4	*Networks*
	Theorem

4.1 Star – Delta and Delta –Star Conversions

These methods of conversions may be used for solving complicated networks having number of branches.

i. **Star(Y or T) Connection Diagram**

In this method of connection the three ends of electrical element (resistor) are joined together at point N as shown in figure (4-1)

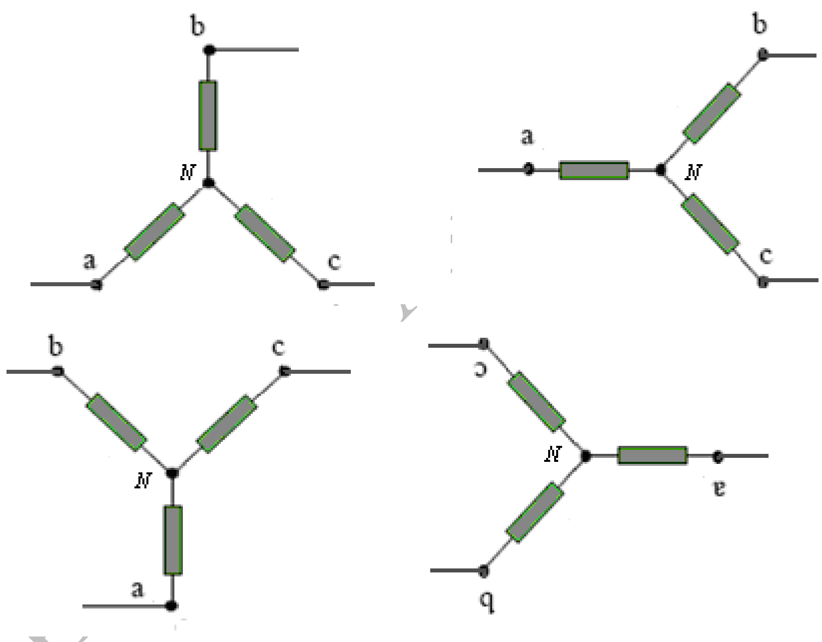


Figure (4-1) the Connection Diagram of Star.

ii. **Delta (Δ or π) Connection**

In this type of connection the electrical element (resistor) are joined as shown in figure (4-2)

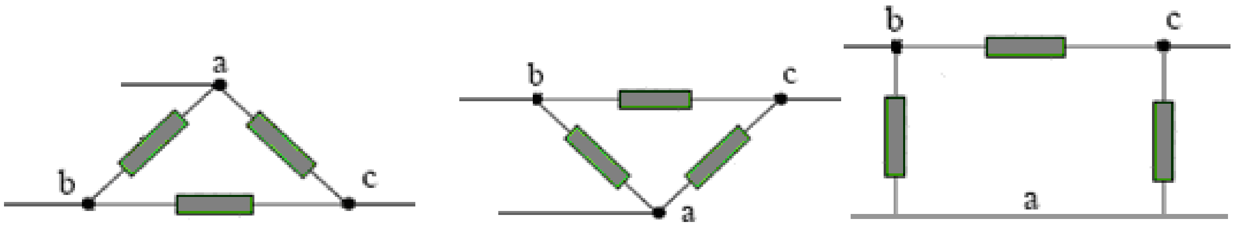


Figure (4-2) the Schematic Diagram of Delta Connections.

4.2 Star to Delta Conversion

If we have three resistances (R1, R2and R3) connected in star shape as in figure (4-3), Then the conversion to delta shape can be easily done by using the following equations:-

$$R_A = \frac{R_1R_2 + R_1R_3 + R_2R_3}{R_3}$$

$$R_B = \frac{R_1R_3 + R_1R_2 + R_3R_2}{R_2}$$

$$R_C = \frac{R_2R_3 + R_2R_1 + R_3R_1}{R_1}$$

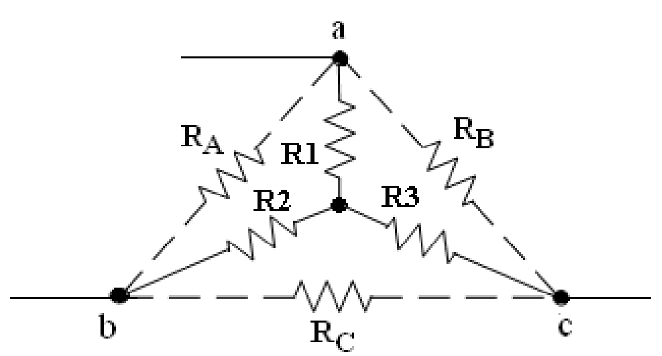


Figure (4-3) Star to Delta Conversion.

