ch 7

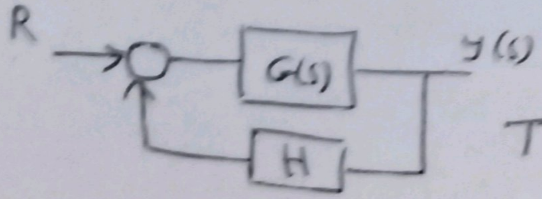
ص 175

P1

Lec - 03 -

Stability by Nyquist Criterion

Consider the following Control system



$$T(s) = \frac{Y(s)}{R(s)} = \frac{G(s)}{1 + G(s)H(s)}$$

$$T(j\omega) = \frac{G(j\omega)}{1 + G(j\omega)H(j\omega)}$$

* علشان يكون النظام مستقر يجب ان تكون كل Roots (Poles & Zeros) تقع بالجانب الايسر والتي تم ايجادها من قانون ch/Eq.

$$F(s) = 1 + GH = 0$$

* poles of T(s) [the overall T.F] equal to zeros of F(s) [ch/Eq.]

Ex: $H = 1$, $G(s) = \frac{1}{(s+2)(s+1)}$

sol $G(s)H(s) = \frac{1}{(s+2)(s+1)}$

$$T.F = \frac{\frac{1}{(s+2)(s+1)}}{1 + \frac{1}{(s+2)(s+1)}} = \frac{\frac{1}{\cancel{(s+2)(s+1)}}}{\frac{(s+2)(s+1)+1}{\cancel{(s+2)(s+1)}}}$$

$$F(s) = 1 + GH = 0$$

$$= 1 + \frac{1}{(s+1)(s+2)}$$

$$F(s) = \frac{\boxed{s^2 + 3s + 3}}{(s+1)(s+2)} \quad \text{zeros of } F(s)$$

$$T.F = \frac{1}{\underbrace{\begin{matrix}(s+2)(s+1)+1 \\ s^2 + 3s + 3\end{matrix}}_{\text{poles of T.F}}}$$

$\therefore$ poles of $T(s)$ = zeros of $F(s)$

لذلك نستخدم القانون التالي لمعرفة استقرار النظام بطريقة Nyquist

$Z = N + P \Longrightarrow Z$: No. of zeros of $\underline{\underline{F(s)}}$ at R.H.S

closed loop

Lec-03 -

Nyquist Criterion

P: No. of Poles of O.L. (GH) at R.H.S

N: No. of rotation of the plot about the point $-1+j0$ critical point
عدد الحلقات

N → +ve → clockwise باتجاه عقرب الساعة

N → -ve → counter clockwise عكس عقرب الساعة

For stability, ~~the system~~ the zeros Z must be zero.

* The Nyquist plot is obtained by [1] drawing the polar plot of $G(j\omega)H(j\omega)$ then [2] drawing its mirror image, ω varies from ∞ to $-\infty$ and [3] No. of N of the point $-1+j0$ is observed تلاحظ

* تنص طريقة Nyquist على ان النظام في حالة C.L.T.F يكون stable اذا كان عدد Zeros لـ ch/Eq والمساوية لعدد Poles $T(s)$ هو صفر.

Ex 1: For the Control system shown below, check weather the system is stable or not

