rotor. **The applications for this design type are extremely high-inertia type loads**.

The typical torque speed characteristics for all design types (A, B, C, D) are illustrated in figure 10.

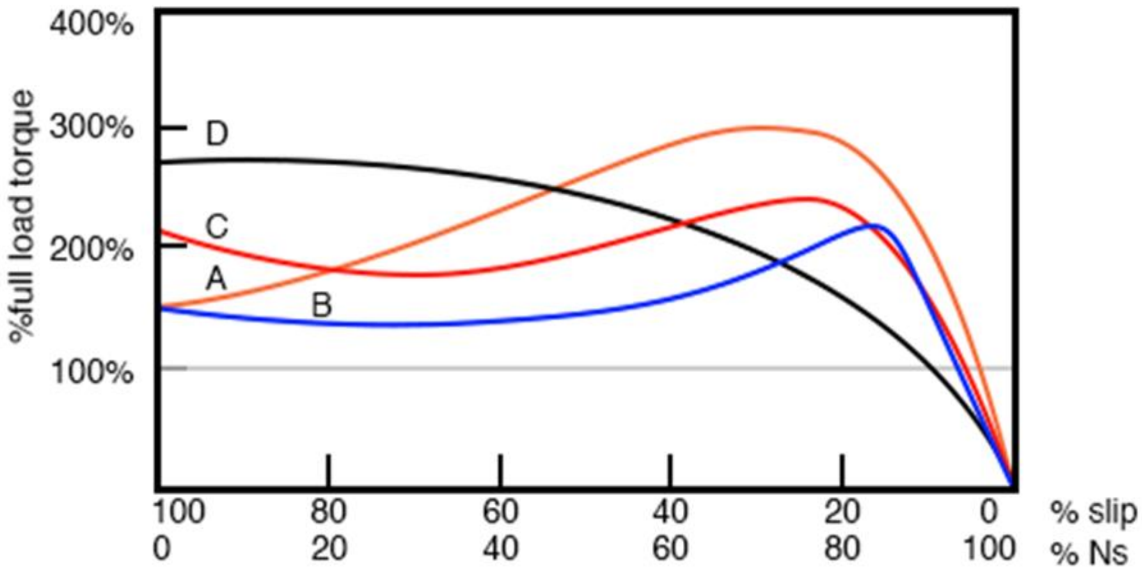

Figure 10 Typical Characteristics for A, B, C, D Motor Design Types

Notes about 3-phase Induction Motor Torque:

I. To select a motor for a given mechanical load, the torque speed characteristic of the motor must be compared to that of the mechanical load. Manufacturers of motors will provide the characteristics of a motor (starting torque to rated torque ratio, maximum torque to rated torque ratio).

II. The starting torque of the motor must be larger than the load requirement, or the motor will not be able to start rotating. As a result, it will draw its locked-rotor current. This may cause a damage for the motor.

III. The maximum torque developed by a motor indicates the capability of the machine to overcome high transient load torques. A motor with a maximum torque that is relatively low may stall when a sudden load torque exceeds the motor's maximum torque.

Solved Examples

Ex: 1, A 220-V, three-phase, two-pole, 50-Hz induction motor is running at a slip of 5 percent. Find:

(a) The speed of the magnetic fields in revolutions per minute
(b) The speed of the rotor in revolutions per minute
(c) The slip speed of the rotor
(d) The rotor frequency in hertz

SOLUTION

(a) The speed of the magnetic fields is

$$n_{sync} = \frac{120f}{P} = \frac{120(50\text{ Hz})}{2} = 3000\text{ r/min}$$

(b) The speed of the rotor is

$$n_m = (1-s)\ n_{sync} = (1-0.05)(3000\text{ r/min}) = 2850\text{ r/min}$$

(c) The slip speed of the rotor is

$$n_{slip} = sn_{sync} = (0.05)(3000\text{ r/min}) = 150\text{ r/min}$$

(d) The rotor frequency is

$$f_r = \frac{n_{slip}P}{120} = \frac{(150\text{ r/min})(2)}{120} = 2.5\text{ Hz}$$

Ex: 2, A three-phase, 60-Hz, four-pole induction motor runs at a no-load speed of 1790 r.p.m and a full-load speed of 1720 r.p.m. Calculate the slip and the frequency of the rotor circuit at no-load and full-load conditions?

Solution

$$s_{nl} = \frac{n_{sync} - n_{nl}}{n_{sync}} \times 100\% = \frac{1800-1790}{1800} \times 100\% = 0.56\%$$

$$f_{r,nl} = sf = (0.0056)(60\text{ Hz}) = 0.33\text{ Hz}$$

The slip and electrical frequency at full load conditions is

$$s_{fl} = \frac{n_{sync} - n_{nl}}{n_{sync}} \times 100\% = \frac{1800-1720}{1800} \times 100\% = 4.44\%$$

$$f_{r,fl} = sf = (0.0444)(60\text{ Hz}) = 2.67\text{ Hz}$$