And the resistance R_A, R_B and R_C are connected in delta as in figure (4-4)

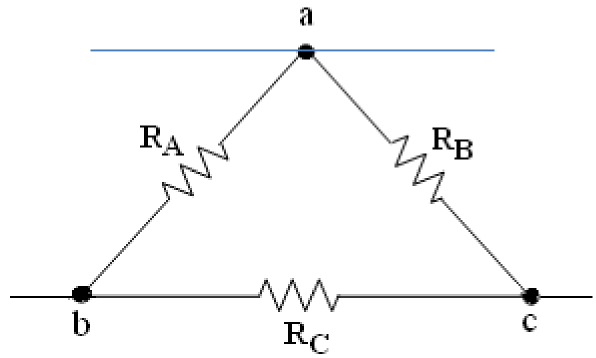


Figure (4-4) the Delta Shape after Conversion.

4.3 Delta – Star Conversions

If we have three resistances (R_A, R_B and R_C) connected in delta shape as in figure (4-5), Then the conversion to star shape can be easily done by using the following equations:-

$$R_1 = \frac{R_A R_B}{R_A + R_B + R_C}$$

$$R_2 = \frac{R_A R_C}{R_A + R_B + R_C}$$

$$R_3 = \frac{R_C R_B}{R_A + R_B + R_C}$$

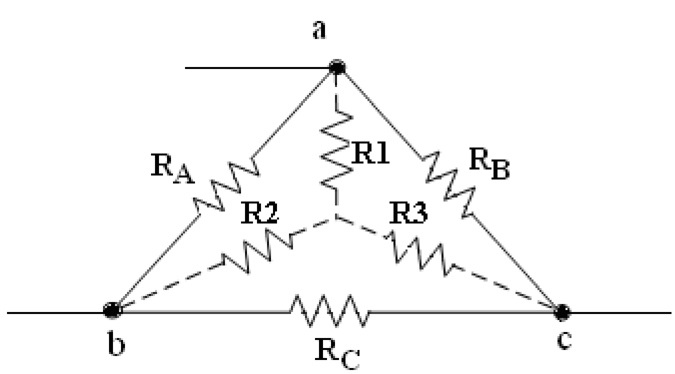


Figure (4-5) the Delta to Star Conversion.

And the resistances R1, R2and R3 are connected in star as in figure (4- 6)

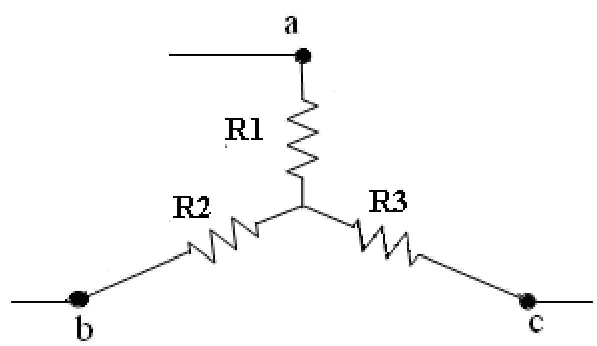
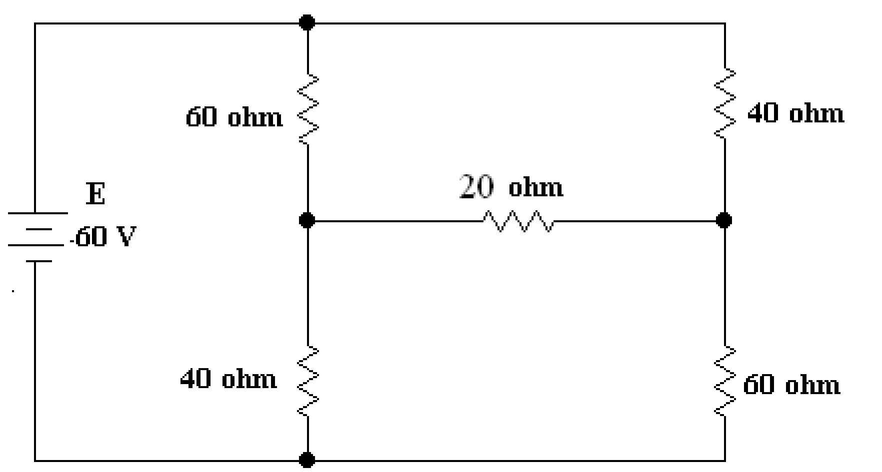


Figure (4-6) the Star- Shape after Conversion.