$$G(s) = \frac{2}{(1+s)(1+0.5s)^2}$$

Sol: $G(j\omega) = \dfrac{2}{(1+j\omega)(1+0.5j\omega)^2}$

Lec - 03 -

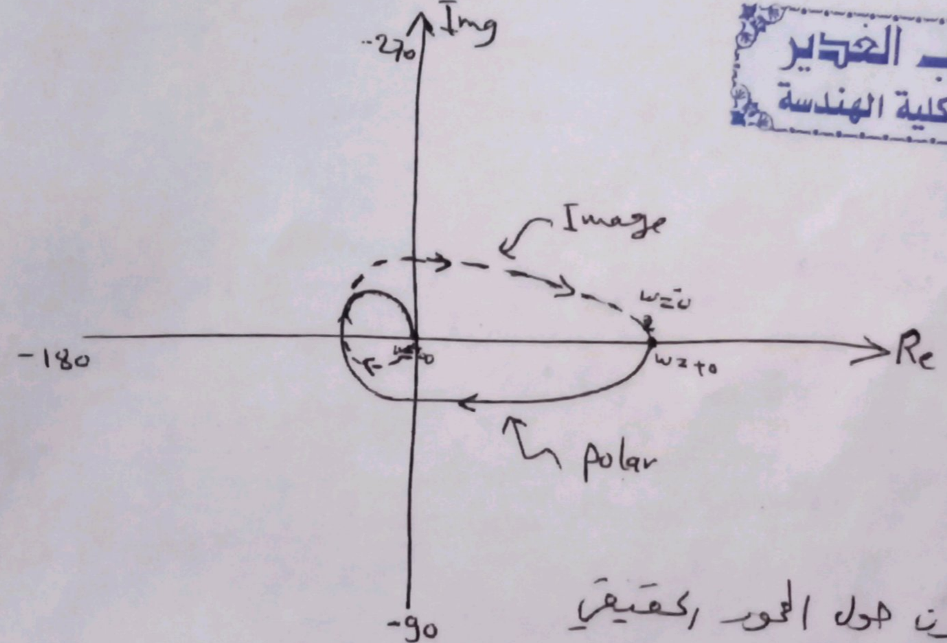
Nyquist Criterion

$$|G(j\omega)| = \frac{2}{\sqrt{1+\omega^2}\sqrt{1+0.25\omega^2}\sqrt{1+0.25\omega^2}}$$

$$\phi = -\tan^{-1}\omega - 2*\tan^{-1}0.5\omega$$

$$G(j0) = \frac{2}{(1+0)(1+0.5*0)^2} = 2\angle 0$$

$$G(j\infty) = \frac{2}{(1+\infty)(1+0.5*\infty)^2} = 0\angle -270$$



التناظر يكون حول المحور الحقيقي

لايجاد نقطة التقاطع مع المحور الحقيقي

$$G(j\omega) = \frac{2}{(1+j\omega)(1+0.5j\omega)^2} * \frac{(1-j\omega)(1-0.5j\omega)^2}{(1-j\omega)(1-0.5j\omega)^2}$$

$$= \frac{2\left[(1-j\omega)(1-j\omega-0.25\omega^2)\right]}{(1+\omega^2)(1+0.25\omega^2)^2}$$

$$= \frac{2\left[1-j\omega-0.25\omega^2-j\omega-\omega^2+0.25j\omega^3\right.}{(1+\omega^2)(1+0.25\omega^2)^2}$$

Lec-03

Nyquist method

$$Rel = X = \frac{2(1-1.25w^2)}{(1+w^2)(1+0.25w^2)^2}$$

$$Img. = y = \frac{2(0.25w^3-2w)}{(1+w^2)(1+0.25w^2)^2} = 0$$

$$0.25w^3 - 2w = 0 \Rightarrow w = 0$$
$$w^2 = 8 \Rightarrow \boxed{w = 2.83 \text{ rad/sec}}$$

sub in Rel

$\therefore Rel = -0.4436$

$Z = N + P$ $\quad P = 0$

$N = 0$

$Z = 0$ The system is stable

Ex2: (Problem 7.2) Comment on the stability of the system whose open loop T.F $G(s)H(s) = \frac{1}{s(1+2s)(1+s)}$

sol $G(jw) = \frac{1}{jw(1+2jw)(1+jw)}$, $|G(jw)| = \frac{1}{w\sqrt{1+4w^2}\sqrt{1+w^2}}$

$\phi = -90 - \tan^{-1}2w - \tan^{-1}w$

$$\left.\begin{aligned} G(j0) &= \frac{1}{0(1+2\times0)(1+0)} = \infty\angle{-90} \\ G(j\infty) &= \frac{1}{\infty(1+2\infty)(1+\infty)} = 0\angle{-270} \end{aligned}\right\} \text{Polar}$$

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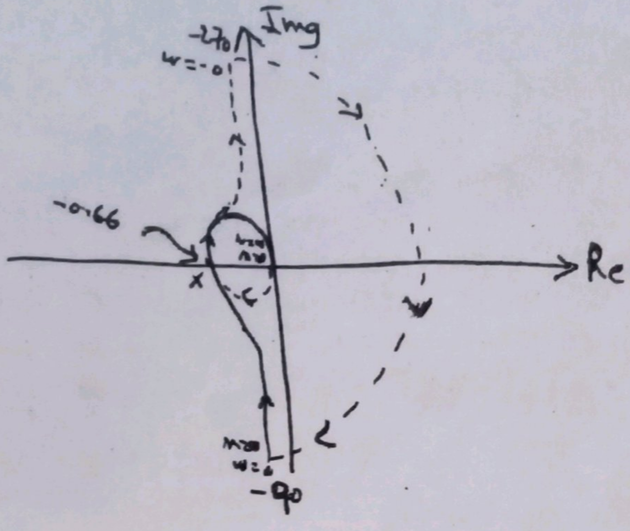
الرسم موجود هنا

