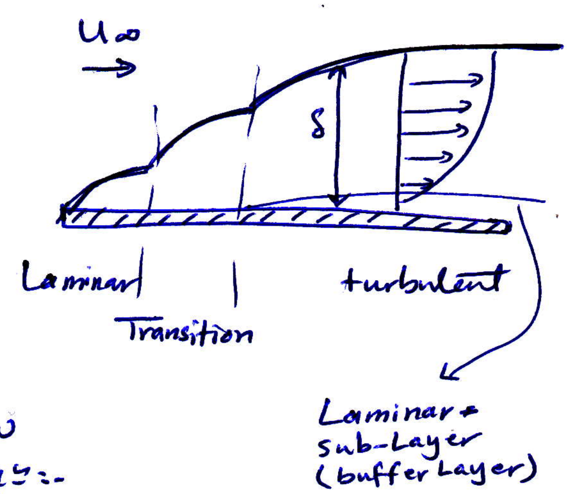
Turbulent Boundary Layer Heat Transfer

Thickness of B.L



The average profile in turbulent boundary layer can be described by one-Seventh law:

$$\left(\frac{u}{u_\infty}\right) = \left(\frac{y}{\delta}\right)^{1/7}$$

The thickness of B.L in turbulent flow can be obtained from the following eqn:

$$\boxed{\frac{\delta}{x} = \underbrace{0.381\, Re^{-0.2}}_{\text{Fully turbulent}} - 10256\, Re^{-1}} \quad [5*10^{5} < Re < 10^{7}]$$

Laminan then turbulent

Heat Transfer coefficient

The colburn analogy between heat & momentum transfer is

$$J_{f_x} = St_x\, Pr^{2/3}$$

The local friction factor (J_f) in turbulent flow on flat plate can be calculated from

$J_f = 0.0296\, Re^{-0.2}$	for $5*10^{5} < Re < 10^{7}$
$J_f = 0.185\, (\log Re_x)^{-2.584}$	for $10^{7} < Re < 10^{9}$

The average friction coefficient for a flat plate with a Laminar B.L. up to $5*10^{5}$ and turbulent thereafter can be calculated from

$$\overline{J_f} = \frac{0.455}{(\log Re_L)^{2.584}} - \frac{850}{Re_L} \qquad \text{for } 5*10^{5} < Re < 10^{9}$$

or $$\overline{J_f} = \frac{0.037}{Re^{0.2}} - \frac{850}{Re_L} \qquad \text{for } 5*10^{5} < Re < 10^{7}$$

CE

Applying the Colburn analogy, we obtain the local turbulent heat transfer as:

$$St \cdot Pr^{2/3} = 0.0296\, Re^{-0.2} \qquad \text{for } 5*10^{5} < Re < 10^{7}$$

$$St \cdot Pr^{2/3} = 0.185\, (\log Re_x)^{-2.584} \qquad \text{for } 10^{7} < Re_x < 10^{9}$$

The average heat transfer coefficient over the entire Laminar turbulent B.L is:-

$$\bar{J}_f = \bar{St} \cdot Pr^{2/3}$$

$$\bar{St} \cdot Pr^{2/3} = 0.037\, Re^{-0.2} - 850\, Re^{-1} \qquad \text{for } 5*10^{5} < Re < 10^{7}$$

$$St = \frac{Nu}{Re \cdot Pr}$$

$$\bar{Nu} = \frac{\bar{h} L}{K} = Pr^{1/3} \left[0.037\, Re^{0.8} - 850 \right]$$

Constant Heat flux

For constant heat flux in turbulent flow region, the local value of Nusselts no. is 4% higher than that for constant wall temp..

$$Nu_x \Big|_{\text{constant } q_w} = 1.04\, Nu_x \Big|_{\text{constant } T_w}$$

(Turbulent flow only)

Summary for Equations for Turbulent Flow Over Flat Plate

(A) Constant Wall temp.

Equations	~~Restriction~~ Restriction
① $St \cdot Pr^{2/3} = 0.0296\, Re_x^{-0.2}$	$5 \times 10^5 < Re_x < 10^7$
② $St \cdot Pr^{2/3} = 0.185\, (\log Re_x)^{-2.584}$	$10^7 < Re_x < 10^9$
③ $\overline{St}\, Pr^{2/3} = 0.037\, Re_L^{-0.2} - 850\, Re^{-1}$	$5 \times 10^5 < Re_x < 10^7$
or $Nu_L = Pr^{1/3} \left[0.037\, Re_L^{0.8} - 850 \right]$	
$Nu_L = Pr^{1/3} \left[0.2275\, Re_L (\log Re_L)^{-2.584} - 850 \right.$	$5 \times 10^5 < Re_L < 10^9$

(B) Constant Heat Flux

$$Nu_x \Big|_{cons.\, q_w} = 1.04\, Nu_x \Big|_{const.\, T_w}$$

Example: Air at $20^{\circ}C$ and 100 Kpa flows over a flat plate at $35\frac{m}{s}$. The plate is 0.75 m long and is maintained at $60^{\circ}C$. Assuming a unit depth in Z-direction, Calculate the heat transfer from the plate?