Example 4.1:-For the network shown in the figure find the total resistance.



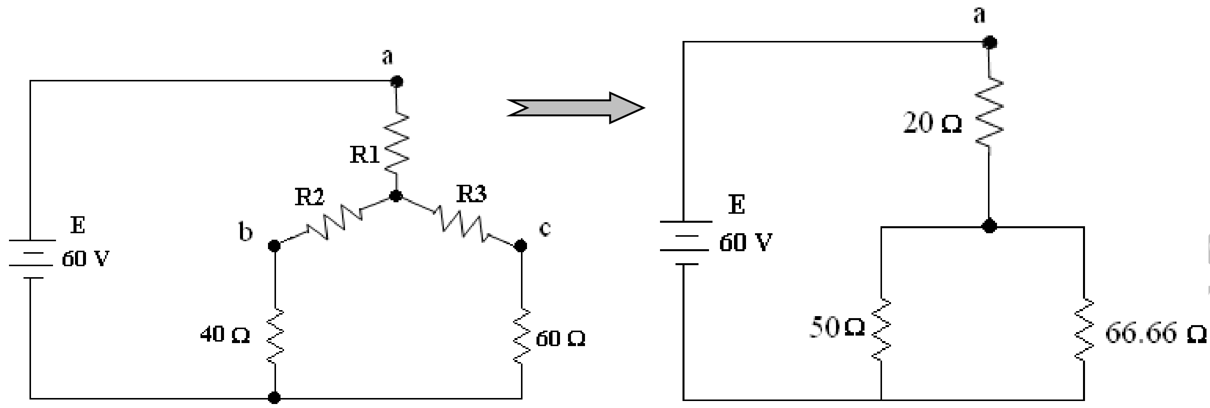
Sol: - using delta to star conversion, then

$$R_1 = \frac{R_A R_B}{R_A + R_B + R_C} = \frac{60 \times 40}{60 + 40 + 20} = 20\Omega$$

$$R_2 = \frac{R_A R_C}{R_A + R_B + R_C} = \frac{60 \times 20}{60 + 40 + 20} = 10\Omega$$

$$R_3 = \frac{R_C R_B}{R_A + R_B + R_C} = \frac{20 \times 40}{60 + 40 + 20} = 6.66\Omega$$

And the circuit is redrawn as in figures below



$$R_T = \left[\frac{66.66 \times 50}{66.66 + 50}\right] + 20 = 28.57 + 20 = 48.57\Omega$$

4.4The Superposition Theorem

State that: - In **any linear circuit containing multiple independent sources, the current through an element in this circuit (or voltage across) is the algebraic sum of the currents through (or voltages across) that element due to each independent source acting alone.**

Therefore if we have an electric circuit contains of (n – number) of sources then to calculate I_1 (for example) using superposition theorem, it will be

$$I_1 = (\pm I_{11}) + (\pm I_{12}) + (\pm I_{13}) + --- + (\pm I_{1n})$$

Where n = number of sources

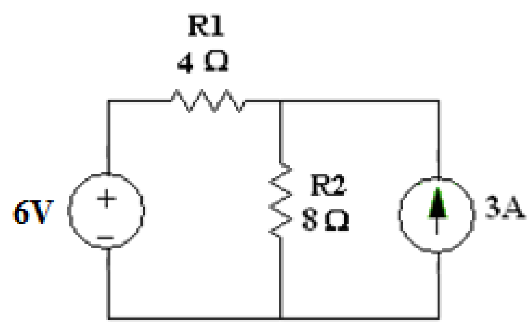
Steps of Applying Superposition Principle:-

1- Deactivate all independent sources except one source. If the internal resistance of the sources is neglected then the voltage source is replaced with a "Short Circuit" (R= 0 Ω) and the current source is replaced with an open circuit (R = ∞) respectively then find the corresponding (V or I) due to that active source.

2- Repeat step (1) for each of the other independent sources.