ملاحظة يبدأ الرسم من نهاية Img وينتهي في بداية رسم polar ويكون مع عقرب الساعة



$$G(jw) = X + jy$$

$$= \frac{1}{jw(1+2jw)(1+jw)} * \frac{-jw(1-2jw)(1-jw)}{-jw(1-2jw)(1-jw)}$$

$$= \frac{-jw(1\overbrace{-jw-2jw}^{-3jw}-2w^2)}{w^2(1+4w^2)(1+w^2)}$$

$$= \frac{-jw-3w^2+2jw^3}{w^2(1+4w^2)(1+w^2)} = \frac{-3w^2}{w^2(1+4w^2)(1+w^2)} + j\frac{-w+2w^3}{w^2(1+4w^2)(1+w^2)}$$

$$y = 0 \Rightarrow -w+2w^3 = 0 \Rightarrow w(2w^2-1) = 0$$

$$w = 0$$

$$w^2 = \frac{1}{2}$$

$$w = \mp\sqrt{\frac{1}{2}}$$

$$w = 0.707 \text{ rad/sec}$$

sub in Rel

$$\therefore Rel = \frac{-3\cancel{w^2}}{\cancel{w^2}(1+4w^2)(1+w^2)} = \frac{-3}{(1+4*\frac{1}{2})(1+\frac{1}{2})} = \frac{-3}{(3)*1.5}$$

$$\boxed{X = -0.66}$$

* النقطة -1+j0 غير محاطة

$$Z = N + P$$

$\because P = 0 , N = 0 \quad \therefore Z = 0$ the sys. is stable

Ex3: (prob 7.3), the open loop T.F with $H(s)=1$ is

$$G(s)H(s)=\frac{s+2}{(s+1)(s-1)} \qquad \boxed{s=1}$$

note :- اذا كان احد poles او zero ~~قيمة~~ موجبة فان زاوية له تحسب بالقانون التالي

$$\text{poles} \Rightarrow -\left(180-\tan^{-1}\frac{Img}{Real}\right)$$

$$\text{Zeros} \Rightarrow +\left(180-\tan^{-1}\frac{Img}{Real}\right)$$

Comment on the stability

sol

$$G(j\omega)=\frac{(j\omega+2)}{(j\omega+1)(j\omega-1)} \qquad M=|G(j\omega)|=\frac{\sqrt{1+0.25\omega^2}}{\sqrt{1+\omega^2}\,\sqrt{(-1)^2+\omega^2}}$$

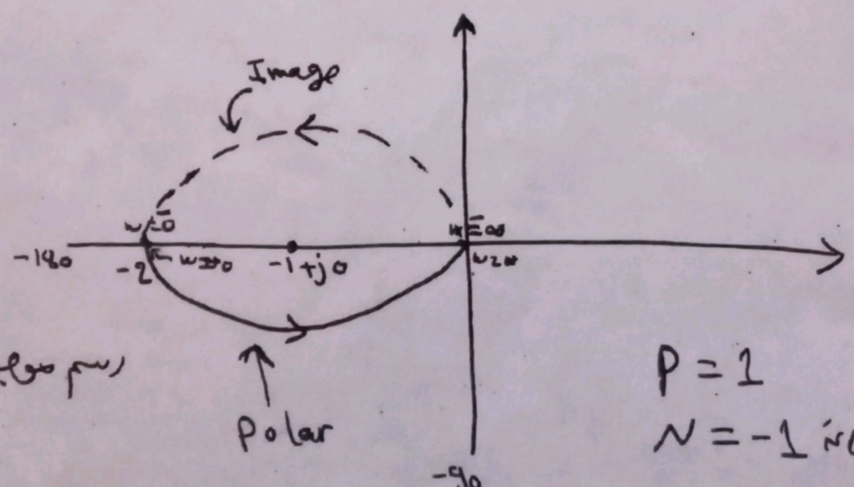
$$=\frac{2\,(0.5j\omega+1)}{(j\omega+1)(j\omega-1)}$$

$$\phi=\tan^{-1}0.5\omega-\tan^{-1}\omega-\left(180-\tan^{-1}\frac{\omega}{1}\right)$$

