Introduction:

If a product is to meet or exceed customer expectations, generally it should be produced by a process that is stable or repeatable. More precisely, the process must be capable of operating with little variability around the target or nominal dimensions of the product's quality characteristics.

Statistical process control (SPC) is a major statistical tool for monitoring a production process to make sure that it works stably. The stability of the production process is reflected by the conformance of the quality characteristics of its products to their designed requirements. This statistically based process information can provide a greater understanding of the process by providing a graphical interpretation of the variation in the process.

Basic principles of the control charts

A control chart is one of the primary techniques of statistical process control (SPC). A typical control chart is shown in Figure 1. The control chart is a graphical display of a quality characteristic that has been measured or computed from a sample versus the sample number or time. The chart contains a **center line** that represents the average value of the quality characteristic corresponding to the in-control state. (That is, only chance causes are present.) Two other horizontal lines, called the **upper control limit (UCL)** and the **lower control limit (LCL)**, are also shown on the chart. These control limits are chosen so that if the process is in control, nearly all of the sample points will fall between them. As long as the points plot within the control limits, the process is assumed to be in control, and no action is necessary. However, a point that plots outside of the control limits is interpreted as evidence that the process is out of control, and investigation and corrective action are required to find and eliminate the assignable cause or causes responsible for this behavior. It is customary to connect the sample points on the control chart with straight-line segments, so that it is easier to visualize how the sequence of points has evolved over time.

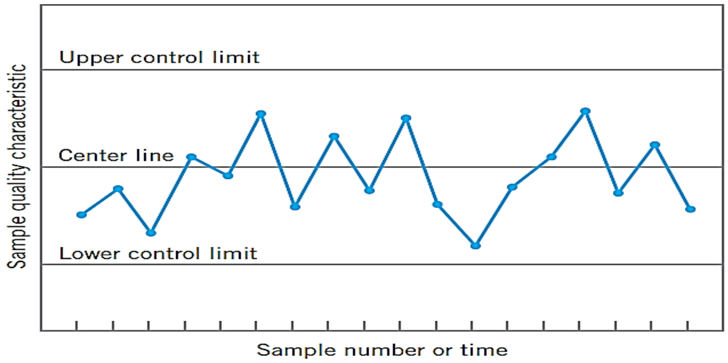


Figure 1: A typical control chart

Types of variation

Variation is a natural and commonly occurring phenomenon but not all variation is created equal. A process may contain variation that is common or inherent to the process and, there may also exist variation that is NOT common or inherent to the process. Variation that is NOT common would be a result of a special cause outside of the normal process conditions. Control charts in Figure 2 represent the variation of a product thickness in a process over time. The graphs are from the same piece of equipment, but on two different days.