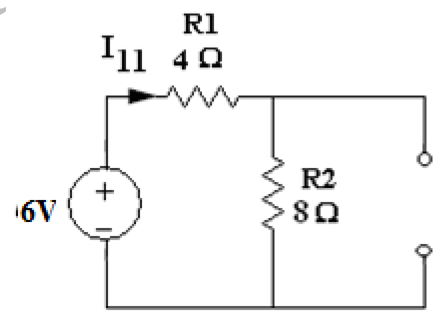
3- Find the total contribution (V or I) by adding algebraically all the currents or voltages due to the independent sources.

Example 4.2:- In the network shown in the figure, find the magnitude and direction of the current I_1 using superposition theorem.



Sol: - using superposition theorem

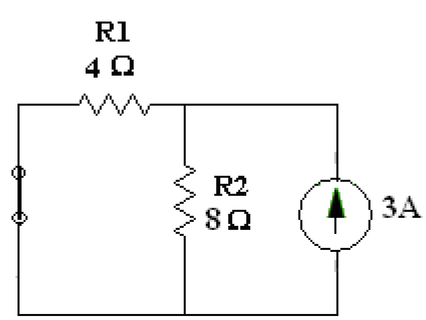
1- The voltage source (6V) is activated (ON) and the current source is OFF as shown in the figure



Hence:-

$$I_{11} = \frac{V}{R_1 + R_2} = \frac{6}{12} = 0.5A \rightarrow$$

2- The current source(3A) is activated (ON) and the voltage source is OFF as shown in the figure



Hence:-

$$I_{12} = 3 \times \frac{R_2}{R_1 + R_2} = 3 \times \frac{8}{12} = 2A \leftarrow$$

Therefore from steps (1 and 2) respectively the total I_1 is

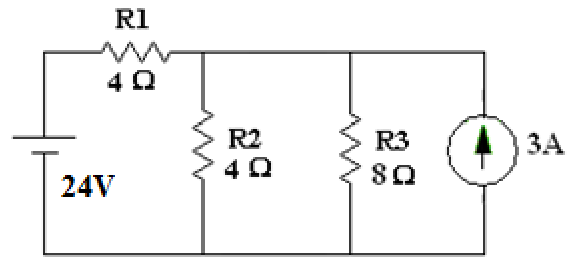
$$I_1 = I_{11} - I_{12} = 0.5 - 2 = 1.5A \leftarrow$$

H.W:-

In the circuit shown in the figure, using superposition theorem, find the following:-

1- The magnitude and the direction of the current through R_3.

2- The voltage across R_1.



4.5 Thevenin's Theorem

Thevenin's Theorem states that: - **it is possible to simplify a complex linear circuit to an equivalent circuit consists of single voltage source and series resistance connected to a load.**

The Thevenin's equivalent circuit is shown in figure (4-7),

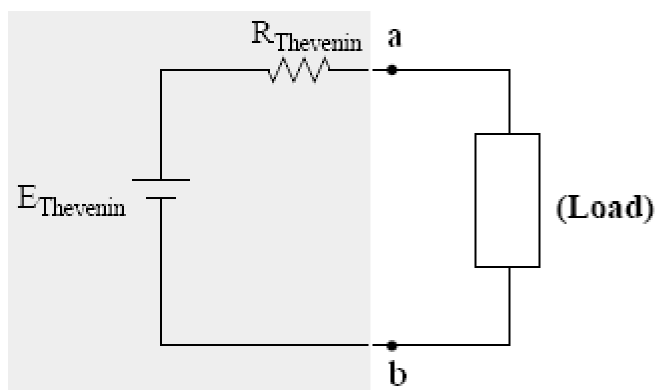


Figure (4-7) the Thevenin's equivalent circuit.

Steps of Applying Thevenin's Theorem:-

1- Remove the load from the original circuit.

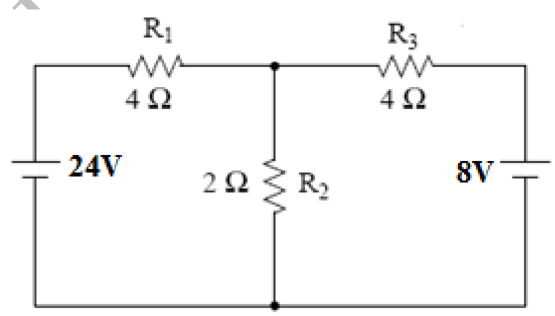
2- Mark the remaining two terminals (a, b).