$$G(j0)=\frac{0+2}{(0+1)(0-1)}=-2\angle-180^\circ$$

$$G(j\infty)=\frac{(\infty+2)}{(\infty+1)(\infty-1)}=0\angle-90$$

} Polar



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رسم موجود ص

$P=1$

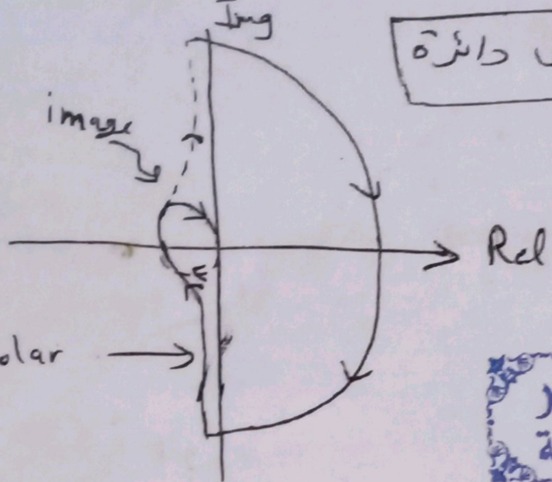
$N=-1$ عكس عقارب الساعة

$Z=P+N$

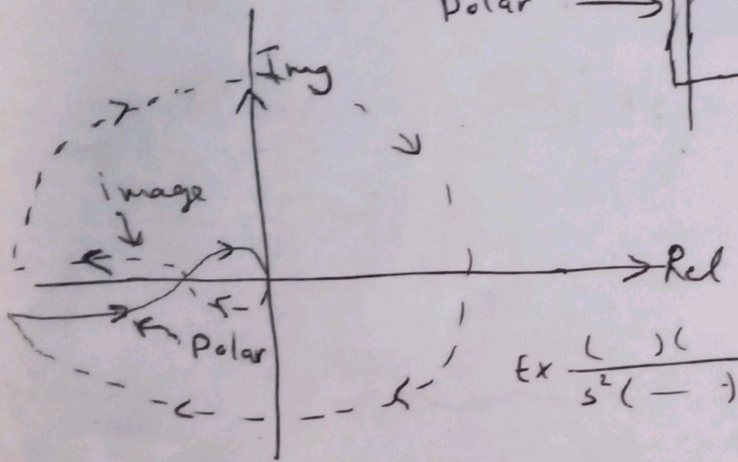
$=0$ the sys. is stable

ملاحظة: يكون رسم Nyquist على شكل انصاف دوائر او دائرة ونصف قطرها يساوي ∞ ويعتمد على عدد poles الواقعة في نقطة الاصل

Ex $\dfrac{(\quad\;)}{s(\quad\;)(-\,-\;)}$

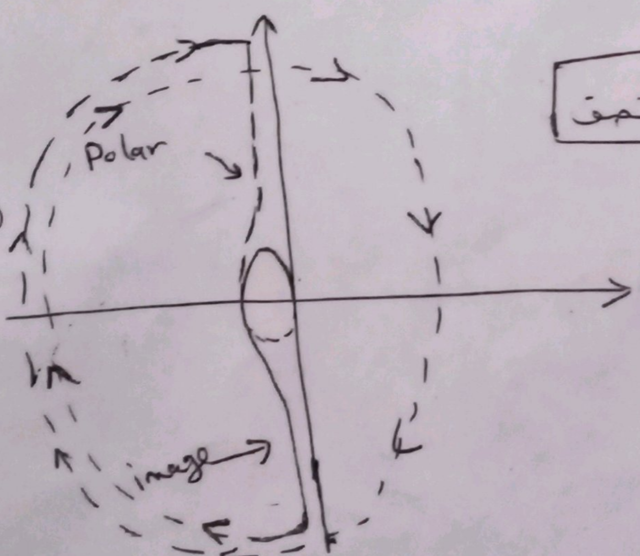


* s ⟵ نصف دائرة



* s^2 ⟵ دائرة

Ex $\dfrac{(\quad\;)(\quad\;)}{s^2(\,-\,)(-\,-\;)}$



* s^3 ⟵ دائرة ونصف

Ex $\dfrac{(\quad\;)(\;-\;)}{s^3(-\,-\;)}$

Ex 4: Comment on the stability for sys. with O.L.T.F

$$G(s)\,H(s) = \frac{k\,(s+3)}{s\,(s-1)}$$ (prob. 7.4 Page 182)