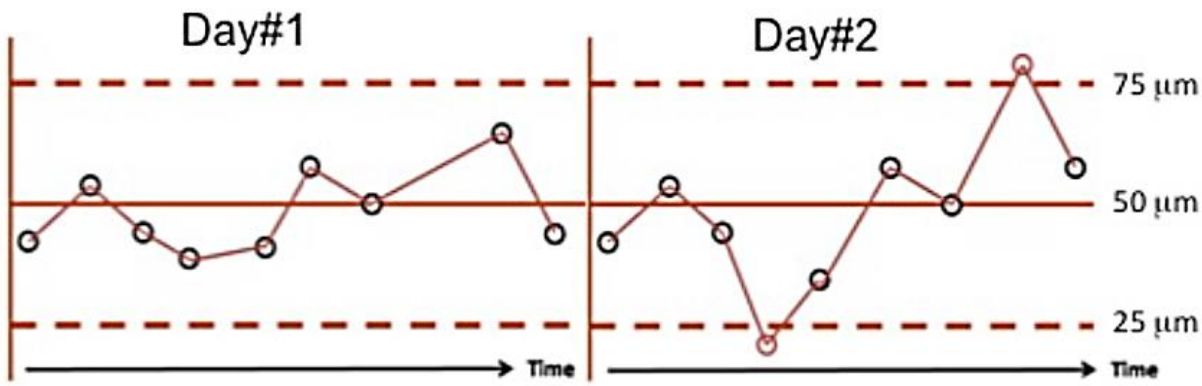


Figure 2: the variation of a product thickness in a process over time

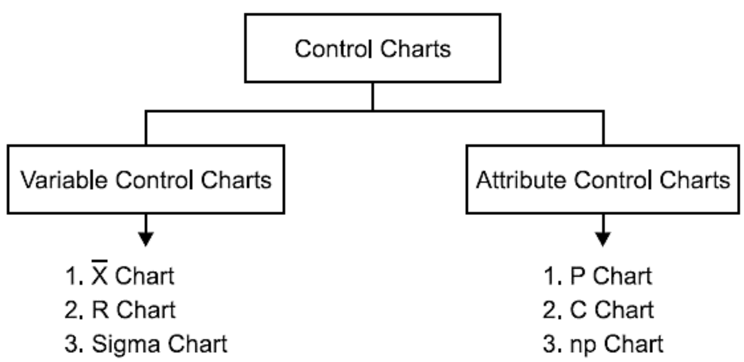
The control chart day 1 is "in control" and the variation that you see is expected because it has been statistically determined that this is the variability inherent to the process. This variation is referred to as "common cause variation". However, the control chart day 2 tells a different story. Notice the two points outside of the dashed lines. These points represent variation that is neither predictable nor inherent to the system. This type of variation is due to an "assignable cause" that can be known, or most often is unknown. Such variation is referred to as "assignable cause variation" or "special cause variation". From this example, control charts used in SPC help determine which type of variation (common or special cause) is present. As a result, control charts can help determine the best course of action.

Types of control charts

SPC is implemented through control charts that are used to monitor the output of the process and indicate the presence of problems requiring further action. Control charts can be used to monitor processes where output is measured as either *variables* or *attributes*. There are two types of control charts: Variable control chart and attribute control chart.

1. **Variable control charts:** It is one by which it is possible to measures the quality characteristics of a product. The variable control charts are **X-BAR** chart, **R-** chart, **SIGMA** chart.

2. **Attribute control chart:** It is one in which it is not possible to measures the quality characteristics of a product, *i.e.*, it is based on visual inspection only like good or bad, success or failure, accepted or rejected. The attribute control charts are **p-charts, np-charts, c-charts, u-charts.** It requires only a count of observations on characteristics *e.g.*, the number of nonconforming items in a sample.



Control charts for variables

As the name indicates, these charts will use variable data of a process. $\bar{X}$ chart given an idea of the central tendency of the observations. These charts will reveal the variations between sample observations. R chart gives an idea about the spread (dispersion) of the observations. This chart shows the variations within the samples.

$\bar{X}$ chart and R-Chart: The formulas used to establish various control limits are as follows:

If $x_1, x_2, \ldots\ldots x_n$ is a sample of size n, then the average of this sample is:

$$\bar{x} = \frac{x_1 + x_2 + \cdots + x_n}{n}$$

Let be the average of each $x_1, x_2, \ldots\ldots x_m$ sample. Then

$$\bar{\bar{x}} = \frac{\bar{x}_1 + \bar{x}_2 + \cdots + \bar{x}_m}{m}$$

Thus, $\bar{\bar{X}}$ would be used as the center line on the chart. To construct the control limits. We will use the range method. If $x_1, x_2, \ldots, x_n$ is a sample of size n, then the range of the sample is the difference between the largest and smallest observations.

$$R = x_{\max} - x_{\min}$$

Let $R_1, R_2, \ldots, R_m$ be the ranges of the m samples. The average range is:

$$\bar{R} = \frac{R_1 + R_2 + \cdots + R_m}{m}$$

The formulas for constructing the control limits on the chart are as follows:

Control Limits for the $\bar{x}$ Chart

$$\text{UCL} = \bar{\bar{x}} + A_2\bar{R}$$

$$\text{Center line} = \bar{\bar{x}}$$

$$\text{LCL} = \bar{\bar{x}} - A_2\bar{R}$$

The constant A_2 is tabulated for various sample sizes

Process variability may be monitored by plotting values of the sample range R on a control chart. The center line and control limits of the R chart are as follows:

Control Limits for the *R* Chart

$$\text{UCL} = D_4\overline{R}$$

$$\text{Center line} = \overline{R}$$

$$\text{LCL} = D_3\overline{R}$$

The constants D_3 and D_4 are tabulated for various values of n

Table1: Factors for $\bar{x}$ - and R –Charts

Number of Data Points in Subgroup	Factors for $\bar{x}$-Charts	Factors for R-Charts	
(n)	A_2	LCL D_3	UCL D_4
2	1.88	0	3.27
3	1.02	0	2.57
4	0.73	0	2.28
5	0.58	0	2.11
6	0.48	0	2.00
7	0.42	0.08	1.92
8	0.37	0.14	1.86
9	0.34	0.18	1.82
10	0.31	0.22	1.78
11	0.29	0.26	1.74
12	0.27	0.28	1.72
13	0.25	0.31	1.69
14	0.24	0.33	1.67
15	0.22	0.35	1.65
16	0.21	0.36	1.64
17	0.20	0.38	1.62
18	0.19	0.39	1.61
19	0.19	0.40	1.60
20	0.18	0.41	1.59

Example 1: The process for this example makes precision spacers that are nominally 100 millimeters thick. The process operates on a two-shift basis and appears to be quite stable. Fifty spacers per hour are produced. To develop a control chart for the process, we will measure the first 10 spacers produced after 9:00 a.m., 1:00 p.m., 5:00 p.m., and 9:00 p.m. We will do this for 3 days, for a total of 120 data points in 12 subgroups.

