4. Extreme Values and Saddle Points:

Difinition: Local Maximum/Minimum

Let $f(x,y)$ be defined on a region R contaning the point (a,b). Then:

1- $f(a,b)$ is a local maximum value of f if $f(a,b) \geq f(x,y)$ for all domain points (x,y) in an open disk centered at (a,b).

2- $f(a,b)$ is a local minimum of f if $f(a,b) \leq f(x,y)$ for all domain points (x,y) in an open disk centered (a,b).

Difinition: Critical point

An interior point of the domain of a function $f(x,y)$ where both f_x and f_y are zero or both of f_x and f_y do not exist is a critical point of f.

Note

Not all critical points gives rise to local minima/maxima.

Difinition: Saddle point

A differentiable function $f(x,y)$ has a saddle point at a critical point (a,b) if in every open disk centered at (a,b) there are domain points (x,y) where $f(x,y) > f(a,b)$ and domain points where $f(x,y) < f(a,b)$. The corresponding point $(a,b,f(a,b))$ on surface $z = f(x,y)$ is called a saddle point of the surface.

Theorem 1: First Derivative Test for Local Extreme Values

If $f(x,y)$ has a local maximum or minimum value at an interior point (a,b) of its domain and if the first partial derivatives exist there, then

$f_x(a,b) = 0$ and $f_y(x,b) = 0$.

Theorem 2: Second Derivative Test for Local Extremum Values

Suppose that $f(x,y)$ and its first and second partial derivatives are continous throughout a disk centered at (a,b) and that $f_x(a,b) = 0$ and $f_y(a,b) = 0$. Then,

(i) f has a local maximum at (a,b) if

$f_{xx} < 0$ and $f_{xx} f_{yy} - f_{xy}^2 > 0$ at (a,b).