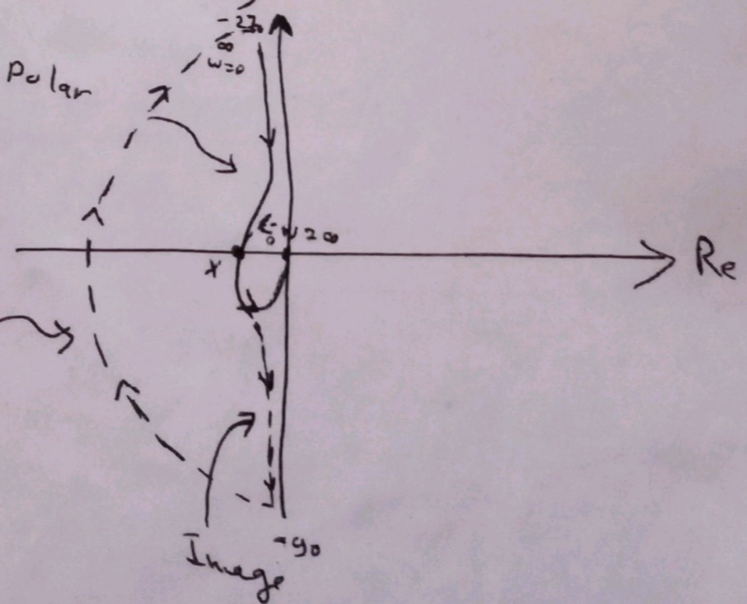
Sol $G(jw) = \dfrac{3k(0.33jw+1)}{jw(jw-1)}$

$$M = |G(jw)| = \frac{3k\sqrt{1+(0.33w)^2}}{w\sqrt{(-1)^2+w^2}}$$

$$\phi = \underbrace{+\tan^{-1} 0.33w}_{\text{Zero زاوية}} \underbrace{-90}_{\text{Poles at origin}} - \underbrace{(180-\tan^{-1} w)}_{\text{(jw-1) زاوية}}$$

$$\left.\begin{aligned} G(j0) &= \frac{3k(0.33\times 0+1)}{0(0-1)} = \infty \angle -270 \\ G(j\infty) &= \frac{3k(0.33\infty+1)}{\infty(\infty-1)} = 0 \angle -90 \end{aligned}\right\} \text{Polar}$$

$$\left.\begin{aligned} G(j\bar{0}) &= \infty \angle 270 \\ G(j\bar{\infty}) &= 0 \angle 90 \end{aligned}\right\} \text{Image}$$



لوجود s بالمقام لذلك
نرسم نصف دائرة
وباتجاه عقرب الساعة

To find x

$$G(jw) = X(w) + jY(w) = \frac{3k(0.33jw+1)}{jw(jw-1)} * \frac{-jw(-jw-1)}{-jw(-jw-1)}$$

$$= \frac{3k(0.33jw+1)*(-w^2+jw)}{w^2(w^2+1)}$$

$$X(w)+jY(w) = \frac{-3k*1.33w^2}{w^2(w^2+1)} + j\,\frac{3k(w-0.33w^3)}{w^2(w^2+1)}$$

$$Y(w)=0 \Rightarrow \frac{3k*w(1-0.33w^2)}{w^2(w^2+1)} = 0$$

$$3k(1-0.33w^2)=0$$

$$3k - kw^2 = 0 \Rightarrow w^2 = \frac{3k}{k}$$

$$w = \mp\sqrt{3}$$

Sub w in Rel part (X) $\qquad w=\sqrt{3}$ يوخذ فقط

$$X\Big|_{w=\sqrt{3}} = \frac{-3k*1.33\cancel{w^2}}{\cancel{w^2}(w^2+1)} = \frac{-3k*1.33}{(3+1)} = -k$$

when $k=1 \Rightarrow \boxed{X=-1}$ * النقطة $-1+0j$ تقع على نقطة التقاطع
The system is critical stable

when $k>1 \Rightarrow X<-1$ * النقطة $-1+0j$ تقع داخل loop

$Z = N+P$ $\qquad P=1$
$\qquad N=-1$ عدد الاحاطات عكس عقرب الساعة

$\therefore Z=-1+1=0$
the sys. is stable

when $k<1 \Rightarrow X>-1 \longrightarrow$ النقطة $-1+0j$ تقع داخل نصف الدائرة

$P=1 \qquad N=1 \qquad Z=P+N=2$ unstable