Date	Subgroup #	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	x_9	x_{10}	Sum Σx	Mean Value $\bar{x}$	Rng. R
		Measured Values												
7/6	1	101	98	102	101	99	100	98	101	100	102	1002	100.2	4
7/6	2	103	100	101	98	100	104	102	99	101	98	1006	100.6	6
7/6	3	103	101	99	102	100	99	102	98	103	100	1007	100.7	5
7/6	4	96	99	102	99	101	102	98	100	99	97	993	99.3	6
7/7	5	99	102	100	99	103	101	102	98	100	100	1004	100.4	5
7/7	6	101	103	99	100	99	98	100	100	99	100	999	99.9	5
7/7	7	100	103	101	98	99	100	99	102	100	98	1000	100.0	5
7/7	8	97	101	102	100	99	96	99	100	103	98	995	99.5	7
7/8	9	102	97	100	101	103	98	100	102	99	101	1003	100.3	6
7/8	10	100	105	99	100	98	102	97	97	99	101	998	99.8	8
7/8	11	101	99	98	101	104	100	98	100	102	98	1001	100.1	6
7/8	12	100	103	101	98	99	100	100	99	98	102	1000	100.0	5
												Total	1,200.8	68

$k = 12$, $\bar{\bar{x}} = 100.067$, $\bar{R} = 5.667$

The raw data are recorded in columns x_1 through x_{10}.

1- Calculate the mean (average) values for each subgroup. This is done by dividing the sum of x_1 through x_{10} by the number of data points in the subgroup.

2- The average $\bar{\bar{X}}$ of the subgroup average $\bar{X}$ is calculated by summing the values of x and dividing by the number of subgroups (k). In this case,

$$\begin{aligned} \bar{\bar{x}} &= 1{,}200.8 \div 12 \\ &= 100.067 \end{aligned}$$

3- The range (R) for each subgroup is calculated by subtracting the smallest value of x from the largest value of x in the subgroup.

4- From the R values, calculate the average of the subgroup ranges. In this case,

$$\overline{R} = 68 \div 12$$
$$= 5.667$$

5- Calculate the UCL and LCL values for the $\bar{x}$ -chart. From table 1, for n=10 in our example A_2=0.31

$$\text{UCL} = \overline{\overline{x}} + A_2\overline{R}$$
$$\text{Center line} = \overline{\overline{x}}$$
$$\text{LCL} = \overline{\overline{x}} - A_2\overline{R}$$

6- Applying our numbers to the UCL and LCL formulas, we have:

$$\text{UCL}_{\overline{\overline{x}}} = 100.067 + (0.31 \times 5.667)$$
$$= 100.067 + 1.75677$$
$$= 101.82377$$

$$\text{LCL}_{\overline{\overline{x}}} = 100.067 - 1.75677$$
$$= 98.31023$$

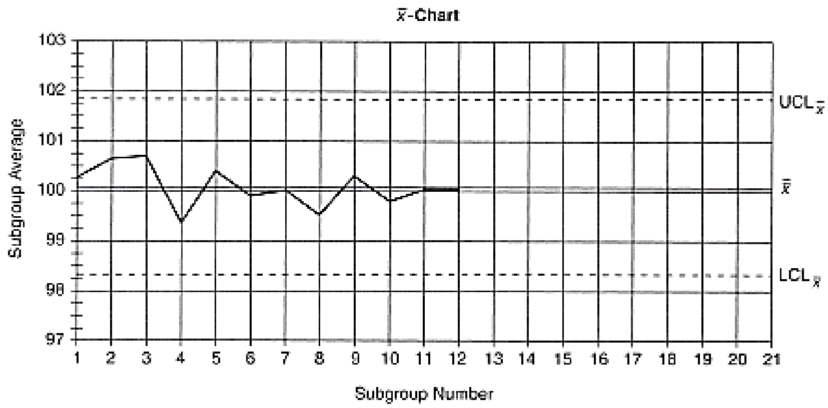
7- Calculate the UCL and LCL values for the R -chart.