3- Find $R_{Thevenin}$ or (R_{Th}) with replacing all sources in the circuit with the corresponding equivalent circuit, then find the resultant resistance.

4- Find $E_{Thevenin}$ or (E_{Th}) by first returning all the sources to the original position, then find the open circuit voltage between the two terminals a, b.

5- Draw Thevenin's equivalent circuit as shown in figure (4-7).

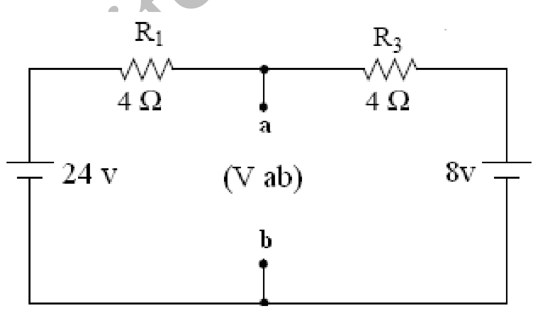
Example 4.3:- In the network shown in the figure find the magnitude and direction of the current passing through R_2 using Thevenin's theorem.



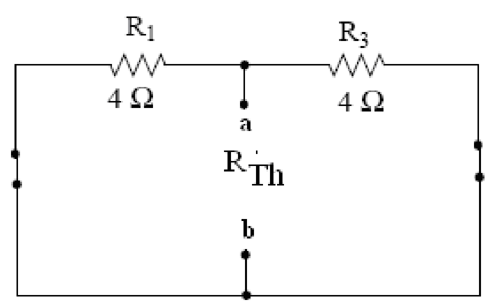
Sol:-

1- Removing R_2 from the circuit

2- Mark the two terminals (a,b)



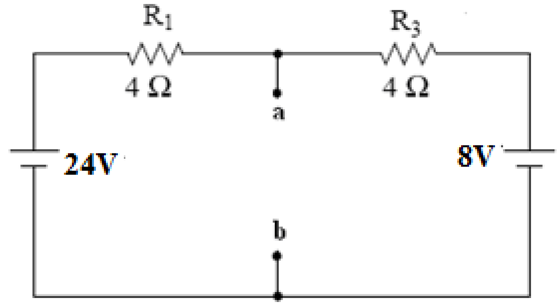
3- To find R_{Th}



$$\frac{1}{R_{Th}} = \frac{1}{R_1} + \frac{1}{R_3} = \frac{1}{4} + \frac{1}{4} = \frac{1}{2}$$

$$\therefore R_{Th} = 2\Omega$$

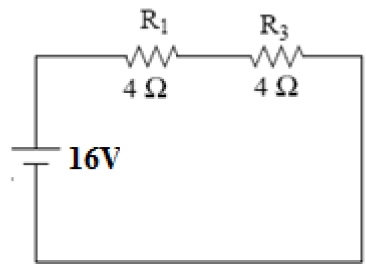
4- To find ($V_{ab} = E_{Th}$)



$$I = \frac{E_T}{R_1 + R_3} = \frac{16}{8} = 2A$$

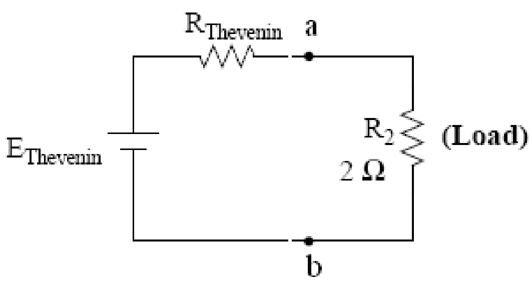
$V_3 = I_3 \times R_3 = 2\times4 = 8V$

And $V_{ab} = V_3 + 8 = 8 + 8 = 16V$



5-From Thevenin's equivalent circuit

$$I_2 = \frac{E_{Th}}{R_{Th} + R_2} = \frac{16}{2+2} = 4A$$



4.6 <u>Norton's Theorem</u>

State that: - ***Any two-terminal linear bilateral dc network can be replaced by an equivalent circuit consisting of a current source and a parallel resistor, as shown in Figure (4-8)***

