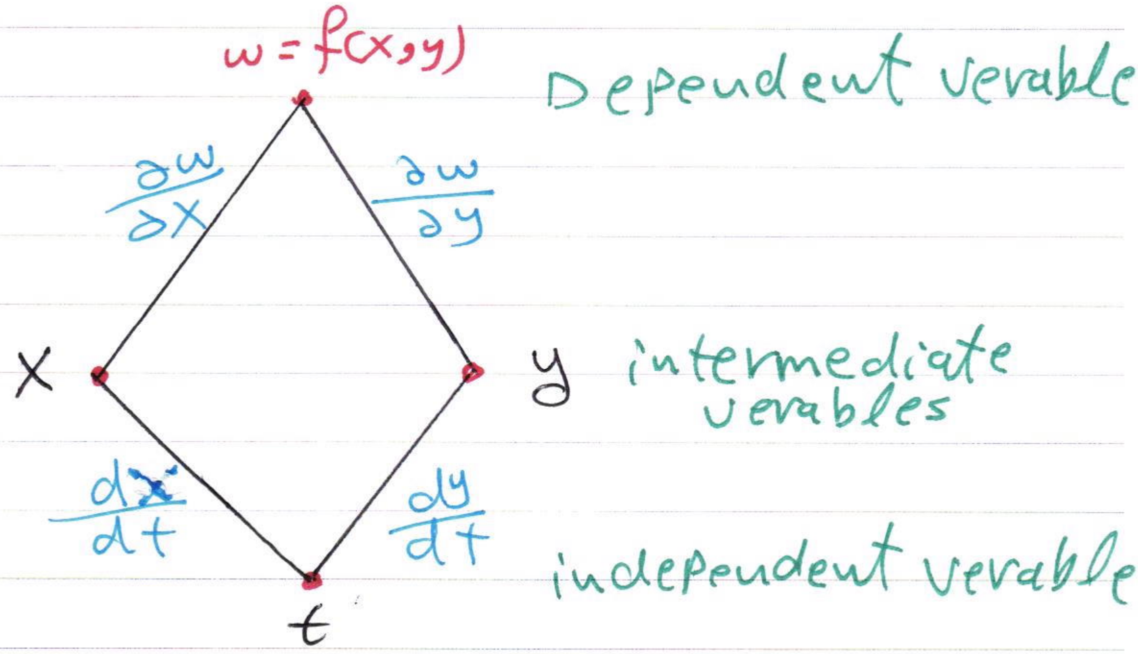
3- Chain Rule:

if $w = f(x, y)$ has continous partial derivatives f_x and f_y and if $x = x(t)$, $y = y(t)$ are the differentiable functions of t, then the composite $w = f(x(t), y(t))$ is a differentiable function of t and

$$\frac{dw}{dt} = \frac{\partial w}{\partial x}\frac{dx}{dt} + \frac{\partial w}{\partial y}\cdot\frac{dy}{dt}$$



$$\frac{dw}{dt} = \frac{\partial w}{\partial x}\frac{dx}{dt} + \frac{\partial w}{\partial y}\cdot\frac{dy}{dt}$$

Example 1: Use the Chain Rule to find the derivative of $w = xy$ with respect to t along the path $x = \cos(t)$ & $y = \sin(t)$ what is the value of the drevative at $t = \frac{\pi}{2}$?

Solution:

$$\frac{dw}{dt} = \frac{\partial w}{\partial x} \cdot \frac{dx}{dt} + \frac{\partial w}{\partial y} \cdot \frac{dy}{dt}$$

$$\frac{\partial w}{\partial x} = y$$

$$\frac{dx}{dt} = -\sin(t)$$

$$\frac{\partial w}{\partial y} = x$$

$$\frac{dy}{dt} = \cos(t)$$

$$\frac{dw}{dt} = \sin(t) \cdot (-\sin(t)) + \cos(t) \cdot \cos(t)$$

$$\frac{dw}{dt} = -\sin^2(t) + \cos^2(t)$$

$$\frac{dw}{dt} = \cos(2t)$$

We can check the result with a more direct calculation as a function of t

$$w = xy = \cos(t) \cdot \sin(t) = \frac{1}{2}\sin(2t)$$

$$\frac{dw}{dt} = \frac{1}{\cancel{2}}\cancel{2}\cos(2t) = \cos(2t)$$

at a given value of $t = \frac{\pi}{2}$

$$\frac{dw}{dt} = \cos(\cancel{2}\frac{\pi}{\cancel{2}}) = -1$$

For two independent verables :

$$\frac{\partial w}{\partial u} = \frac{\partial w}{\partial x}\frac{\partial x}{\partial u} + \frac{\partial w}{\partial y}\cdot\frac{\partial y}{\partial u}$$

$$\frac{\partial w}{\partial v} = \frac{\partial w}{\partial x}\frac{\partial x}{\partial v} + \frac{\partial w}{\partial y}\frac{\partial y}{\partial v}$$