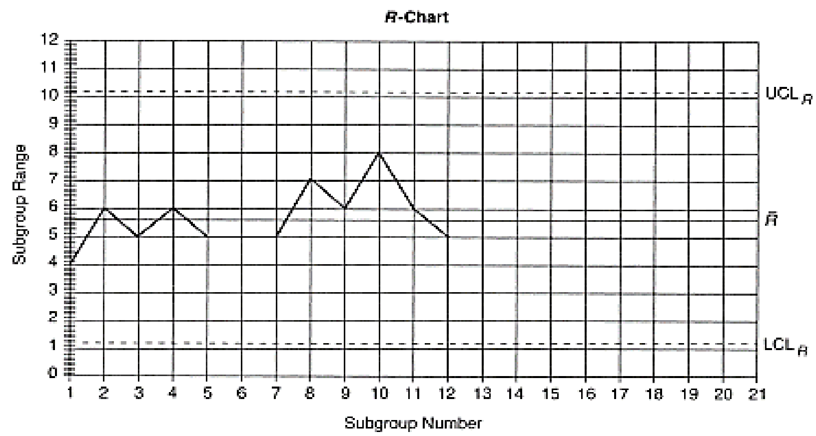
$$\text{UCL} = D_4\overline{R}$$
$$\text{Center line} = \overline{R}$$
$$\text{LCL} = D_3\overline{R}$$

Factors *D* 3 and *D* 4 are found in Table 1. With n = 10 in our example, D_3 = 0.22 and D_4 = 1.78. Applying the numbers to the LCL *R* and UCL *R* formulas, we have:

$$\text{UCL}_R = 1.78 \times 5.667 \qquad \text{LCL}_R = 0.22 \times 5.667$$
$$= 10.08726 \qquad = 1.24674$$

8- Upper and lower control limits are drawn on both charts as dashed lines, and $\bar{\bar{X}}$ and $\bar{R}$ centerlines are placed on the appropriate charts as solid lines. Then the data are plotted—subgroup averages ($\bar{x}$) on the $\bar{x}$ -chart and subgroup ranges (R) on the R -chart.





Control charts for attributes

Attributes data are concerned not with measurement but with something that can be counted. For example, the number of defects is attributes data. Whereas the $\bar{x}$ -chart and R -chart are used for certain kinds of variables data, where measurement is involved.

The p –Chart

The p -chart is used when the data are the fraction defective of some set of process output. It may also be shown as percentage defective. The points plotted on a p -chart are the fraction of nonconforming units or defective pieces found in the sample of n pieces. The formulas of P-chart are:

$$p = \frac{nonconforming\ units\ in\ subgroup}{number\ inspected\ in\ subgroup}$$

$$\bar{p} = \frac{Total\ number\ of\ nonconforming\ units\ in\ all\ samples}{Total\ number\ of\ units\ in\ all\ samples}$$

Control Limits for the P-Chart

$$UCL = \bar{p} + 3\sqrt{\frac{\bar{p}(1-\bar{p})}{n}}$$

$$\text{Center line} = \bar{p}$$

$$LCL = \bar{p} - 3\sqrt{\frac{\bar{p}(1-\bar{p})}{n}}$$

Where n = sample size (number inspected in subgroup)

Example 2: The following are the inspection results of 10 lots, each lot being 300 items. Number defectives in each lot is 25, 30, 35, 40, 45, 35, 40, 30, 20 and 50. Calculate the average fraction defective, construct P-chart and state whether the process is in control.

Date	*Number of pieces inspected* (a)	*Number of defective pieces found* (b)	*Fraction defective* $p = (b)/(a)$	*% Defective loop*
November 4	300	25	0.0834	8.34
November 5	300	30	0.1000	10.00
November 6	300	35	0.1167	11.67
November 7	300	40	0.1333	13.33
November 8	300	45	0.1500	15.00
November 10	300	35	0.1167	11.67
November 11	300	40	0.1333	13.33
November 12	300	30	0.1000	10.00
November 13	300	20	0.0666	6.66
November 14	300	50	0.1666	16.66
Total Number = 10	3000	350		

where

$$\bar{p} = \frac{\text{Total number of defective pieces found}}{\text{Total number of pieces inspected}}$$

$$\bar{p} = \frac{350}{3000} = \mathbf{0.1167}$$

and

$$n = \text{number of pieces inspected every day} = 300$$

Therefore, $\sqrt{\frac{\bar{p}\,(1-\bar{p})}{n}} = \sqrt{\frac{0.1167 \times (1-0.1167)}{300}}$

$= \sqrt{\frac{0.1167 \times 0.8333}{300}} = \mathbf{0.01852}$

and $3.\sqrt{\frac{\bar{p}\,(1-\bar{p})}{n}} = 0.01852 \times 3 = \mathbf{0.05556}$

