

Partial Derivatives :

1- First order Partial derivatives :

If $f(x, y)$ is a function of two variables, then $\frac{\partial}{\partial x} f(x, y)$ is defined as the derivative of the function $g(x) = f(x, y)$, where y is considered a constant. It is called Partial derivative of f with respect to x . The partial derivative with respect to y is defined similarly.

The first partial derivatives of the function $f(x, y)$ with respect to the variables x and y is given by :

$$\frac{\partial f}{\partial x} = \lim_{h \rightarrow 0} \frac{f(x+h, y) - f(x, y)}{h}$$

$$\frac{\partial f}{\partial y} = \lim_{h \rightarrow 0} \frac{f(x, y+h) - f(x, y)}{h}$$

Notation: From now on we will employ the following shorter notation for the partial derivatives of $f(x, y)$

$$\frac{\partial f}{\partial x} = f_x$$

$$\frac{\partial f}{\partial y} = f_y$$

We will denote by $f_x(x_0, y_0)$ and $f_y(x_0, y_0)$ the partial derivatives at the point (x_0, y_0) .

Example 1: Find the value of $\frac{\partial f}{\partial x}$ and

$\frac{\partial f}{\partial y}$ at the point (4, -5) if

$$f(x, y) = x^2 + 3xy + y - 1$$

solution:

$$\frac{\partial f}{\partial x} = \frac{\partial}{\partial x} [x^2 + 3xy + y - 1]$$

$$= 2x + 3y + 0 - 0 - 0 + 0 = 2x + 3y$$

$$\left. \frac{\partial f}{\partial x} \right|_{(4, -5)} = 2(4) + 3(-5) = -7$$

$$\frac{\partial f}{\partial y} = \frac{\partial}{\partial y} [x^2 + 3xy + y - 1]$$

$$= 0 + 3x + 1 - 0 = 3x + 1$$

$$\left. \frac{\partial f}{\partial y} \right|_{(4, -5)} = 3(4) + 1 = 13$$

Example 2: Find f_y if $f(x,y) = y \sin(xy)$

Solution:

$$f_y = \frac{\partial}{\partial y} (y \sin(xy))$$

$$= y \times \cos(xy) + \sin(xy) \quad (1)$$

$$= xy \cos(xy) + \sin(xy)$$

Example 3: Find f_x and f_y if

$$f(x,y) = \frac{2y}{y + \cos x}$$

$$f_x = \frac{\partial}{\partial x} \left(\frac{2y}{y + \cos x} \right) = \frac{(y + \cos x)(0) - 2y(-\sin x)}{(y + \cos x)^2}$$

$$f_x = \frac{-2y \sin x}{(y + \cos x)^2}$$

$$f_y = \frac{\partial}{\partial y} \left(\frac{2y}{y + \cos x} \right) = \frac{(y + \cos x)(2) - 2y(1)}{(y + \cos x)^2}$$