Sol^n

$$T_f = \frac{20+60}{2} = 40^{\circ}C = 313\,K$$

$$\rho = \frac{PM_w}{RT} = \frac{100\,(28.9)}{8.314\,(313)} = 1.11\ Kg/m^3$$

$$Pr = 0.73\ ,\ K = 0.02723\ \frac{W}{mK}\ ,\ Cp = 1.007\ \frac{KJ}{Kg.K}$$

$$\mu = 1.906 * 10^{-5}\ Kg/m^3$$

$$Re_L = \frac{\rho U_\infty L}{\mu} = \frac{(1.11)(35)(0.75)}{1.906 * 10^{-5}} = 1.53 * 10^{6} < 10^{7}$$

$$\overline{Nu} = Pr^{1/3}\left[0.037\,Re_L^{0.8} - 850\right]$$

$$\Rightarrow \bar{h} = \frac{0.02723}{0.75}\left[0.73^{1/3}\left(0.037(1.53 * 10^{6})^{0.8} - 850\right)\right]$$

$$= 78.5\ W/m^2K$$

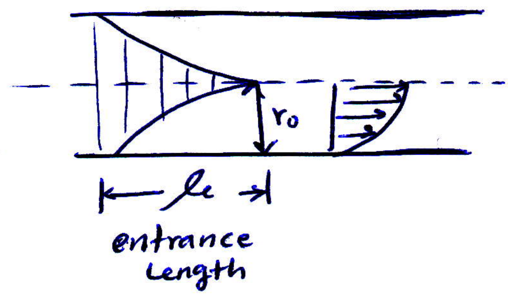
$$q = \bar{h}\,A\,(T_w - T_\infty)$$

$$= 78.5 * (0.75 * 1)(60-20)$$

$$= 2355\ Watt$$

Laminar Flow in Tubes :

For the Laminar flow in tubes,
The parbolic velocity distribution
exists as shown in the figure.



$$\frac{u}{u_o} = 1 - \frac{r^2}{r_o^2}$$

where u_o is the center line velocity.
r_o is the radius of the tube.

The temp. distribution in laminar tube flow may be written in terms of the temp. in the center of the tube (T_c)

$$T - T_c = \frac{1}{\alpha} \frac{\partial T}{\partial x} \frac{u_o r_o^2}{4} \left[\left(\frac{r}{r_o}\right)^2 - \frac{1}{4}\left(\frac{r}{r_o}\right)^4 \right]$$

The Bulk Temp. :

The convection heat transfer in tube flow is usually calculated from

$$q = hA(T_w - T_b)$$

where T_w is the wall temp. & T_b is called "the mean bulk temp.".

The bulk temp. is defined as the energy average fluid temp.

$$T_b = \frac{\int_0^{r_o} \rho \, 2\pi r dr \, u \, C_p T}{\int_0^{r_o} \rho \, 2\pi r dr \, u \, C_p}$$

After integration, we obtain

$$T_b = T_c + \frac{7}{97} \frac{u_o r_o^2}{\alpha} \frac{\partial T}{\partial x}$$

The heat transfer coefficient is calculated from

$$q = hA(T_b - T_w) = -KA \left.\frac{\partial T}{\partial x}\right|_{r=r_o}$$

$$\therefore \; h = \frac{k\,(dT/dx)|_{r=r_o}}{(T_w - T_b)}$$

~~For~~ The radial temp. gradient is given by:

$$\frac{dT}{dr} = \frac{1}{\alpha}\,\frac{dT}{dx}\,\frac{u_o r_o^2}{4}\left[\frac{2r}{r_o^2} - \frac{r^3}{r_o^4}\right]$$

$$\left.\frac{dT}{dr}\right|_{r=r_o} = \frac{u_o}{\alpha}\,\frac{dT}{dr}\left.\left(\frac{r}{2} - \frac{r^3}{4r_o^2}\right)\right|_{r=r_o} = \frac{u_o r_o}{4\alpha}\,\frac{dT}{dx}$$

substitute to find h

$$h = \frac{k\,u_o r_o\,(dT/dx)}{4\alpha\,(T_w - T_\infty)} = \frac{k\,u_o r_o\,(dT/dx)}{4\alpha\,\frac{r_o^2 u_o}{\alpha}\,\frac{dT}{dr}\left(\frac{3}{16} - \frac{7}{97}\right)}$$

$$= \frac{k}{4 r_o \left(\frac{11}{96}\right)} \Rightarrow h = \frac{24}{11}\,\frac{k}{r_o} \Rightarrow Nu_d = \frac{48}{11}$$

$\therefore \; Nu_d = 4.364$ (Laminar Flow in Tubes at Constant heat flux)