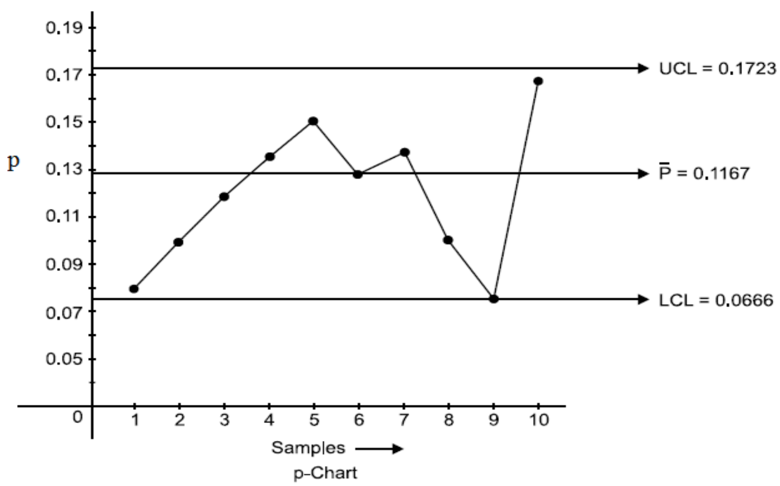
Thus,

$$\text{UCL} = 0.1167 + 0.05556 = 0.17226 = 0.1723 \text{ (Approx.)}$$
$$\text{LCL} = 0.1167 - 0.05566 = 0.06114 = 0.0611 \text{ (Approx.)}$$

$$\text{UCL} = \bar{p} + 3\sqrt{\frac{\bar{p}(1-\bar{p})}{n}}$$

$$\text{Center line} = \bar{p}$$

$$\text{LCL} = \bar{p} - 3\sqrt{\frac{\bar{p}(1-\bar{p})}{n}}$$



Conclusion: All the samples are within the control limit and we can say process is under control.

The c – chart

The c -charts are used when the data are concerned with the number of defects in a piece. The formulas of c-chart are:

$$\bar{c} = \frac{total\ number\ of\ defects}{number\ of\ samples}$$

Control Limits for the C Chart

$$UCL = \bar{c} + 3\sqrt{\bar{c}}$$

$$\text{Center line} = \bar{c}$$

$$LCL = \bar{c} - 3\sqrt{\bar{c}}$$

Example 3: The table below presents the number of nonconformities observed in 26 successive samples of 100 printed circuit boards. Note that, for reasons of convenience, the inspection unit is defined as 100 boards. Set up a *c* chart for these data.

Sample Number	Number of Nonconformities	Sample Number	Number of Nonconformities
1	21	14	19
2	24	15	10
3	16	16	17
4	12	17	13
5	15	18	22
6	5	19	18
7	28	20	39
8	20	21	30
9	31	22	24
10	25	23	16
11	20	24	19
12	24	25	17
13	16	26	15

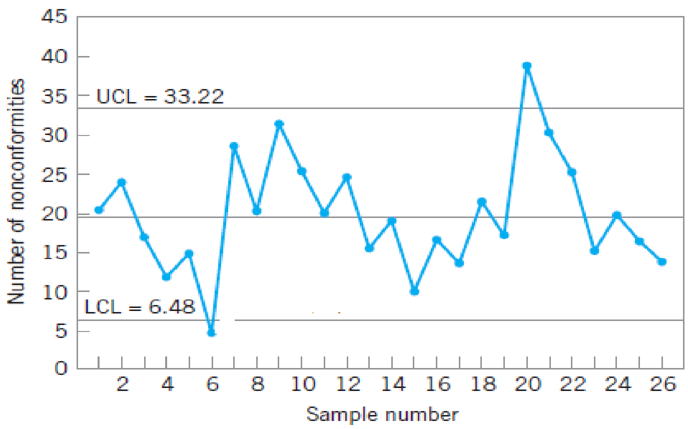
Since the 26 samples contain 516 total nonconformities, we estimate *c* by

$$\bar{c} = \frac{516}{26} = 19.85$$

$$\text{UCL} = \bar{c} + 3\sqrt{\bar{c}} = 19.85 + 3\sqrt{19.85} = 33.22$$

$$\text{Center line } = \bar{c} = 19.85$$

$$\text{LCL} = \bar{c} - 3\sqrt{\bar{c}} = 19.85 - 3\sqrt{19.85} = 6.48$$



Conclusion: Two points plot outside the control limits samples 6 and 20.