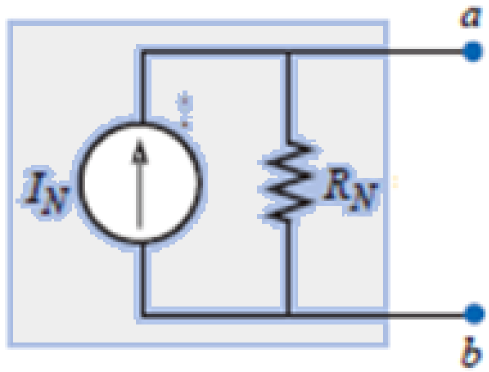


Figure (4-8) Norton's Equivalent Circuit

<u>Steps of Applying Norton's Theorem:-</u>

1. Remove that portion of the network across which the Norton equivalent circuit is found.

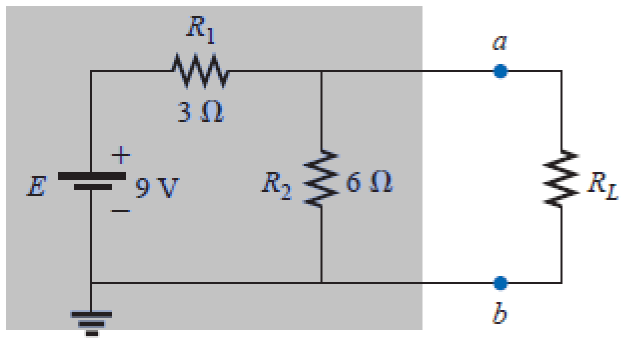
2. Mark the terminals of the remaining two-terminal network.

3. Calculate R_N by first setting all sources to zero (voltage sources are replaced with short circuits and current sources with open circuits) and then finding the resultant resistance between the two marked terminals. (If the internal resistance of the voltage and/or current sources is included in the original network, it must remain when the sources are set to zero.)

4. Calculate IN by first returning all sources to their original position and then finding the short-circuit current between the marked terminals. It is the same current that would be measured by an ammeter placed between the marked terminals.

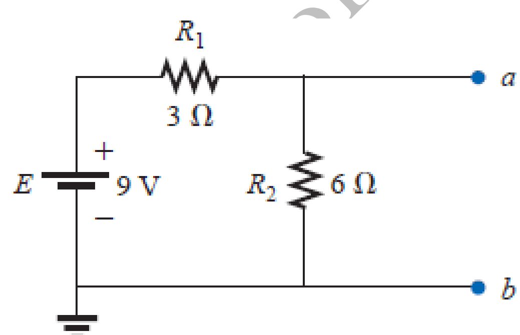
5. Draw the Norton equivalent circuit with the portion of the circuit previously removed replaced between the terminals of the equivalent circuit.

Example 4.4:- Find the Norton equivalent circuit for the network in the shaded area of the figure

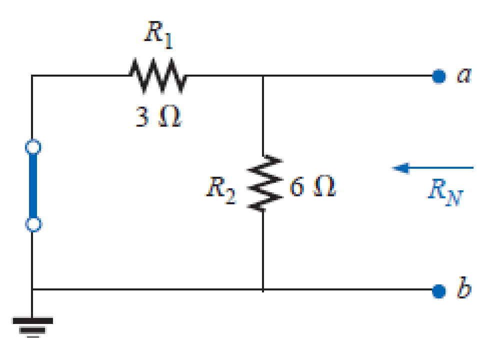


Solution:-

Steps 1 and 2

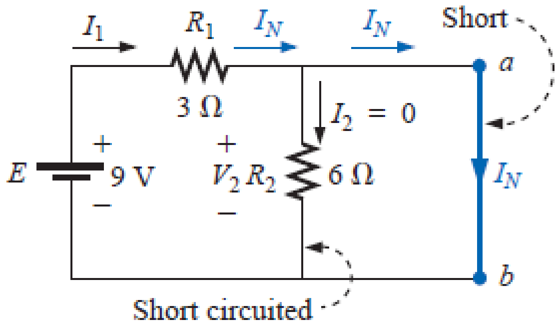


Step 3



$$R_N = R_1 \parallel R_2 = 3\ \Omega \parallel 6\ \Omega = \frac{(3\ \Omega)(6\ \Omega)}{3\ \Omega + 6\ \Omega} = \frac{18\ \Omega}{9} = \mathbf{2\ \Omega}$$

Step 4



$$V_2 = I_2R_2 = (0)6\ \Omega = 0\ \text{V}$$

$$I_N = \frac{E}{R_1} = \frac{9\ \text{V}}{3\ \Omega} = \mathbf{3\ A}$$

Step 5